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The Glulam Handbook

Structural design of glulam structures

Volume 3

SWEDISH WOOD

The Glulam Handbook Volumes 1–3 are the result of a collaboration between glulam manufacturers and the industry organisations in Finland, Norway and Sweden. *The Glulam Handbook Volumes 1–3* are available in three languages – English, Finnish, Norwegian and Swedish. The content of these versions is adapted to meet Eurocode 5 and the associated national annexes, NA.

The Glulam Handbook Volume 4 is available in Swedish and English. It was produced by Swedish Wood and funded by the Swedish glulam manufacturers.

This publication is the third of the four-part Glulam Handbook.

- Volume 1 contains facts about glulam and planning guidance.
- Volume 2 provides calculations for the structural dimensioning of glulam.
- Volume 3 gives a number of example calculations for the most common glulam structures.
- Volume 4 provides knowledge on the planning and assembly of glulam structures.

Further knowledge, information and practical instructions on wood, glulam, CLT and wood construction are available on Wood Campus, **woodcampus.co.uk**, which is constantly updated with new knowledge and practical experiences. Wood Campus is an extensive resource with tables, drawings and illustrations.

Welcome to **woodcampus.co.uk**.

Information on wood, glulam, CLT and wood construction can also be found at **www.swedishwood.com**.

Stockholm, March 2024

Johan Fröbel
Swedish Wood

Cover: Universeum, Gothenburg, Sweden.

Preface

The purpose of *The Glulam Handbook Volume 3* is to assist the reader in the design of glulam structures. The volume is intended as a complement to *The Glulam Handbook Volume 2*. Particular emphasis has been given to the design of large-span glulam structures and the associated connections. Reinforcing methods used to enhance the mechanical properties of glulam in weak zones are also covered in the document. The Volume is divided into two separate parts, namely:

- *The first part, pages 6 – 79*, which is an introduction containing formulae, data and design methods. Part A should enable the design process of glulam structures to be made more quickly by using tabulated or graphical values for the most relevant design formulae contained in the European building codes.
- *The second part, pages 81 – 219*, which contains 22 thoroughly worked examples concerning the design of various glulam structures.

The Glulam Handbook Volume 3 is mainly related to the European standard EN 1995-1-1:2004 (Eurocode 5: Design of timber structures — Part 1-1: General — Common rules and rules for buildings). Some basic rules for the design of steel members and connections are presented according to the European standard EN 1993-1-1:2005 (Eurocode 3: Design of steel structures — Part 1-1: General rules and rules for buildings). In addition, the rules given here are based on the Swedish application rules connected to EN 1995-1-1, described in the document EKS 10 (BFS 2015:6). However, in case of lack of rules or questionable design methods present in Eurocode 5, other design approaches are proposed. For example, for the design of (i) beams with holes, (ii) loads attached close to the tension side of a beam, (ii) reinforcements to prevent cracking due to stresses perpendicular to the grain, etc. the German code DIN EN 1995-1-1/NA:2013-08 (Nationaler Anhang — National festgelegte Parameter — Eurocode 5: Bemessung und Konstruktion von Holzbauten, Teil 1-1: Allgemeines — Allgemeine Regeln und Regeln für den Hochbau) is used. The Swiss SIA 265:2012, as well as approaches based on research results and practical experience are also adopted in this document.

The Glulam Handbook Volume 3 is primarily intended for structural engineers and engineering students. The interpretations of the building codes, research reports, industry literature, etc. are those of the authors and are intended to reflect current structural design practice. The material presented is suggested as a guide only; final design responsibility lies with the structural engineer.

The main author of *The Glulam Handbook Volume 3* has been Prof. Roberto Crocetti, Lund University. The development of the design examples was primarily done by Mr. Simone Rossi, PhD student at Trento University, Italy, with the collaboration of Dr. Tiziano Sartori, also from Trento University and Mr. Luca Costa, former Master's student at Lund University. Prof. emeritus Kolbein Bell, NTNU, Trondheim, Norway has read the manuscript and made some corrections/suggestions.

Lund, mars 2016

Roberto Crocetti, professor
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Cow shed, Lyrestad, Sweden.

Contents

Rules and formulae for design according to Eurocode 5

1 Convention for member axes

The Glulam Handbook Volume 3 uses a coordinate system where:

- the x-axis corresponds to the element's longitudinal axis.
- the y-axis runs along the element's cross-sectional plane (perpendicular to the force of gravity).
- the z-axis runs along the element's cross-sectional plane (parallel to the force of gravity).

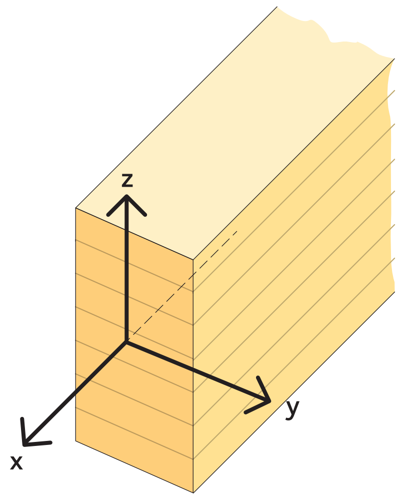


Figure 1.1 Convention for member axes

2 Moisture-related movement

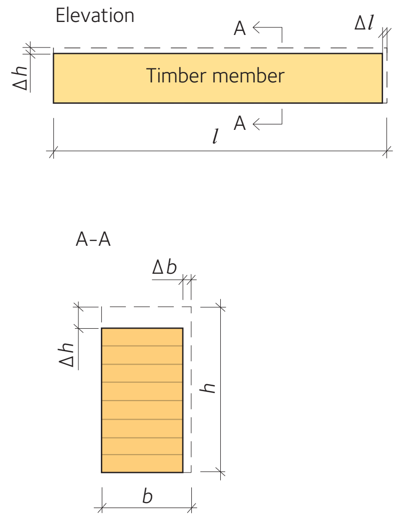
Table 2.1 **Changes in dimensions of a glulam member (applies below the fibre saturation point (30 %))**

$$\Delta h \left(\text{or } \Delta b\right) = \alpha_{\perp} \times \frac{\Delta\omega}{100} \times h \left(\text{or } \times b\right)$$

$$\Delta l = \alpha_{II} \times \frac{\Delta\omega}{100} \times l$$

where:

$\alpha_{\perp}$ = 0.24	is the deformation coefficient in the direction perpendicular to the grain
$\alpha_{//}$ = 0.01	is the deformation coefficient in the direction parallel to the grain
h, b, l	are the depth, width and length of the member respectively
Δh, Δb, Δl	are the change in dimension for depth, width and length of the member respectively
$\Delta\omega$	is the change in moisture content in the timber (in [%]). Common values of $\Delta\omega$ are: - indoor members (heated environment): $\Delta\omega$ = 4 – 6 - indoor members (non-heated environment): $\Delta\omega$ = 2 – 3 - outdoor members: $\Delta\omega$ = 5 – 8



3 Load-duration classes

Table 3.1 **Load-duration classes**

Load-duration class	Accumulated duration	Examples of loading
Permanent (P)	> 10 years	Self-weight
Long-term (L)	6 months – 10 years	Storage
Medium-term (M)	1 week – 6 months	Imposed floor load Snow load
Short-term (S)	< 1 week	Wind load
Instantaneous (I)		Wind gusts Accidental load Single concentrated roof load

Nordens Ark zoo, Hunnebostrand, Sweden.

4 Service classes

Service class 1

The average moisture content for most softwood species will not exceed 12 %, which corresponds to an environment with temperature of 20 °C and relative humidity, RH, exceeding 65 % only a few weeks per year.

Examples: External walls surrounding permanently heated premises that are protected by tight and ventilated external cladding. Members in heated indoor environment.

Service class 2

The average moisture content for most softwood species will not exceed 20 %, which corresponds to an environment with temperature of 20 °C and relative humidity exceeding 85 % only a few weeks per year.

Examples: Wooden members which are ventilated and protected against direct precipitation, such as roof trusses, attic and crawl space floors. Structures in ventilated buildings which are not permanently heated or premises with activities or storage that do not generate moisture, such as summer houses, unheated garages and storages, farm buildings and crawl spaces ventilated by outdoor air.

Service class 3

The average moisture content for most softwood species will exceed 20 %, which gives a higher wood moisture content than that specified for service class 2.

Examples: Wooden members not protected from precipitation or that are in ground contact.

5 Load combinations

Ultimate limit state (ULS)

According to Eurocode 0 (EN 1990), the following requirements must, if relevant, be verified:

- Equilibrium (EQU). To demonstrate that the structure or any part of it, is not unstable, e.g. design of holding-down anchors or bearings subject to uplift in continuous beams.
- Strength (STR). To demonstrate that the structure and its elements will not fail due to stress or instability. If displacements affect the behaviour of the structure, their effect must be taken into account.
- Geotechnical (GEO). To demonstrate that the foundations provide the strength and stiffness required by the structure.
- Fatigue (FAT). To demonstrate that the elements of the structure will not fail in fatigue.

For structures made of timber or wood products, STR is thus normally the deciding factor when the structure's load-carrying capacity needs to be verified in the ultimate limit state.

Table 5.1 Safety factors γ_d for design values according to EKS 10

Safety class	Possible consequences due to member failure	γ_d
1 - Low	Low risk of serious injury	0.83
2 - Medium	Medium risk of serious injury	0.91
3 - High	High risk of serious injury	1

Table 5.2 Design values of actions for equilibrium (EQU) and strength (STR) limit states based on EN 1990.
Safety factors γ_d according to *table 5.1*. Load combination factors ψ are given in *table 5.4, page 11*. For timber structures, STR-2 is normally governing the design at ULS.

Type of load	Load combination		
	STR-1	STR-2	EQU
Permanent load G			
Unfavourable	$\gamma_d \times 1.35 \times G_K$	$\boldsymbol{\gamma_d \times 1.2 \times G_K}$	$\gamma_d \times 1.1 \times G_K$
Favourable	$1.0 \times G_K$	$\boldsymbol{1.0 \times G_K}$	$0.9 \times G_K$
Variable load Q			
Leading load Q_{K1}	–	$\boldsymbol{\gamma_d \times 1.5 \times Q_{K,1}}$	$\gamma_d \times 1.5 \times Q_{K,1}$
Accompanying variable loads ($\Sigma\psi_{0,i} \times Q_{K,i}$)	$\gamma_d \times 1.5 \times \psi_{0,i} \times Q_{K,i}$	$\boldsymbol{\gamma_d \times 1.5 \times \psi_{0,i} \times Q_{K,i}}$	$\gamma_d \times 1.5 \times \psi_{0,i} \times Q_{K,i}$

In order to determine the load combination that governs the design, the load combination equation(s) must be applied with each variable action acting as the leading variable in turn. For example, consider the STR limit state of a simply supported beam loaded by its own weight plus a permanent load, G_k, a medium-term duration variable load, $Q_{k,1}$, and an unrelated short-term variable load, $Q_{k,2}$. Adopting STR-2 the loading conditions that need to be considered in order to determine an effect, LC_i (e.g. a bending moment) are:

Ulls hus, Uppsala, Sweden.

$$\gamma_d \times 1.2 \times G_k \rightarrow LC_1$$
$$\gamma_d \times \left(1.2 \times G_k + 1.5 \times Q_{k,1}\right) \rightarrow LC_2$$
$$\gamma_d \times \left(1.2 \times G_k + 1.5 \times Q_{k,2}\right) \rightarrow LC_3$$
$$\gamma_d \times \left(1.2 \times G_k + 1.5 \times Q_{k,1} + 1.5 \times \psi_{0.2} \times Q_{k,2}\right) \rightarrow LC_4$$
$$\gamma_d \times \left(1.2 \times G_k + 1.5 \times Q_{k,2} + 1.5 \times \psi_{0.1} \times Q_{k,1}\right) \rightarrow LC_5$$

with $k_{mod,perm}$, $k_{mod,med}$ and $k_{mod,short}$ as the modification factors for the permanent, medium-term and short-term actions, respectively, the design value LC_d is given by:

$$LC_d = \max \begin{cases} LC_1 / k_{mod,perm} \\ LC_2 / k_{mod,med} \\ LC_3 / k_{mod,short} \\ LC_4 / k_{mod,short} \\ LC_5 / k_{mod,short} \end{cases}$$

For roof structures the variable loads are, in most cases: $Q_{k,1}$: “snow load” and $Q_{k,2}$: “wind load”. In such a case, LC_2 and sometimes LC_3 (for a relatively large slope of the roof) are normally the governing load combinations.

Serviceability limit state (SLS)

For timber structures the following SLS shall be verified:
- Deformation.
- Vibration.

Deformation

The combinations of actions for SLS are given in Eurocode 0 (EN 1990). They are:
- Characteristic combination.
- Frequent combination.
- Quasi-permanent combination.

According to EN 1995-1-1, section 2.3.2:
The instantaneous deformation, u_{inst} should be calculated for the characteristic load combination, see EN 1990, section 6.5.3(2) a, using an average value for the modulus of elasticity, shear modulus and slip modulus.

The final deformation, u_{fin} should be calculated for the quasi-permanent load combination, see EN 1990, section 6.5.3(2) c.

For structural elements comprising parts, components and connections with the same creep effects and assuming a linear relationship between loads and corresponding deformations, as a simplification the final deformation, u_{fin}, may be calculated as:

$$u_{fin} = u_{fin,G} + U_{fin,Q,1} + \Sigma u_{fin,Q,i}$$

where:

$u_{fin,G} = u_{inst,G}\left(1 + k_{def}\right)$	for a permanent load, G
$u_{fin,Q,1} = u_{inst,Q,1}\left(1 + k_{def}\right)$	for the leading load of the variable loads, Q_1
$u_{fin,Q,i} = u_{inst,Q,i}\left(1 + k_{def}\right)$	for accompanying variable loads, Q_i $(i > 1)$

Service class	1	2	3
k_{def}	0.60	0.80	2.00

Table 5.3 **Load combinations for SLS according to EN 1990, section 6.5.3**
Load combination factors ψ are found in *table 5.4*.

Type of load	Load combination		
	Characteristic [1]	Frequent [2]	Quasi-permanent [3]
Permanent load G	$1.0 \times G_K$	$1.0 \times G_K$	$1.0 \times G_K$
Variable loads Q			
– Leading load Q_{K1}	$1.0 \times Q_{Ki}$	$\psi_{1,1} \times Q_{K,1}$	
– Accompanying variable loads ($\Sigma\psi_{0,i} \times Q_{K,i}$)	$\psi_{0,i} \times Q_{K,i}$	$\psi_{2,i} \times Q_{K,i}$	$\psi_{2,i} \times Q_{K,i}$

[1] corresponds to "permanent damage" (irreversible deformations).
[2] corresponds to "temporary large deformation" (reversible deformations).
[3] corresponds to "loads with long duration" (effects of creep).

Table 5.4 **Load combination factors ψ according to EN 1990**

Load	ψ_0	ψ_1	ψ_2
Imposed loads in buildings, category [1]			
A: Residential areas	0.7	0.5	0.3
B: Office areas	0.7	0.5	0.3
C: Congregation areas	0.7	0.7	0.6
D: Shopping areas	0.7	0.7	0.6
E: Storage areas	1.0	0.9	0.8
F: Traffic area, vehicle weight ≤ 30 kN	0.7	0.7	0.6
G: Traffic area, 30 kN < vehicle weight ≤ 160 kN	0.7	0.5	0.3
H: Roofs	0	0	0
Snow load			
$s_k \geq 3$ kN/m²	0.8	0.6	0.2
$2.0 \leq s_k < 3.0$ kN/m²	0.7	0.4	0.2
$1.0 \leq s_k < 2.0$ kN/m²	0.6	0.3	0.1
Wind load	0.3	0.2	–
Thermal loads (non-fire) in buildings	0.6	0.5	0

[1] Category according to EN 1991-1-1.

6 General design rules for steel structures

House M, Linnaeus University, Växjö, Sweden.

Table 6.1 **Design value for strength of steel structures in ultimate limit state (ULS)**

$$f_{yd} = \frac{f_y}{\gamma_{M0}}$$

$$f_{ud} = \frac{f_u}{\gamma_{M2}}$$

where:

f_{yd}	is the design yield strength.
f_{ud}	is the design ultimate tensile strength .
f_y	is the characteristic yield strength.
f_u	is the characteristic ultimate tensile strength.
γ_{M0} =1.0	is the partial safety factor for resistance of cross section to overall yielding.
γ_{M2}	is the partial safety factor for resistance of net section (γ_{M2} = 1.1) and for resistance of connections (γ_{M2} = 1.2).

Table 6.2 **Material properties**

Modulus of elasticity	E = 210,000 MPa
Shear modulus	G = 81,000 MPa
Coefficient of linear thermal expansion	α = 12 (μm/m)/°C
Poisson's ratio	υ = 0.3
Density	ρ = 7,850 kg/m^3

Table 6.3 **Nominal values of yield strength f_y and ultimate tensile strength f_u for hot rolled structural steel according to the European standard EN 10025-2**

Steel grade	Nominal thickness of the element t [mm]			
	$t \leq 40$ mm		$40 < t \leq 80$ mm	
	f_y [MPa]	f_u [MPa]	f_y [MPa]	f_u [MPa]
S235	235	360	215	360
S275	275	430	255	410
S355	355	510	335	470
S450	440	550	410	550

Table 6.4 Nominal values of yield strength f_{yb} and ultimate tensile strength f_{ub} for bolts according to the European standard EN 15048-1

Bolt grade	f_{yb} [MPa]	f_{ub} [MPa]
4.6	240	400
4.8	320	400
5.6	300	500
5.8	400	500
6.8	480	600
8.8	640	800
10.9	900	1,000

Table 6.5 Nominal area A_n and tensile stress area A_s of bolts

Bolt type	Nominal diameter d_n [mm]	Nominal area A_n [mm²]	Tensile stress area A_s [mm²]
M12	12	113	84
M16	16	201	157
M20	20	314	245
M24	24	452	353
M27	27	572	459
M30	39	707	561

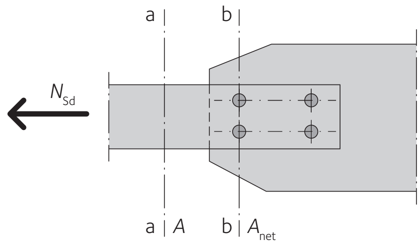
Table 6.6 Net section failure according to EC3, section 6.2.3

Section a-a: gross section

$$N_{sd} \leq N_{Rd} = \frac{A \times f_y}{\gamma_{M0}} \quad (\gamma_{M0} = 1.0)$$

Section b-b: net section

$$N_{sd} \leq N_{Rd} = 0.9 \times \frac{A_{net} \times f_u}{\gamma_{M2}} \quad (\gamma_{M2} = 1.1)$$

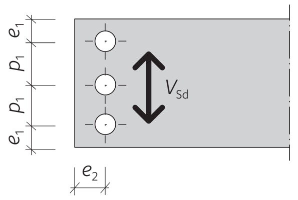


where:

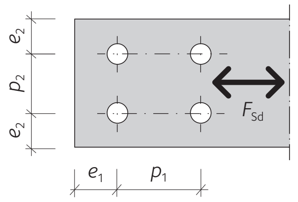
N_{Sd}	is the design axial load.
N_{Rd}	is the design axial strength.
f_y	is the characteristic yield strength.
f_u	is the characteristic value for ultimate tensile strength.
A	is the cross sectional gross area.
A_{net}	is the cross sectional net area.
γ_{M0} and γ_{M2}	are the partial safety factors for resistance (γ_{M0} = 1.0 and γ_{M2} = 1.1).

Table 6.7 **Designation of distances between bolts, (a) for a shear joint, (b) for a tension or compression joint**
(The designations e_2 and p_2 also apply when distances measured are not in the direction of stress).

a)



b)



$1.2 \times d_0 \leq e_1 \leq \max\{12 \times t, 150 \text{ mm}\}$

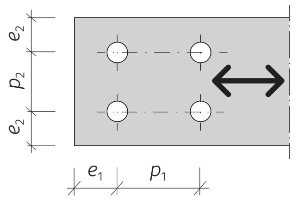
$1.5 \times d_0 \leq e_2 \leq \max\{12 \times t, 150 \text{ mm}\}$

$2.2 \times d_0 \leq p_1 \leq \max\{14 \times t, 200 \text{ mm}\}$

$3.0 \times d_0 \leq p_2 \leq \max\{14 \times t, 200 \text{ mm}\}$

Recommended distances [mm]			
Bolts	p_1; p_2	e_1	e_2
M12	40	30	25
M16	55	40	30
M20	70	50	40
M24	80	60	50
M27	90	70	55
M30	100	75	60

Table 6.8 **Characteristic bearing resistance for bolts referred to plate thickness t_p = 10 mm**
The values apply only for e_2 and p_2 according to "Recommended distances" given in *table 6.7*. Corresponding design values can be obtained by multiplying the characteristic values by $(1/\gamma_{M2})$, where $\gamma_{M2} = 1.2$.



$$F_{b,Rk} = k_1 \times \alpha_b \times f_u \times d \times t$$

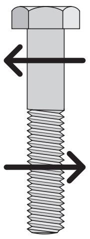
$$\alpha_b = \min \begin{cases} e_1/(3 \times d_0) \\ p_1/(3 \times d_0) - 0.25 \\ f_{ub}/f_u \\ 1 \end{cases}$$

$$k_1 = \min \begin{cases} e_1/(3 \times d_0) - 1.7 \\ 2.5 \end{cases}$$

Bolt diameter d [mm]		12	16	20	22	24	27	30
Hole diameter d_0 [mm]		13	18	22	24	26	30	33
Characteristic bearing resistance [kN] [1)]								
Steel grade	S235	83	107	136	151	166	182	204
	S275	99	127	163	181	198	218	244
	S355	118	151	193	214	235	258	289

[1)] For different plate thickness t than t_p = 10 mm, multiply the values by t/t_p.

Table 6.9 Characteristic shear resistance per bolt and shear plane. Corresponding design values can be obtained by multiplying the characteristic values by $(1/\gamma_{M2})$, where $\gamma_{M2} = 1.2$.



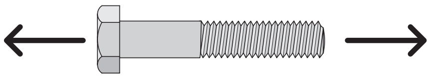
Shear in thread

$$F_{v,Rk} = C_1 \times f_{ub} \times A_s$$

$$C_1 = \begin{cases} 0.6 \text{ for grades 4.6 5.6 and 8.8} \\ 0.5 \text{ for grades 4.8 5.8 6.8 and 10.9} \end{cases}$$

Bolt diameter d [mm]		12	16	20	22	24	27	30
Hole diameter d_0 [mm]		13	18	22	24	26	30	33
Characteristic bearing resistance [kN]								
Bolt grade	4.6	20	38	59	73	85	110	135
	5.6	25	47	73	91	106	138	168
	8.8	40	75	118	145	169	220	269
	10.9	42	78	122	151	176	229	280

Table 6.10 Characteristic tension resistance per bolt. Corresponding design values can be obtained by multiplying the characteristic values by $(1/\gamma_{M2})$, where $\gamma_{M2} = 1.2$.

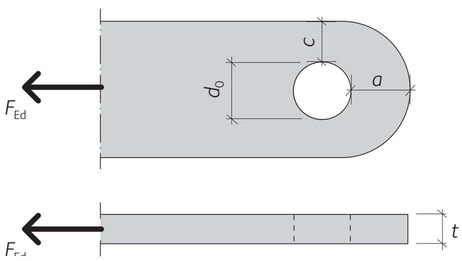


$$F_{t,Rk} = 0.9 \times f_{ub} \times A_s$$

Bolt diameter d [mm]		12	16	20	22	24	27	30
Characteristic bearing resistance [kN]								
Bolt grade	4.6	30	56	88	109	127	165	202
	5.6	38	71	110	136	159	207	252
	8.8	61	113	176	218	254	330	404
	10.9	76	141	220	272	318	413	505

Table 6.11 Geometrical requirements for pin ended members, according to EN 1993-1-8

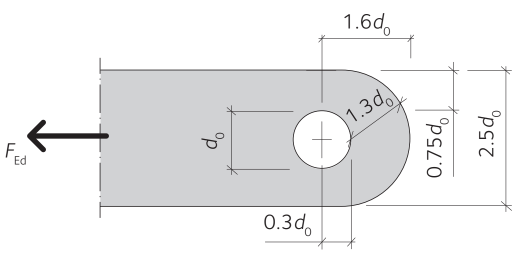
Given thickness t and pin hole diameter d



$$a \geq \frac{F_{E,d} \times \gamma_{M0}}{2 \times t \times f_y} + \frac{2 \times d_0}{3}$$

$$c \geq \frac{F_{E,d} \times \gamma_{M0}}{2 \times t \times f_y} + \frac{d_0}{3}$$

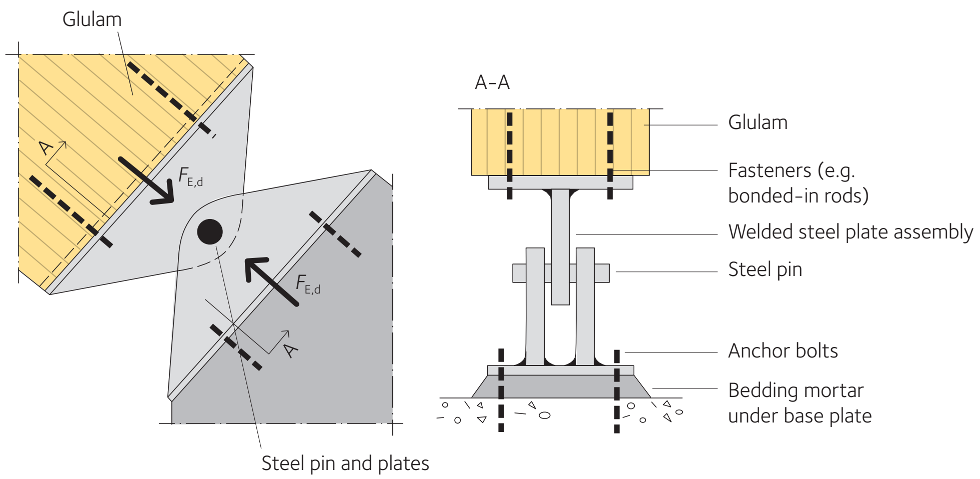
Given geometry



$$t \geq 0.7 \times \sqrt{\frac{F_{E,d} \times \gamma_{M0}}{f_y}}$$

$$d_0 \leq 2.5 \times t$$

Table 6.12 **Design criteria for pin connections, according to EN 1993-1-8**

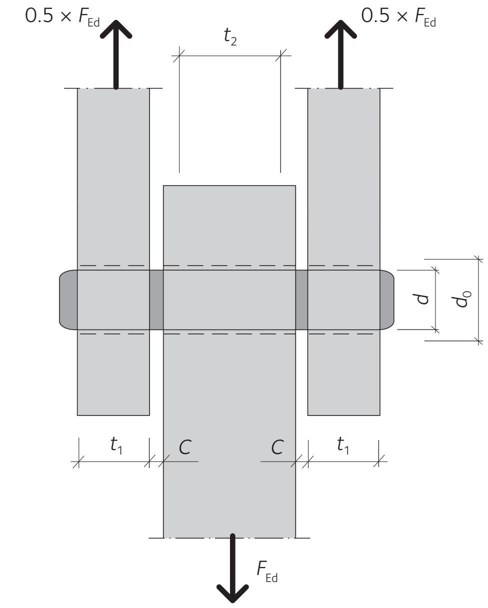


Failure mode	Design requirement
Shear resistance of the pin (per shear plane)	$F_{v,Rd} = \dfrac{0.6 \times A \times f_{u,p}}{\gamma_{M2}}$
The load-carrying capacity of the pin with compression on the hole edge (per steel plate)	$F_{b,Rd} = \dfrac{1.5 \times t \times d \times f_y}{\gamma_{M0}}$
Bending resistance of the pin	$M_{Rd} = \dfrac{1.5 \times W \times f_{y,p}}{\gamma_{M0}}$
Combined shear and bending resistance of the pin	$\left(\dfrac{M_{Ed}}{M_{Rd}}\right)^2 + \left(\dfrac{F_{v,Ed}}{F_{v,Rd}}\right)^2 \leq 1$ [1)]

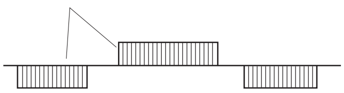
d	is the diameter of the pin.
f_y	is the lowest of the yield strengths of the pin and the connected parts.
$f_{u,p}$	is the ultimate strength of the pin.
$f_{y,p}$	is the yield strength of the pin.
t	is the thickness of the connected part.
A	is the cross sectional area of the pin ($=\pi \times d^2/4$).
W	is the elastic section modulus of the pin ($=\pi \times d^3/32$).

[1)] For the determination of M_{Ed} and $F_{v,Ed}$, see *table 6.13, page 17.*

Table 6.13 **Bending moment and shear force in a pin with two shear planes**



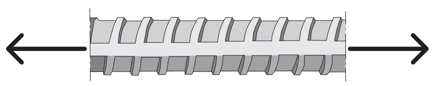
$$\sigma_{\text{bear}} = \frac{F_{\text{Ed}}}{t_2 \times d} \left(= \frac{F_{\text{Ed}}}{2 \times t_1 \times d} \right)$$



$$M_{\text{Ed}} = \frac{F_{\text{Ed}}}{8} \times (t_2 + 2 \times t_1 + 4 \times c)$$

$$F_{\text{V,Ed}} = \frac{1}{2} \times F_{\text{Ed}}$$

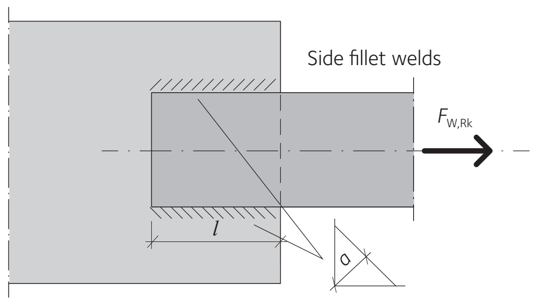
Table 6.14 **Characteristic tension resistance of some special bars used in timber structures**
Corresponding design values can be obtained by multiplying the characteristic values by $(1/\gamma_{M2})$, where $\gamma_{M2} = 1.2$.



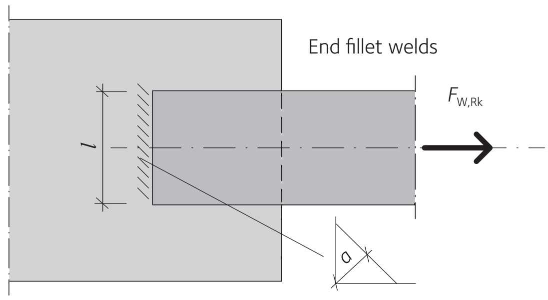
$$F_{\text{t,Rk}} = f_{\text{ub}} \times A_{\text{s}}$$

Nominal bar diameter d [mm]		26.5	28	32	36	40	50
Diameter over threads [mm]		30	32	36	40	45	56
Nominal area A_s [mm^2]		551	616	804	1,018	1,256	1,963
		Characteristic tension resistance [kN]					
Bar type (f_y/f_u)	GWS (950/1050)	579	–	844	1,069	1,319	–
	GEWI (500/550)	–	339	442	–	691	1,080

Table 6.15 Characteristic bearing resistance of fillet welds. Corresponding design values can be obtained by multiplying the characteristic values by $(1/\gamma_{M2})$, where $\gamma_{M2} = 1.2$.



$$F_{W,Rk} = \frac{f_u}{\sqrt{3}} \times \frac{a \times l}{\beta_w} \quad ^{1)}$$



$$F_{W,Rk} = \frac{f_u}{\sqrt{2}} \times \frac{a \times l}{\beta_w}$$

Throat thickness *a* [mm]			3	4	5	6	7	8	9
Fillet weld resistance in [kN] for 100 mm weld length [2)]									
Steel grade	S235 (β_w = 0.8)	Side	78	104	130	156	182	208	234
		End	95	127	159	191	223	255	286
	S275 (β_w = 0.85)	Side	88	117	146	175	204	234	263
		End	108	143	179	215	250	286	322
	S355 (β_w = 0.9)	Side	98	131	163	196	229	262	294
		End	120	160	200	240	280	320	361

1) Resistance of a single fillet weld.
2) For different lengths than 100 mm, multiply the values by *l*/100.

7 Design of timber structures

Table 7.1 Design value for strength of wood products in the ultimate limit state

$$f_d = \frac{k_{mod} \times f_k}{\gamma_M}$$

where:

f_d	is the design value of a strength parameter.
f_k	is the characteristic value of a strength parameter.
k_{mod}	is the modification factor taking into account the effect of load duration and moisture content (service class).
γ_m	is the partial coefficient for material, *see table 7.2*.

Table 7.2 Partial coefficient γ_M for materials in ultimate limit state according to EN 1995-1-1, section 2.4.1

Material	γ_M
Solid timber	1.3
Glulam	1.25
Laminated veneer lumber (LVL), plywood, Oriented strand board (OSB)	1.2
Wood connections	1.3

Table 7.3 Strength modification factors k_{mod} for different service classes and load-duration classes according to EN 1995-1-1, section 3.1.3

Material	Standard	Service class	Load duration class				
			P	L	M	S	I
Solid wood	EN 14081-1	1	0.60	0.70	0.80	0.90	1.10
		2	0.60	0.70	0.80	0.90	1.10
		3	0.50	0.55	0.65	0.70	0.90
Glulam	EN 14080	1	0.60	0.70	0.80	0.90	1.10
		2	0.60	0.70	0.80	0.90	1.10
		3	0.50	0.55	0.65	0.70	0.90
	EN 14374	1	0.60	0.70	0.80	0.90	1.10
	EN 14279	2	0.60	0.70	0.80	0.90	1.10
		3	0.50	0.55	0.65	0.70	0.90
Plywood	EN 636	1	0.60	0.70	0.80	0.90	1.10
		2	0.60	0.70	0.80	0.90	1.10
		3	0.50	0.55	0.65	0.70	0.90
Oriented strand board (OSB)	EN 300						
	OSB/2	1	0.30	0.45	0.65	0.85	1.10
	OSB/3	1	0.40	0.50	0.70	0.90	1.10
	OSB/4	1	0.40	0.50	0.70	0.90	1.10
	OSB/3	2	0.30	0.40	0.55	0.70	0.90
	OSB/4	2	0.30	0.40	0.55	0.70	0.90

Table 7.4 Characteristic strength and stiffness of combined glued laminated timber according to EN 14080. Properties in MPa and densities in kg/m³. *See The Glulam Handbook Volume 2, section 1.3.5, page 19,* for information regarding resawn glulam.

Property	GL20c	GL22c	GL24c	GL26c	GL28c	GL30c	GL32c
Strength values							
Bending parallel to grain $f_{m,k}$ 1)	20	22	24	26	28	30	32
Tension parallel to grain $f_{t,0,k}$	15.0	16.0	17.0	19.0	19.5	19.5	19.5
Tension perpendicular to grain $f_{t,90,k}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Compression parallel to grain $f_{c,0,k}$	18.5	20.0	21.5	23.5	24.0	24.5	24.5
Compression perpendicular to grain $f_{c,90,k}$	2.5	2.5	2.5	2.5	2.5	2.5	2.5
Shear strength $f_{v,k}$ (shear and torsion)	3.5	3.5	3.5	3.5	3.5	3.5	3.5
Rolling shear strength $f_{r,k}$	1.2	1.2	1.2	1.2	1.2	1.2	1.2
Stiffness values for capacity analysis							
Elastic modulus $E_{0,05}$	8,600	8,600	9,100	10,000	10,400	10,800	11,200
Elastic modulus $E_{90,05}$	250	250	250	250	250	250	250
Shear modulus G_{05}	540	540	540	540	540	540	540
Stiffness values for deformation calculations							
Elastic modulus $E_{0,mean}$	10,400	10,400	11,000	12,000	12,500	13,000	13,500
Elastic modulus $E_{90,mean}$	300	300	300	300	300	300	300
Shear modulus G_{mean}	650	650	650	650	650	650	650
Density							
Density ρ_k	355	355	365	385	390	390	400
Density ρ_{mean}	390	390	400	420	430	430	440

1) The bending strength about the weak axis can be assumed to be the same as the bending about the strong axis. *See The Glulam Handbook Volume 2, section 1.3.4, page 16* for more information.

Table 7.5 Characteristic strength and stiffness of homogeneous glued laminated timber according to EN 14080. Properties in MPa and densities in kg/m³. *See The Glulam Handbook Volume 2, section 1.3.5, page 19,* for information regarding resawn glulam.

Property	GL20h	GL22h	GL24h	GL26h	GL28h	GL30h	GL32h
Strength values							
Bending parallel to grain $f_{m,k}$ 1)	20	22	24	26	28	30	32
Tension parallel to grain $f_{t,0,k}$	16.0	17.6	19.2	20.8	22.4	24.0	25.6
Tension perpendicular to grain $f_{t,90,k}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Compression parallel to grain $f_{c,0,k}$	20	22	24	26	28	30	32
Compression perpendicular to grain $f_{c,90,k}$	2.5	2.5	2.5	2.5	2.5	2.5	2.5
Shear strength $f_{v,k}$ (shear and torsion)	3.5	3.5	3.5	3.5	3.5	3.5	3.5
Rolling shear strength $f_{r,k}$	1.2	1.2	1.2	1.2	1.2	1.2	1.2
Stiffness values for capacity analysis							
Elastic modulus $E_{0,05}$	7,000	8,800	9,600	10,100	10,500	11,300	11,800
Elastic modulus $E_{90,05}$	250	250	250	250	250	250	250
Shear modulus G_{05}	540	540	540	540	540	540	540
Stiffness values for deformation calculations							
Elastic modulus $E_{0,mean}$	8,400	10,500	11,500	12,100	12,600	13,600	14,200
Elastic modulus $E_{90,mean}$	300	300	300	300	300	300	300
Shear modulus G_{mean}	650	650	650	650	650	650	650
Density							
Density ρ_k	340	370	385	405	425	430	440
Density ρ_{mean}	370	410	420	445	460	480	490

1) The bending strength about the weak axis can be assumed to be the same as the bending about the strong axis. *See The Glulam Handbook Volume 2, section 1.3.4, page 16* for more information.

8 Design for ULS of members subjected to tension, compression, bending and shear – Members not prone to instability

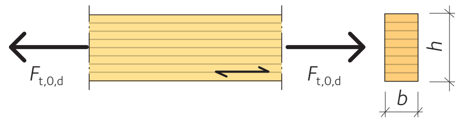
Table 8.1 Tension parallel to the grain according to EN 1995-1-1, section 6.1.2

$$\sigma_{t,0,d} = \frac{F_{t,0,d}}{A_n} \leq f_{t,0,d}$$

where:

$\sigma_{t,0,d}$	is the design tensile stress along the grain.
$f_{t,0,d}$	is the design tensile strength along the grain. *
$F_{t,0,d}$	is the design tensile load along the grain.
A_n	is the cross sectional net area .

$h \leq 231$ mm	$f_{t,k} \times 1.1$
231 mm < h < 600 mm	$f_{t,k} \times \left(\frac{600}{h}\right)^{0.1}$
$h \geq 600$ mm	$f_{t,k}$



* Characteristic tensile strength as a function of h.

Table 8.2 Compression parallel to the grain according to EN 1995-1-1, section 6.1.4

$$\sigma_{c,0,d} = \frac{F_{c,0,d}}{A_n} \leq f_{c,0,d}$$

where:

$\sigma_{c,0,d}$	is the design compression stress along the grain.
$f_{c,0,d}$	is the design compression strength along the grain.
$F_{c,0,d}$	is the design compression load along the grain.
A_n	is the cross sectional net area.

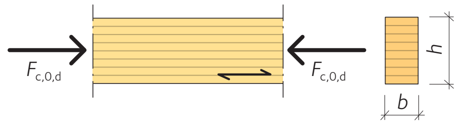


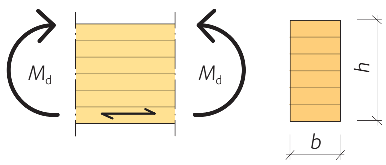
Table 8.3 Bending about one principal axis according to EN 1995-1-1, section 6.1.6

$$\sigma_{m,d} = \frac{M_d}{W_n} \leq f_{m,d}$$

where:

$\sigma_{m,d}$	is the design bending stress stress about y.
$f_{m,d}$	is the design bending strength. *
M_d	is the design bending moment.
W_n	is the net section modulus.

$h \leq 231$ mm	$f_{m,k} \times 1.1$
231 mm < h < 600 mm	$f_{m,k} \times \left(\frac{600}{h}\right)^{0.1}$
$h \geq 600$ mm	$f_{m,k}$



* Characteristic bending strength as a function of h.

Table 8.4 **Bending about two principal axes according to EN 1995-1-1, section 6.1.6**

$$\frac{M_{y,d}}{W_{n,y} \times f_{m,y,d}} + 0.7 \times \frac{M_{z,d}}{W_{n,z} \times f_{m,z,d}} \leq 1$$

and

$$0.7 \times \frac{M_{y,d}}{W_{n,y} \times f_{m,y,d}} + \frac{M_{z,d}}{W_{n,z} \times f_{m,z,d}} \leq 1$$

where:

$f_{m,y,d}$ and $f_{m,z,d}$	are the design bending strengths about y and z, respectively.
W_{ny} and W_{nz}	are the net section moduli about y and z, respectively.
$M_{y,d}$ and $M_{z,d}$	are the design bending moments about y and z, respectively.

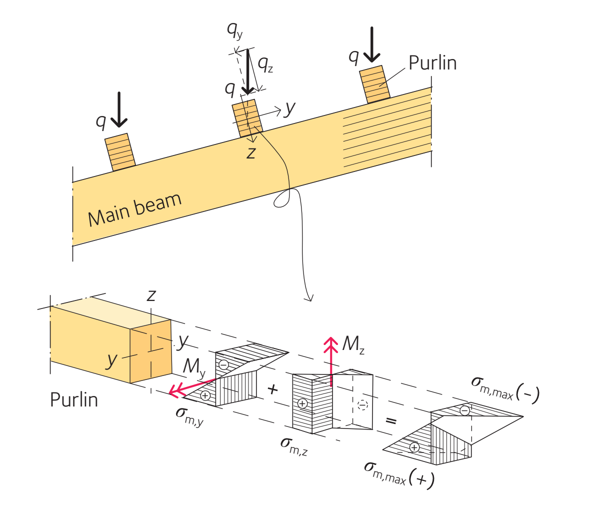
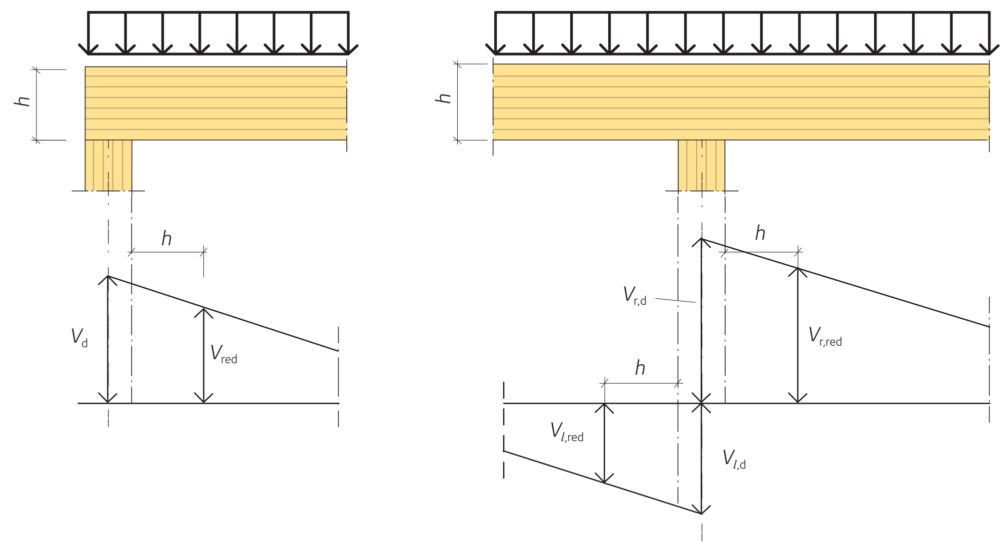


Table 8.5 **Shear force at support zones**



At supports, the design shear force due to distributed load can be reduced according to the figure above. Furthermore, at the supports the contribution to the total shear force of possible concentrated loads acting on the top side of the beam and within a distance "h" from the edge of the support may be disregarded.

Table 8.6 Shear due to bending according to EN 1995-1-1, section 6.1.7

$$\tau_d = \frac{V_d \times S}{I \times b} \leq k_{cr} \times f_{v,d}$$

where:

τ_d	is the design shear stress.
$f_{v,d}$	is the design shear strength.
V_d	is the design shear load.
S	is the maximum static moment (for a rectangular cross section $S = b \times h^2/8$).
I	is the moment of inertia about the neutral axis (for a rectangular cross section $I = b \times h^3/12$).
k_{cr}	= 0.67 and 0.857 for glulam unprotected from precipitation and solar radiation and for glulam protected from precipitation and solar radiation, respectively.

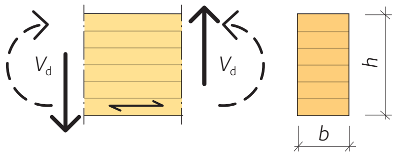
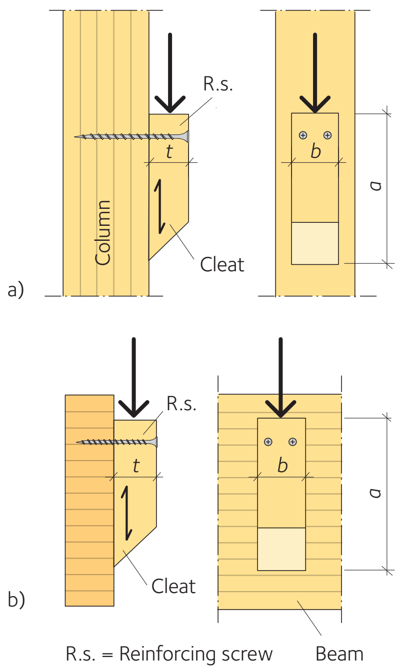


Table 8.7 Shear due to transmission of concentrated loads

$$\tau_d = \frac{V_d}{A^*} \leq \begin{cases} 0.5 \times f_{v,d} & \text{if shear is // grain (figure a)} \\ f_{r,d} & \text{if shear is} \perp \text{grain (figure b)} \end{cases}$$

where:

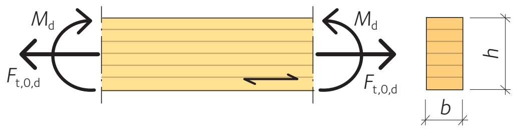
τ_d	is the design shear stress.
$f_{v,d}$	is the design shear strength along the grain.
$f_{r,d}$	is the design shear strength perpendicular to the grain (rolling shear strength).
V_d	is the design shear load.
A*	is the shear area ($A^* = b \times d$, where $d = \min\{a, 8 \times t\}$).



Note: The gluing of the cleat should always occur under controlled humidity and temperature conditions, preferably at the glulam mill. Furthermore, it is always recommended to use reinforcing screw(s) where tension perpendicular to the grain is likely to occur, as shown in the figures to the right.

Table 8.8 Combined bending and axial tension according to EN 1995-1-1, section 6.2.3

$$\frac{F_{t,0,d}}{A_n \times f_{t,0,d}} + \frac{M_d}{W_n \times f_{m,d}} \leq 1$$

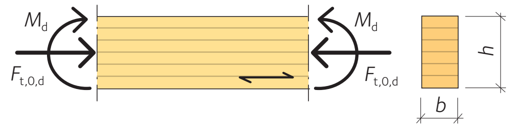


where:

$f_{t,0,d}$	is the design tensile strength along the grain.
$f_{m,d}$	is the design bending strength.
$F_{t,0,d}$	is the design tensile load along the grain.
M_d	is the design bending moment.
A_n	is the cross sectional net area.
W_n	is the net section modulus.

Table 8.9 Combined bending and axial compression according to EN 1995-1-1, section 6.2.4

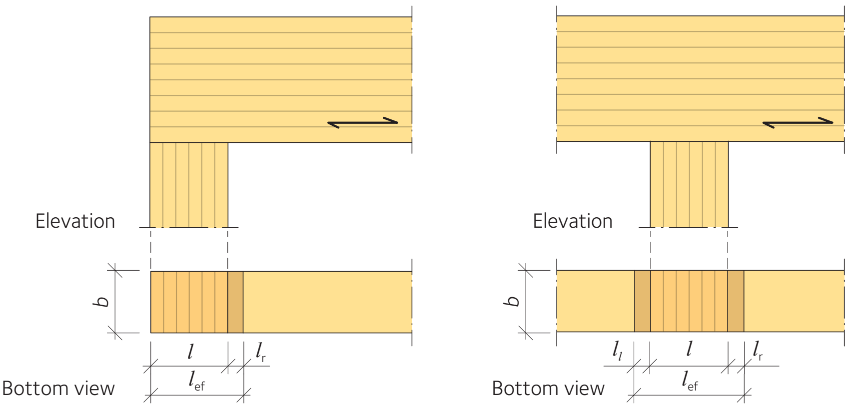
$$\left(\frac{F_{c,0,d}}{A_n \times f_{c,0,d}}\right)^2 + \frac{M_d}{W_n \times f_{m,d}} \leq 1$$



where:

$f_{c,0,d}$	is the design compression strength along the grain.
$f_{m,d}$	is the design bending strength.
$F_{c,0,d}$	is the design compression load along the grain.
M_d	is the design bending moment.
A_n	is the cross sectional net area.
W_n	is the net section modulus.

Table 8.10 Effective contact area in compression perpendicular to the grain according to EN 1995-1-1, section 6.1.5



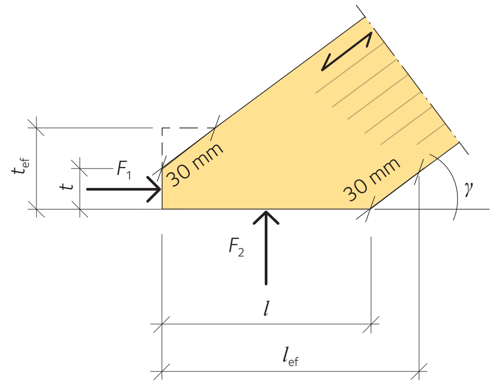
$$l_{ef} = l + l_l + l_r$$

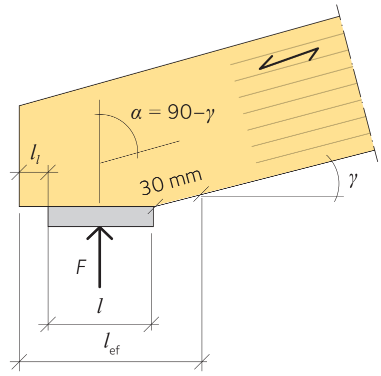
$$A_{ef} = b \times (l + l_l + l_r)$$

where:

l_{ef}	is the effective support length in compression perpendicular to the grain.
A_{ef}	is the effective contact area in compression perpendicular to the grain.
b	is the the width of the beam.
l	is the support length.
l_l and l_r	are the fictitious additional contact lengths (= min {30 mm; l}).

Table 8.11 **Effective contact lengths in compression at an angle *α* to the grain, according to Colling, F., Holzbau Grundlagen, Bemessungshilfen 2. überarbeitete Auflage, Vieweg+Teubner, Wiesbaden 2008**





$l_{ef} = l + 30 \times \cos\gamma$	$t_{ef} = t + 30 \times \sin\gamma$	$l_{ef} = l + \min\begin{pmatrix} l_l \\ 30 \times \sin\alpha \end{pmatrix} + 30 \times \cos\gamma$

Table 8.12 **Compression perpendicular to the grain according to EN 1995-1-1, section 6.1.5**
(k_1 is a factor recommended by the authors of *The Glulam Handbook Volume 3* but is not included in EN 1995-1-1).

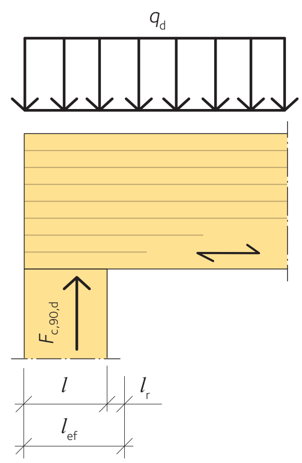
$$\sigma_{c,90,d} = \frac{F_{c,90,d}}{l_{ef} \times b} \leq k_{c,90} \times k_1 \times f_{c,90,d}$$

where:

$\sigma_{c,90,d}$	is the design compression stress perpendicular to the grain.
$f_{c,90,d}$	is the design compression strength perpendicular to the grain.
$F_{c,90,d}$	is the design compression load perpendicular to the grain.
l_{ef}	is the effective support area, *see table 8.10*.
$k_{c,90}$	is a magnification factor ($k_{c,90}$ = 1.75). [1]
k_1	is a factor that takes into account the ratio between permanent load and live load g_k/q_k. **Note** that Eurocode 5 assumes k_1 = 1, for all cases.

Values of k_1

$g_k/q_k \leq 0.4$	$g_k/q_k > 0.4$
$k_1 = \frac{f_{c,90,k}}{f_{c,90,d}}$	$k_1 = 1$

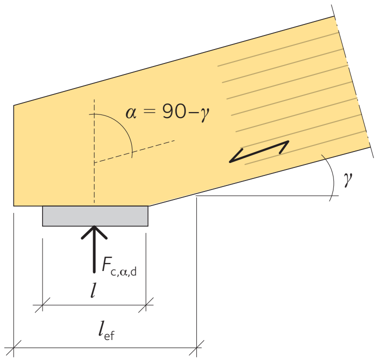


[1] If l > 400 mm, the effective length can be taken as l_{ef} = 400 mm + l_r, and the magnification factor can be set to $k_{c,90}$ = 1.75. Support lengths with l > 600 mm are not recommended.
Note that Eurocode 5 recommends $k_{c,90}$ = 1.75 only if $l \leq$ 400 mm. If l > 400 mm, Eurocode 5 recommends $l_{ef} = l$ and $k_{c,90}$ = 1.0.

According to EKS 10, in many cases γ_M = 1.0 and k_{mod} = 1.0 can be applied, which gives $f_{c,90,k} = f_{c,90,d}$ = 2.5 MPa. For cases where compression of the glulam can be considered to affect the load-carrying capacity, e.g. local compression in trusses or where deformations have a significant effect on the function (e.g. in buildings with more than two floors), γ_M = 1.25 should be used. For glulam structures in service class 3, it is recommended that k_{mod} is selected according to *table 7.3, page 19*.

Table 8.13 **Compression at an angle α to the grain according to EN 1995-1-1, section 6.1.5**
(k_1 is a factor recommended by the authors of *The Glulam Handbook Volume 3* and is not included in EN 1995-1-1)

$$\sigma_{c,\alpha,d} = \frac{F_{c,\alpha,d}}{l_{ef} \times b} \leq f_{c,\alpha,d} = \frac{f_{c,0,d}}{\frac{f_{c,0,d}}{k_{c,90} \times k_1 \times f_{c,90,d}} \times \sin^2\alpha + \cos^2\alpha}$$



where:

$\sigma_{c,\alpha,d}$	is the design compression stress at an angle α to the grain.
$f_{c,\alpha,d}$	is the design compression strength at an angle α to the grain.
$f_{c,90,d}$	is the design compression strength perpendicular to the grain.
$f_{c,0,d}$	is the design compression strength along the grain.
$F_{c,\alpha,d}$	is the design compression load at an angle α to the grain.
l_{ef}	is the effective support area, *see table 8.11, page 25*.
$k_{c,90}$	is a magnification factor ($k_{c,90}$ = 1.75 if $l \leq$ 400 mm).
k_1	is a factor that takes into account the ratio between permanent load and live load g_k/q_k. **Note** that Eurocode 5 assumes k_1 = 1, for all cases.

Values of k_1

Ratio permanent load to live load: $g_k/q_k \leq 0.4$.	Ratio permanent load to live load: $g_k/q_k > 0.4$.
$k_1 = \frac{f_{c,90,k}}{f_{c,90,d}}$	$k_1 = 1$

Values of $f_{c,\alpha,d}$. Glulam in strength class GL30c. Service class: 1. Load duration: medium term.

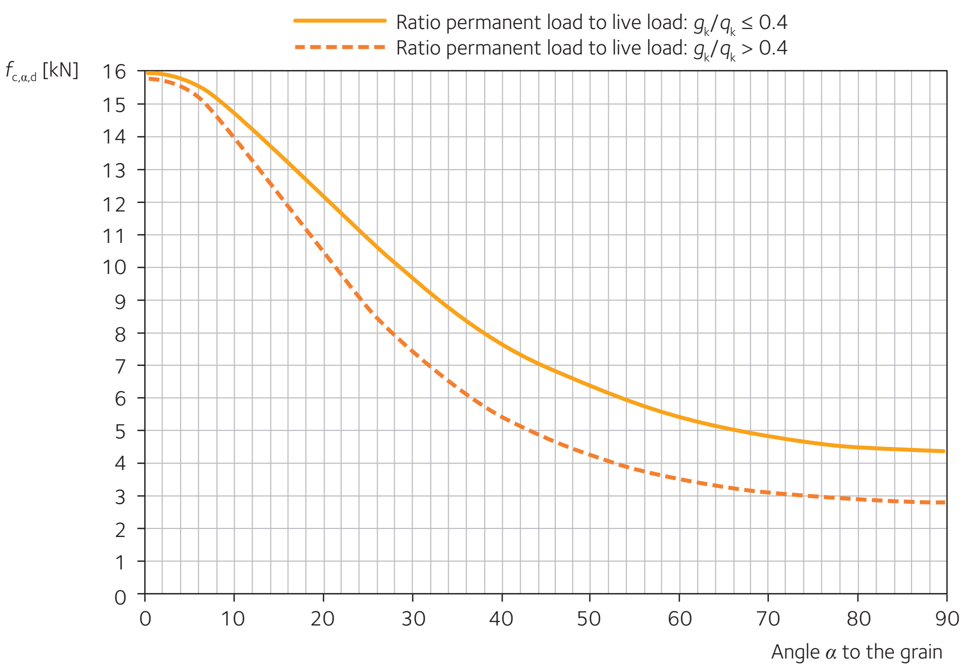


Table 8.14 **Reinforcement of the support zone by means of self-tapping screws, according to Bejtka, I. et al**

$$R_{90,k} = \min \begin{cases} k_{c,90} \times b \times l_{ef,1} \times f_{c,90,k} + n \times \min\left(R_{ax,k}; R_{kl,k}\right) \\ b \times l_{ef,2} \times f_{c,90,k} \end{cases}$$

$$R_{90,d} = \frac{k_{mod} \times R_{90,k}}{\gamma_m}$$

where:

$R_{90,k}$	is the characteristic compressive capacity perpendicular to the grain of the support.
$k_{c,90}$	= 1.75 magnification factor ($k_{c,90}$ = 1.75 if $l_{sup} \leq$ 400 mm).
$f_{c,90,k}$	is the characteristic compressive strength perpendicular to the grain.
b	is the beam width.
l_{ef}	is the length of the screw threaded part.
l_{sup}	is the support length.
$l_{ef,1}$	= l_{sup} + 30 mm.
$l_{ef,2}$	$= l_{sup} + 0.25 \times l_{ef} \times e^{3.3 \times \frac{l_{ef}}{h}}$
n	is the number of reinforcing screws.
$R_{ax,k}$	is the characteristic pull-through capacity (≈ withdrawal capacity) of the screw. This value is usually provided by the screw manufacturer.
$R_{kl,k}$	is the characteristic buckling strength of the screw, *see table 8.15, page 28*.
k_{mod}	is the modification factor, *see table 8.3, page 21*.
γ_m	= 1.3

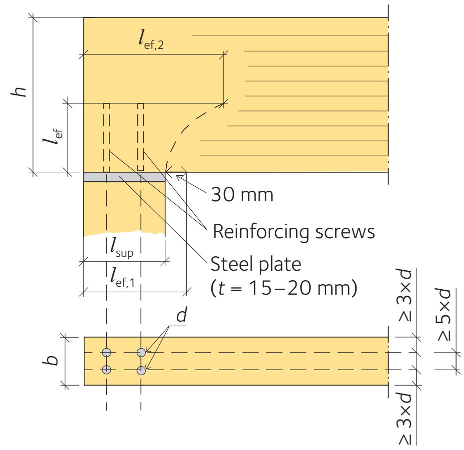
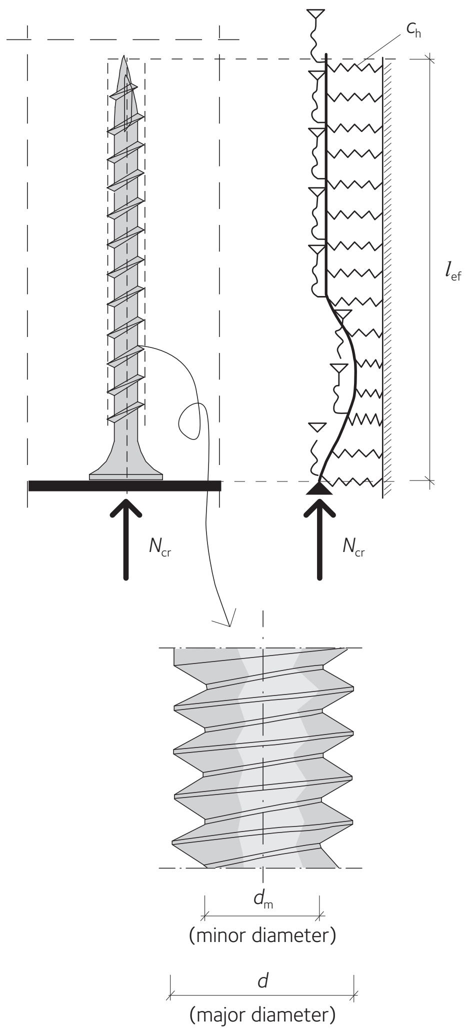


Table 8.15 Reinforcement of the support zone by means of self-tapping screws: buckling capacity of the screw, according to Bejtka, I. et al

$R_{kl,k} = k_c \times \left(\frac{\pi \times d_m^2}{4}\right) \times f_{y,k}$	
where:	
k_c	is the reduction factor for buckling: $k_c = \begin{cases} 1 \text{ for } \lambda_{rel} \leq 0.2 \\ \left(k + \sqrt{k^2 - \lambda_{rel}^2}\right)^{-1} \text{ for } \lambda_{rel} > 0.2 \end{cases}$ $k = 0.5 \times \left[1 + 0.49 \times \left(\lambda_{rel} - 0.2\right) + \lambda_{rel}^2\right]$
λ_{rel}	is the relative slenderness ratio: $\lambda_{rel} = \sqrt{\frac{N_{pl}}{N_{cr}}}$
N_{pl}	is the screw yield strength: $N_{pl} = \frac{\pi \times d_m^2}{4} \times f_{y,k}$
N_{cr}	is the screw buckling load: $N_{cr} = \sqrt{c_h \times E_s \times I_s}$
c_h	is the horizontal stiffness constant: $c_h = \left(0.19 + 0.012 \times d\right) \times \rho_k$
d	is the thread (or major) diameter of the screw
d_m	is the core (or minor) diameter of the screw
$f_{y,k}$	is the yield strength of the screw
E_s	is the modulus of elasticity of the screw material (e.g. for steel: E_s = 210,000 MPa)
I_s	is the moment of inertia of the screw core $I_s = \pi \times (dm)4/64$



9 Design for ULS of members subjected to compression and bending – members prone to instability

Table 9.1 **Buckling lengths for typical compression members**
$\beta = L_E / L$, where L_E is the effective (or buckling) length of the member and L is the geometrical length of the member. The recommended design values of β are modifications of the ideal values, taking into account that perfect fixity is very seldom attained in practice.

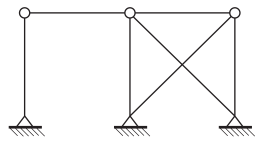
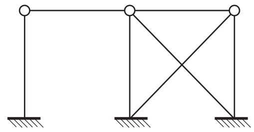
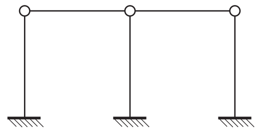
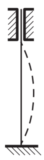
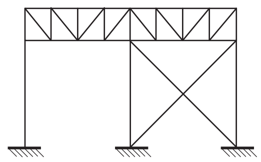
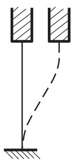
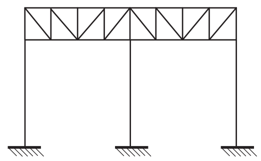
Model	Typical example	Theoretical β value	Recommended β value
		1.0	1.0
		0.7	0.85
		2.0	2.25
		0.5	0.7
		1.0	1.2

Table 9.2 **Definition of buckling load P_{cr} and relative slenderness ratio λ_{rel}**
The considered member is assumed braced in the y-direction, therefore buckling can only occur around the y-axis.

Buckling load

$$P_{cr} = \pi^2 \times \frac{E_{0.05} \times I_y}{L_E^2}$$

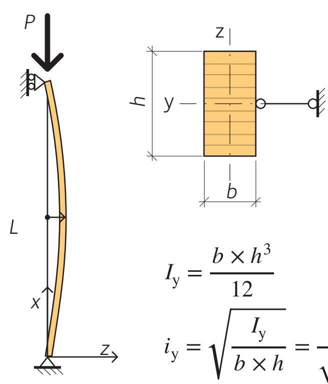
Relative slenderness ratio

$$\lambda_{rel} = \sqrt{\frac{f_{c,0,k} \times A}{P_{cr}}} = \frac{\lambda_y}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}}$$

where:

P_{cr}	is the critical load.
$f_{c,0,k}$	is the characteristic compression strength along the grain.
A	is the gross area of the cross section.
$E_{0,05}$	is the characteristic value of modulus of elasticity.
I_y	is the moment of inertia of the cross section about y.
L_E	is the effective (or buckling) length = $\beta \times L$, *see table 9.1.*

Buckling around the y-axis

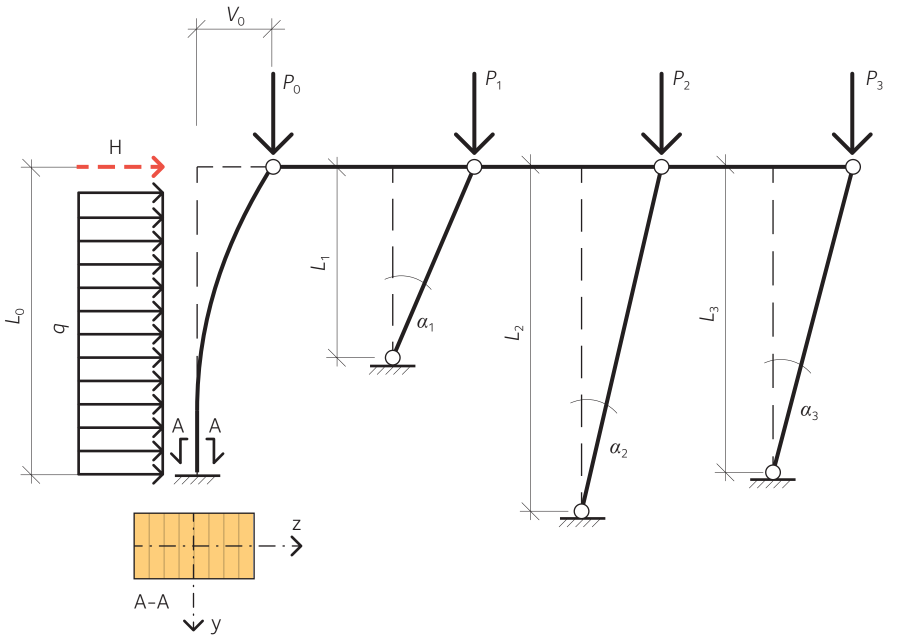


$$I_y = \frac{b \times h^3}{12}$$

$$i_y = \sqrt{\frac{I_y}{b \times h}} = \frac{h}{\sqrt{12}}$$

$$\lambda_y = \frac{\beta \times L}{i_y}$$

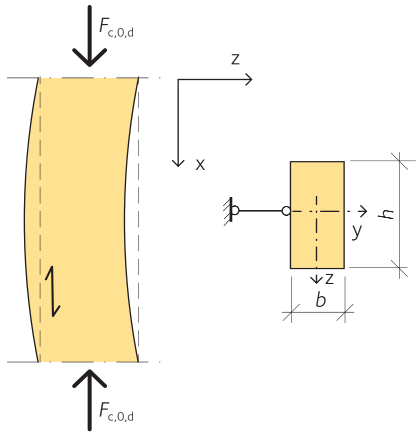
Table 9.3 **Buckling of cantilever column as a part of a system of columns, according to Åkerlund, S., Stål- och träkonstruktioner AK II, Faculty of engineering, Lund University, Sweden, 1984**



Initial out-of-plumb (mean value)	$\alpha_m = 0.003 + 0.015/\sqrt{n}$
Equivalent horizontal force (or destabilising force) acting at the top of the cantilevered column	$H = \dfrac{\alpha_m \times \sum_1^n P_i + v_0 \times \dfrac{1}{1-\frac{P_0}{P_{cr}}} \times \sum_1^n \frac{P_i}{L_i}}{1 - \dfrac{L_0^3}{3 \times E_{0.05} \times I_y} \times \dfrac{1}{1-\frac{P_0}{P_{cr}}} \times \sum_1^n \frac{P_i}{L_i}}$
Stability check of the cantilevered column	*See table 9.1 and 9.2, page 29.*
where:	
n	is the number of doubly-pinned columns (n = 3 for the example shown in the figure above).
v_0	is the horizontal displacement at the top of the cantilevered column due to the applied load only (i.e. without the contribution of "H"). For the case shown in the figure above: $v_0 = \dfrac{q \times L_0^2}{8 \times E \times I_y}$
P_{cr}	is the critical load of the cantilevered column, *see table 9.1 and 9.2, page 29.*
I_y	is the moment of inertia of the cross section about y.
$E_{0,05}$	is the characteristic value of the modulus of elasticity.

Table 9.4 **Members subjected to compression according to EN 1995-1-1, section 6.3.2**

$\sigma_{c,0,d} = \dfrac{F_{c,0,d}}{A_n} \leq k_c \times f_{c,0,d}$	
where:	
$\sigma_{c,0,d}$	is the design compression stress along the grain.
$f_{c,0,d}$	is the design compression strength along the grain.
$F_{c,0,d}$	is the design compression load along the grain.
A_n	is the cross sectional net area.
k_c	is the reduction factor for compression buckling: $k_c = \begin{cases} 1 \text{ for } \lambda_{rel} \leq 0.3 \\ \dfrac{1}{k+\sqrt{k^2-\lambda_{rel}^2}} \quad \text{for } \lambda_{rel} > 0.3 \end{cases}$ where: $k = 0.5 \times \left[1 + 0.1 \times \left(\lambda_{rel} - 0.3\right) + \lambda_{rel}^2\right]$ and with λ_{rel} according to *table 9.2, page 29*.



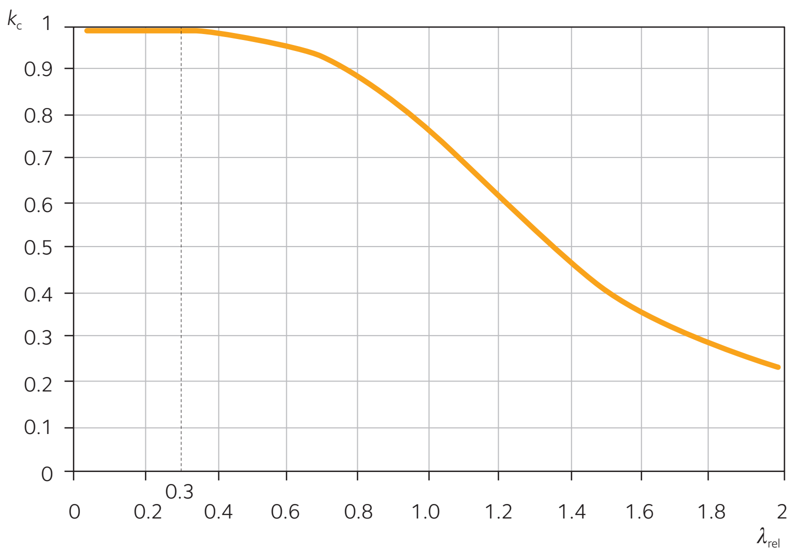
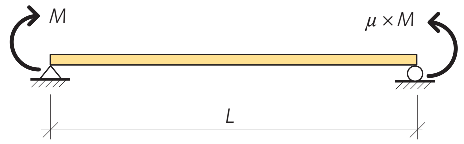
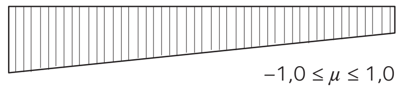
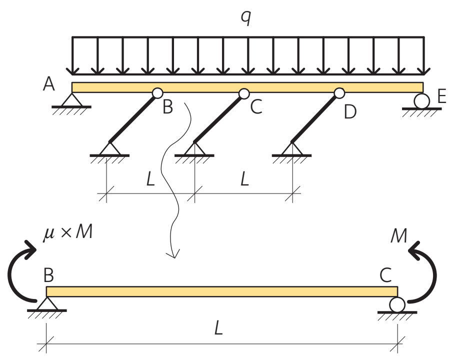
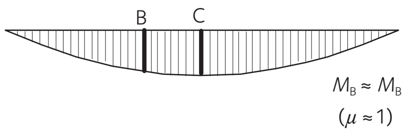
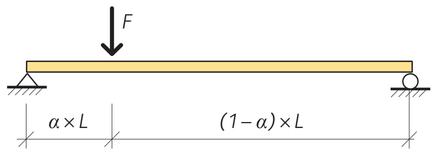
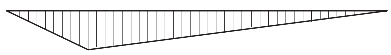
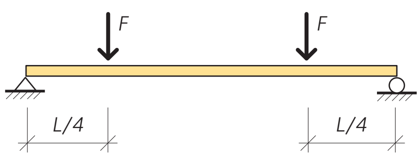
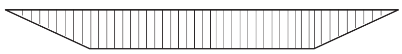
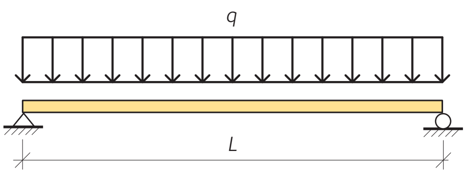
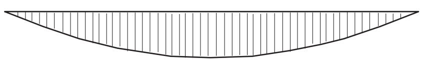
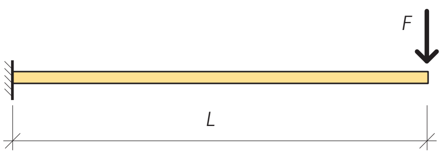
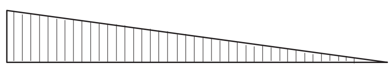
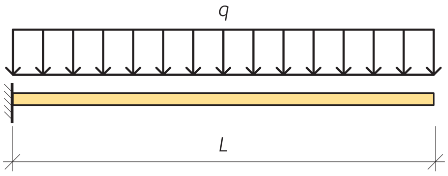
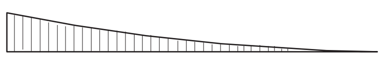


Table 9.5 Effective lengths for typical bending members. Distance between bracings: L. $\beta_{LT} = L_{ef} / L$, where L_{ef} is the effective (or buckling) length of the member and L is the geometrical length of the member. The following boundary conditions have been assumed at supports and/or bracing points. Torsion is restrained but warping is free.

Statical system	Moment diagram	$\beta_{LT} = L_{ef} / L$
		0.6 + 0.4 × μ (≥ 0.4)
		1.0
		0.56 + 0.74 × α × (1 − α) [1]
		1.0 [1]
		0.9 [1]
		0.8 [1]
		0.5 [1]

[1] The ratio between the effective length L_{ef} and the span L is valid for a beam loaded at the centre of gravity. If the load is applied at the top edge of the beam, L_{ef} should be increased by $2 \times h$. If the load is applied at the bottom edge of the beam, L_{ef} should be decreased by $0.5 \times h$, where h is the beam depth.

Table 9.6 **Definition of critical bending moment M_{cr} and relative slenderness ratio $\lambda_{rel,m}$**

Critical bending moment

$$M_{cr} = \frac{\pi}{L_{ef}}\sqrt{(E_{0.05} \times I_z) \times (G_{0.05} \times K_v)}$$

Relative slenderness ratio

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k} \times W}{M_{cr}}} \approx \frac{1}{b} \times \sqrt{\frac{h \times L_{ef} \times f_{m,k}}{0.78 \times E_{0.05}}}$$

where:

M_{cr}	is the critical bending moment.
$f_{m,k}$	is the characteristic bending strength.
b	is the beam width.
h	is the beam depth.
W	is section modulus (= $b \times h^2/6$).
$E_{0,05}$	is the characteristic value of modulus of elasticity.
$G_{0,05}$	is the characteristic value of shear modulus.
K_v	is torsional stiffness factor (≈ $b^3 \times h/3$).
I_z	is the moment of inertia of the cross section about z.
L_{ef}	is the effective (or buckling) length.

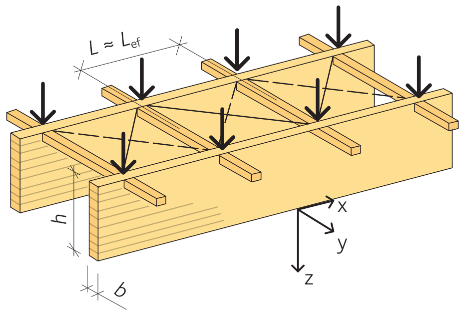
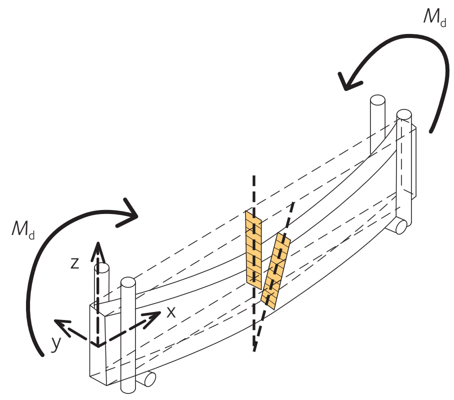


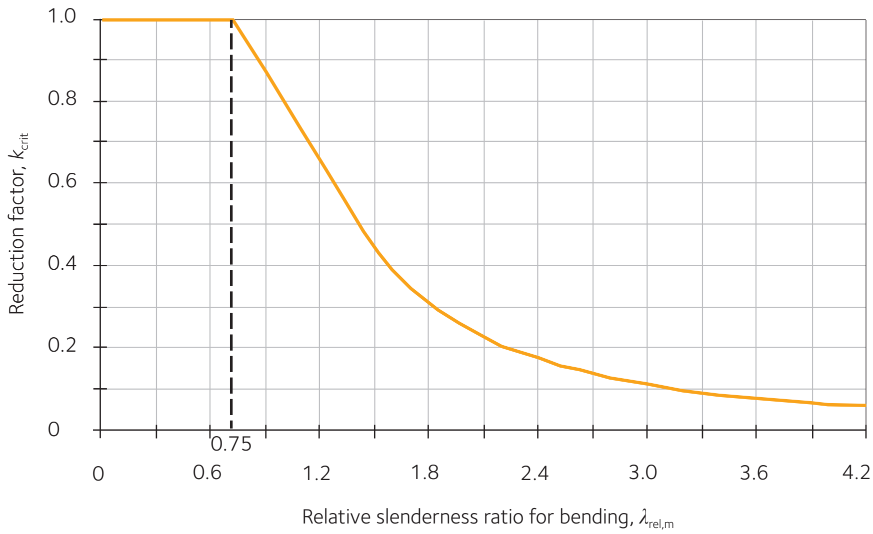
Table 9.7 **Members subjected to bending according to EN 1995-1-1, section 6.3.3**

$$\sigma_{m,d} = \frac{M_d}{W_n} \leq k_{crit} \times f_{m,d}$$



where:

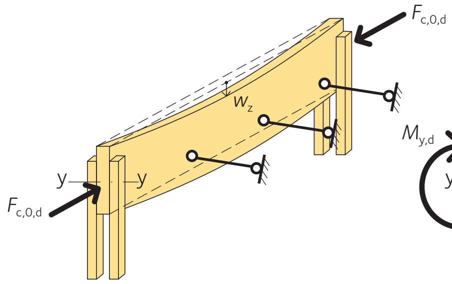
$\sigma_{m,d}$	is the design bending stress.
$f_{m,d}$	is the design bending strength.
M_d	is the design bending moment.
W_n	is the net section modulus.
k_{crit}	is the reduction factor for lateral torsional buckling: [1)] $$k_{crit} = \begin{cases} 1.0 \text{ for } \lambda_{rel,m} \leq 0.75 \\ 1.56 - 0.75 \times \lambda_{rel,m} \text{ for } 0.75 < \lambda_{rel,m} \leq 1.4 \\ \left(\frac{1}{\lambda_{rel,m}}\right)^2 \text{ for } \lambda_{rel,m} > 1.4 \end{cases}$$ and with λ_{rel} according to *table 9.6, page 33*.



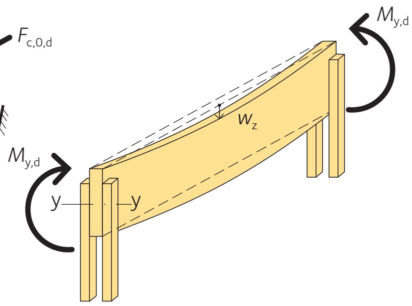
[1)] **Note** that if one of the following two conditions is fulfilled:
- $(L_{ef} \times h)/b2 \leq 140$
- $h / b \leq \sim 4$

Lateral torsional buckling is not likely to occur, i.e. kcrit may be set equal to 1.

Table 9.8 **Members subjected to combined compression and bending (without lateral torsional buckling, i.e. $\lambda_{rel,m} \leq 0.75$) according to EN 1995-1-1, section 6.3.2**

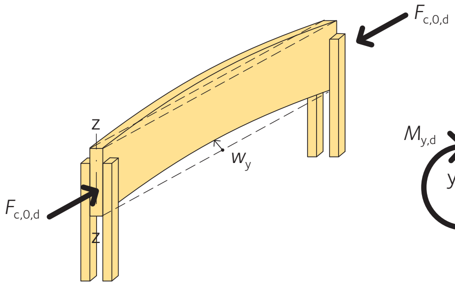


Buckling about y-axis
→ $k_{c,y}$

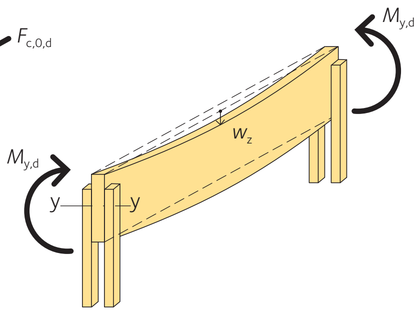


Bending about y-axis
→ $\sigma_{m,y}$

$$\frac{F_{c,0,d}}{k_{c,y} \times (A_n \times f_{c,0,d})} + \frac{M_{y,d}}{W_{n,y} \times f_{m,y,d}} \leq 1$$



Buckling about z-axis
→ $k_{c,z}$

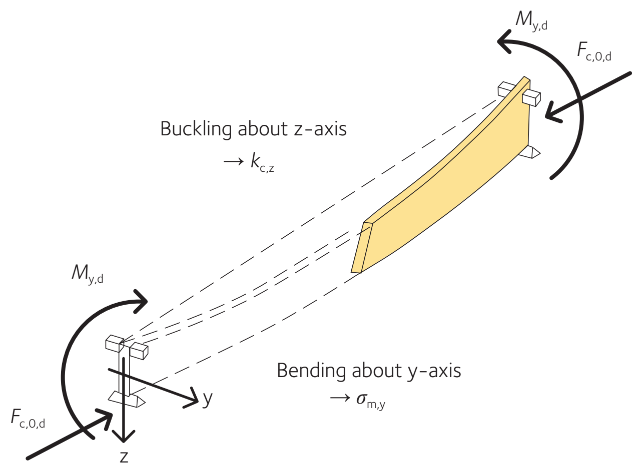


Bending about y-axis
→ $\sigma_{m,y}$

$$\frac{F_{c,0,d}}{k_{c,z} \times (A_n \times f_{c,0,d})} + 0.7 \times \frac{M_{y,d}}{W_{n,y} \times f_{m,y,d}} \leq 1$$

$F_{c,0,d}$	is the design compression load along the grain.
$M_{y,d}$	is the design bending moment about y.
$f_{c,0,d}$	is the design compression strength along the grain.
$f_{m,y,d}$	is the design bending strength about y.
A_n	is the cross sectional net area.
W_{ny}	is the net section modulus about y.
$k_{c,y}$ and $k_{c,z}$	are the reduction factors for compression buckling about y and z respectively

Table 9.9 **Members subjected to combined compression and bending (with possible lateral torsional buckling, i.e. $\lambda_{rel,m} > 0.75$) according to EN 1995-1-1, section 6.3.3**



$$\frac{F_{c,0,d}}{k_{c,z} \times (A_n \times f_{c,0,d})} + \left[\frac{M_{y,d}}{k_{crit} \times \left(W_{n,y} \times f_{m,y,d}\right)}\right]^2 \leq 1$$

$F_{c,0,d}$	is the design compression load along the grain.
$M_{y,d}$	is the design bending moment about y.
$f_{c,0,d}$	is the design compression strength along the grain.
$f_{m,y,d}$	is the design bending strength about y.
A_n	is the cross sectional net area.
W_{ny}	is the net section modulus about y.
$k_{c,z}$	is the reduction factor for compression buckling about z.
k_{crit}	is the reduction factor for lateral torsional buckling.

Notched members

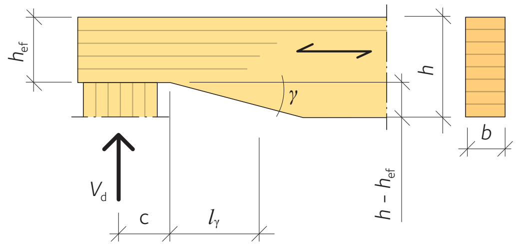
Table 9.10 **Notched members according to EN 1995-1-1, section 6.5**

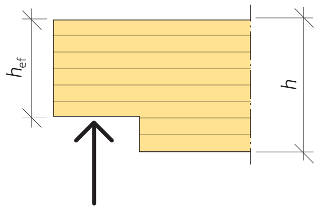
$$\tau_d = \frac{3}{2}\frac{V_d}{b \times h_{ef}} \le k_v \times f_{v,d}$$

$$k_v = \min\left\{1;\ \frac{6.5 \times \left(1 + \frac{1.1 \times i^{1.5}}{\sqrt{h}}\right)}{\sqrt{h} \times \left(\sqrt{\alpha \times (1-\alpha)} + 0.8 \times \frac{c}{h} \times \sqrt{\frac{1}{\alpha} - \alpha^2}\right)}\right\}$$

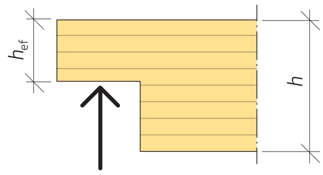
where:

τ_d	is the design shear stress.
$f_{v,d}$	is the design shear strength.
V_d	is the design shear force.
i	$= l_\gamma / (h - h_{ef})$
α	$= h_{ef} / h$





$\frac{h_{ef}}{h} = 0.75 \longrightarrow k_v \approx 0.4 - 0.5$



$\frac{h_{ef}}{h} = 0.50 \longrightarrow k_v \approx 0.3 - 0.4$

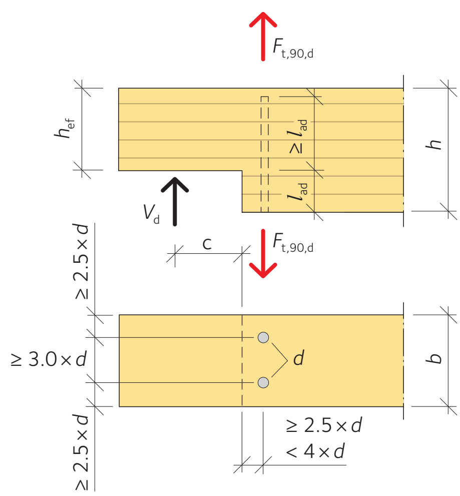
Table 9.11 **Reinforcement of notched members by means of self-tapping screws or bonded-in rods according to DIN EN 1995-1-1/NA**

$$F_{t,90,d} = 1.3 \times V_d \times \left(3 \times (1-\alpha)^2 - 2 \times (1-\alpha)^3\right)$$

Check: $F_{t,90,d} \le n_r \times R_{t,d}$

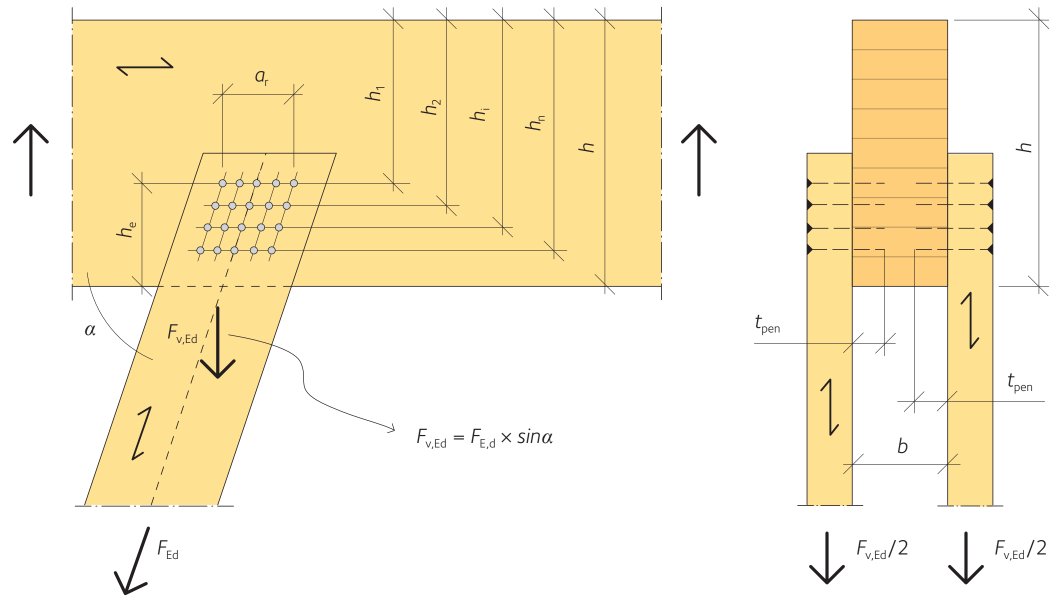
where:

$F_{t,90,d}$	is the design tensile force perpendicular to the grain.
$R_{t,d}$	is the design axial strength of the reinforcing screw/rod, i.e. the smallest value of tensile and withdrawal strength. The bonding length in case of bonded-in rods or the penetration length in case of screws shall be taken equal to $l_{ad} = h - h_{ef}$, see figure to the right.
n_r	is the number of reinforcing screws.
V_d	is the design shear force.
α	$= h_{ef} / h$
d	is the outer thread diameter, $d \le 20$ mm.



Suspended loads at tensioned beam edge

Table 9.12 **Connection forces at an angle to the grain according to DIN EN 1995-1-1/NA**



$$F_{90,\mathrm{Rd}} = k_s \times k_r \times \left(6.5 + \frac{18 \times h_e^2}{h^2}\right) \times (t_{ef} \times h)^{0.8} \times f_{t,90,d}$$

where:

$$k_s = \max\left\{1; 0.7 + \frac{1.4 \times a_r}{h}\right\} \quad \text{and} \quad k_r = \frac{n}{\sum_{i=1}^{n}\left(\frac{h_1}{h_i}\right)^2}$$

Check: $F_{90,\mathrm{Rd}} \geq F_{v,\mathrm{Ed}}$

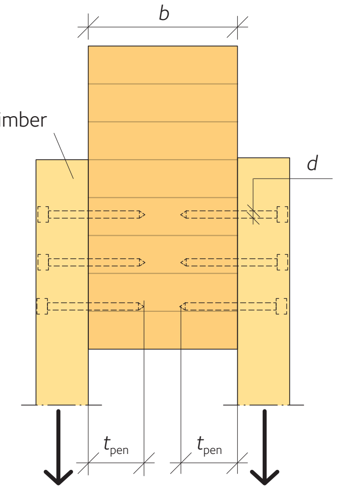
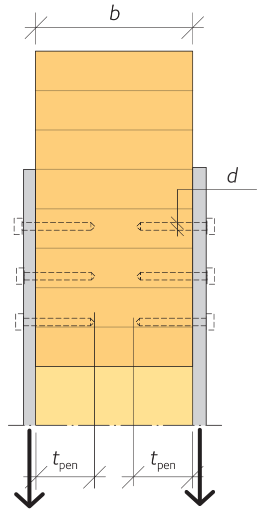
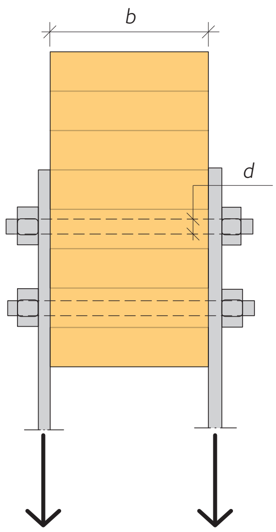
(Cross-connections with $a_r/h > 1$ and $F_{v,ED} > 0.5 \times F_{90,Rd}$ should be reinforced, *see table 9.14, page 39*)

where:

$F_{v,Ed}$	is the design value of the force component perpendicular to the grain, [N].
$F_{90,Rd}$	is the design splitting capacity of the member, [N].
$f_{t,90,d}$	is the design tensile strength perpendicular to the grain, [MPa].
k_s	is a factor to take into account the spacing of fasteners in a row of fasteners parallel to the grain.
k_r	is a factor to take into account multiple rows of fasteners.
h_e	is the loaded edge distance to the centre of the most distant fastener, [mm].
a_r	is the spacing between the centres of the two outermost fasteners in a row of fasteners parallel to the grain, see figure above.
h	is the timber member depth, [mm].
t_{ef}	is the effective depth, in mm, *see table 9.13, page 39.*
n	is the number of rows of fasteners.
h_i	is the unloaded edge distance to the axis of the row of fasteners considered, see figure above.

For cross-connections with $h_e/h > 0.7$, no further verification is required.
Cross-connections with $h_e/h < 0.2$ should only be loaded by forces of short duration (e.g. wind suction).

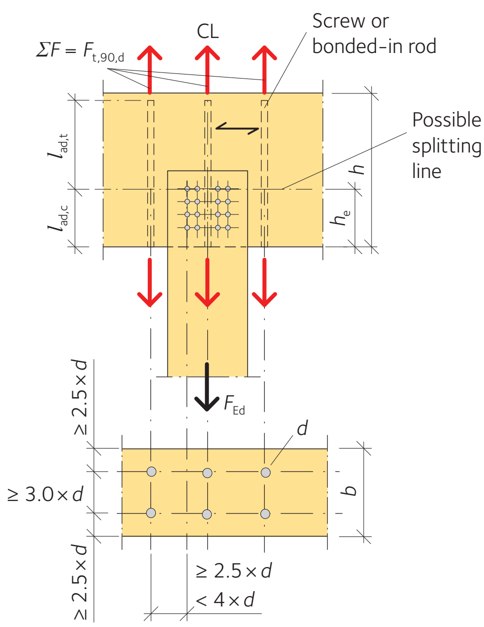
Table 9.13 **The effective depth t_{ef} for different configurations of the connection according to DIN EN 1995-1-1/NA**

Timber-to-timber or panel-to-timber connections with nails or screws	Nailed steel-to-timber connections	Dowelled or bolted connections, either timber-to-timber or steel-to-timber
		
$t_{ef} = \min\left\{b; 2 \times t_{pen}; 24 \times d\right\}$	$t_{ef} = \min\left\{b; 2 \times t_{pen}; 30 \times d\right\}$	$t_{ef} = \min\{b; 12 \times d\}$ [1)]

1) For connections with screws: $t_{ef} = \min\left\{b; 2 \times t_{pen}; 12 \times d\right\}$

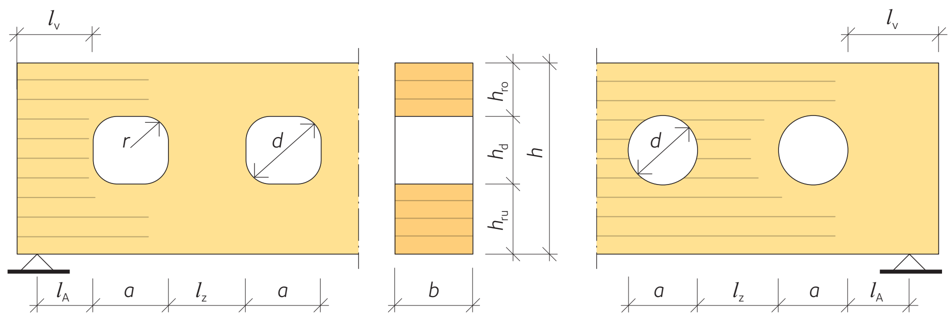
Table 9.14 **Reinforcement to carry tensile stresses perpendicular to the grain in connections with a tensile force component perpendicular to the grain according to DIN EN 1995-1-1/NA**

$F_{t,90,d} = \left[1 - 3 \times \alpha^2 + 2 \times \alpha^3\right] \times F_{v,Ed}$	
Check:	$F_{t,90,d} \leq n_r \times R_{t,d}$
where:	
$F_{t,90,d}$	is the design tensile force perpendicular to the grain.
$F_{v,Ed}$	is the design value of the tensile force component perpendicular to the grain.
$R_{t,d}$	is the design axial strength of the reinforcing screw/rod, i.e. the smallest value of tensile and withdrawal strength. The bonding length in case of bonded-in rods or the penetration length in case of screws shall be taken equal to $l_{ad} = \min(l_{ad,t}; l_{ad,c})$, see figure to the right.
n_r	is the number of reinforcing screws.
α	$= h_e / h$
d	is the outer thread diameter, $d \leq 20$ mm.



Members with holes

Table 9.15 **Members with holes: geometric requirements according to DIN EN 1995-1-1/NA**
Note that DIN EN 1995-1-1/NA proposes $r \geq 15$ mm. The authors of *The Glulam Handbook Volume 3* suggest $r \geq 25$ mm.



Holes with diameter $d \leq 50$ mm can be disregarded in design, if they are placed close to the neutral axis or very close to the upper corner above a support.

$l_v \geq h$	$l_z \geq 1.5h$ or at least 300 mm	$l_A \geq 0.5h$	$h_{ro} \geq 0.35h$ $h_{ru} \geq 0.35h$	$a \leq 0.4h$	$h_d \leq 0.15h$	$r \geq 25$ mm

Table 9.16 **Members with holes: design tensile force at the edge of the hole according to DIN EN 1995-1-1/NA**

$$F_{t,90,d} = F_{t,90,V,d} + F_{t,90,M,d}$$

where:

$$F_{t,V,d} = \frac{V_d \times h_d}{4 \times h} \times \left[3 - \frac{h_d^2}{h^2}\right]$$

and

$$F_{t,M,d} = 0.008 \times \frac{M_d}{h_r}$$

Check: $$\sigma_{t,90,d} = \frac{F_{t,90,d}}{0.5 \times l_{t,90} \times b} \leq k_{t,90} \times f_{t,90,d}$$

where:

V_d	is the design shear force at the edge of the hole (i.e. at a distance "x" from the support).
M_d	is the design bending moment at the edge of the hole (i.e. at a distance "x" from the support).
b	beam width.
h_r	min (h_{ro} + 0.15 × h_d ; h_{ru} + 0.15 × h_d) for circular holes.
h_r	min (h_{ro} ; h_{ru}) for rectangular holes.
$l_{t,90}$	0.35 × h_d + 0.5 × h for circular holes.
$l_{t,90}$	0.5 × (h_d + h) for rectangular holes.
$k_{t,90}$	min (1 ; (450 / h)0.5), where h is the beam depth.

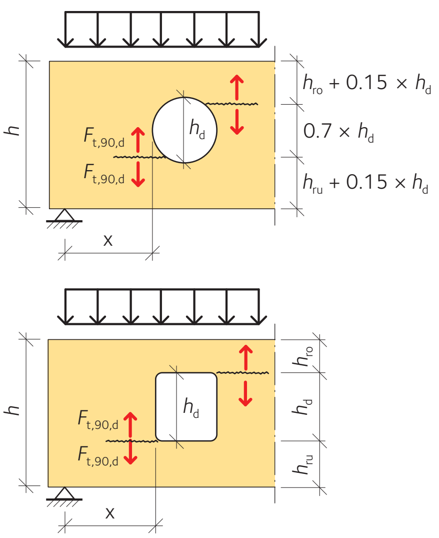
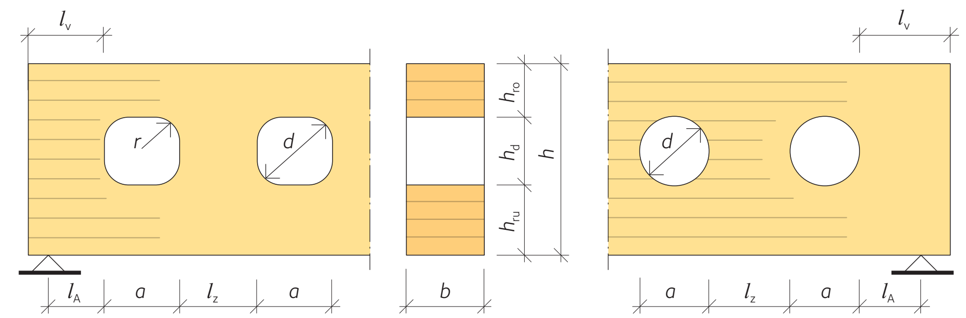


Table 9.17 **Members with holes reinforced by means of screws or bonded-in rods: geometric requirements according to DIN EN 1995-1-1/NA**
Note that DIN EN 1995-1-1/NA proposes $r \geq 15$ mm. The authors of *The Glulam Handbook Volume 3* suggest $r \geq 25$ mm.



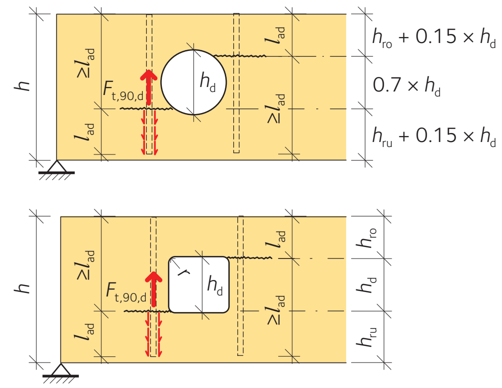
Holes with diameter $d \leq 50$ mm can be disregarded in design, if they are placed close to the neutral axis or very close to the upper corner above a support.

$l_v \geq h$	$l_z \geq 1.0h$ or at least 300 mm	$l_A \geq 0.5h$	$h_{ro} \geq 0.25h$ $h_{ru} \geq 0.25h$	$a \leq 1.0h$ $a \leq 2.5h_d$	$h_d \leq 0.30h$ [1)] $h_d \leq 0.40h$ [2)]	$r \geq 25$ mm

[1)] Applicable to beams with internal reinforcement.
[2)] Applicable to beams with external reinforcement.

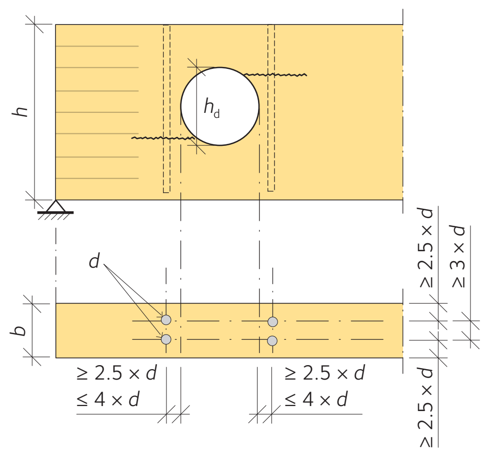
Table 9.18 **Reinforcement of members with holes according to DIN EN 1995-1-1/NA**

Check:	$F_{t,90,d} \leq n_r \times R_{t,d}$
where:	
$F_{t,90,d}$	is the design tensile force perpendicular to the grain, *see table 9.16, page 40.*
$R_{t,d}$	is the design axial strength of the reinforcing screw/bonded-in bar, i.e. the minimum value between tensile and withdrawal strength. The bonding length in case of bonded-in rods or the penetration length in case of screws shall be taken equal to $l_{ad} = \min\{h_{ro} + 0.15;\ h_{ru} + 0.15 \times h_d\}$ for circular holes and the bonding length $l_{ad} = h_{ro}$ or h_{ru}, for rectangular holes, see figure to the right.
n_r	is the number of reinforcing screws at one side (left side or right side) of the hole.



Note that for each side of the hole, only one row of screws should be used in the "critical" cross section, as shown e.g. in *table 9.19*.

Table 9.19 **Placement of the reinforcing screws or bonded-in rods according to DIN EN 1995-1-1/NA**

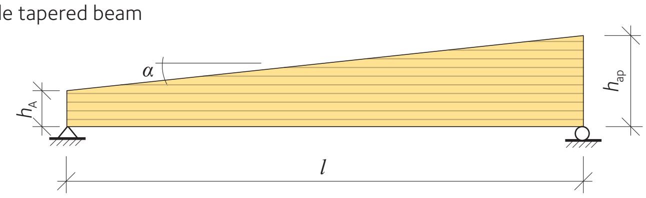


The minimum length of each steel rod is $2 \times l_{ad}$ and the outer thread diameter is limited to $d \leq 20$ mm.

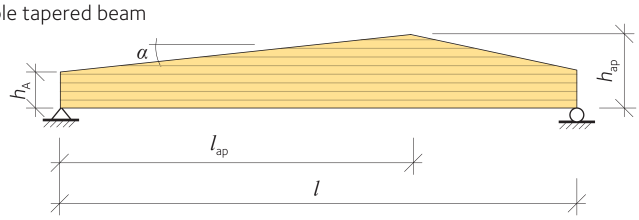
10 Design of cross-sections in members with varying cross-section or curved shape

Table 10.1 Common beam geometries

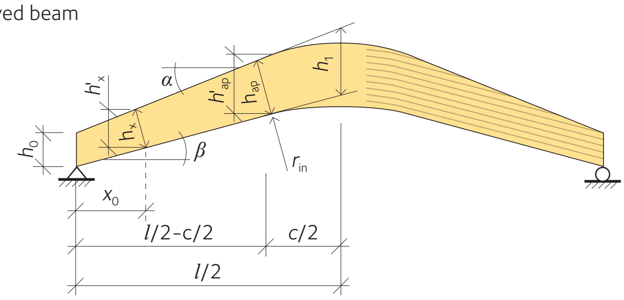
Single tapered beam



Double tapered beam



Curved beam



$\gamma = \alpha - \beta$

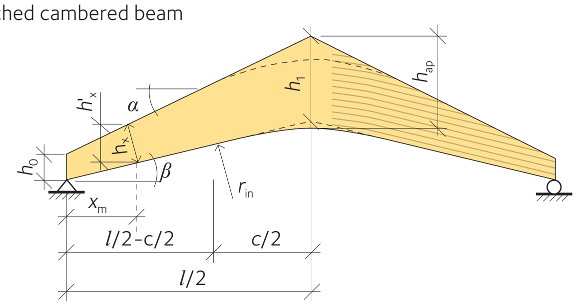
$c/2 = r_{in} \times \sin \beta$

$h_1 = h_0 + (l/2 - c/2) \times (\tan \alpha - \tan \beta)$

$h'_{ap} = h_0 + l/2 \times (\tan \alpha - \tan \beta)$

$h_x \approx h'_x \times \cos \gamma$

Pitched cambered beam



$\gamma = \alpha - \beta$

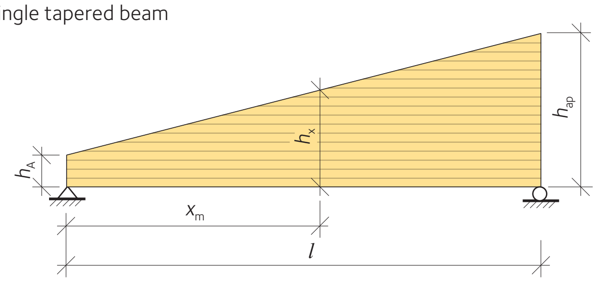
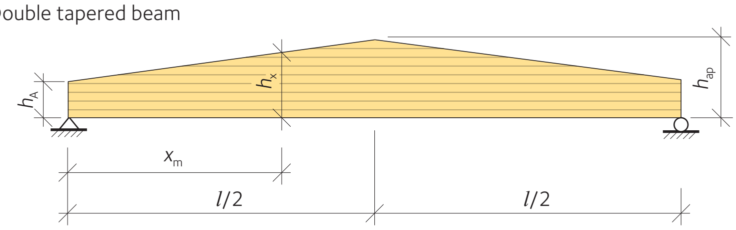
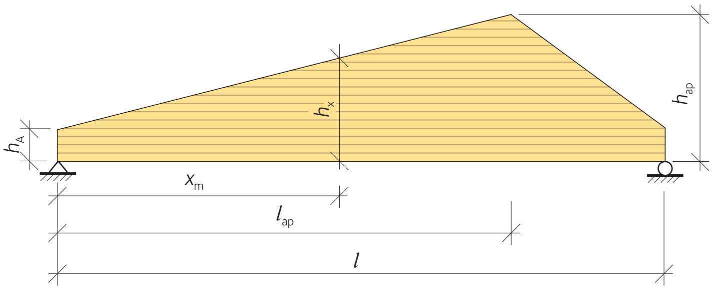
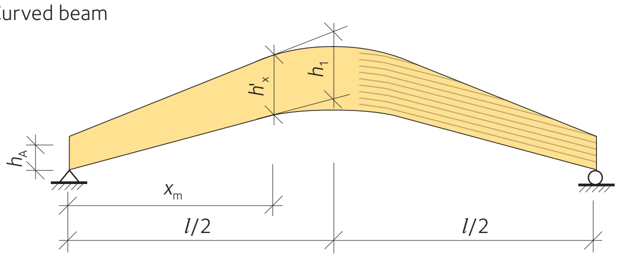
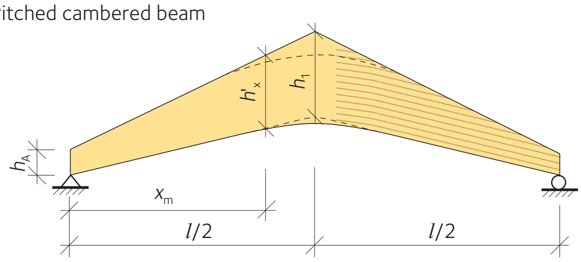
$c/2 = r_{in} \times \sin \beta$

$h_1 = h_0 + l/2 \times (\tan \alpha - \tan \beta)$

$h'_{ap} = h_0 + l/2 \times (\tan \alpha - \tan \beta) + r_{in} \times (1 - \cos \beta)/\cos \beta$

$h_x \approx h'_x \times \cos \gamma$

Table 10.2 **Position of the cross section with highest bending stress, $x = x_m$, in simply supported beams subjected to uniformly distributed load**

Single tapered beam 	$x_m = \dfrac{l}{1 + h_{ap}/h_A}$ $h_x = \dfrac{2 \times h_{ap}}{1 + h_{ap}/h_A}$
Double tapered beam 	$x_m = \dfrac{l \times h_A}{2 \times h_{ap}}$ $h_x = h_A \times \left(2 - h_A/h_{ap}\right)$
Double tapered beam 	$x_m = \dfrac{l_{ap}}{h_{ap}/h_A + 2 \times l_{ap}/l - 1}$ $h_x = h_A + \dfrac{x_m}{l_{ap}} \times \left(h_{ap} - h_A\right)$
Curved beam 	$h'_x = h_A \times (2 - h_A/h_1)$
Pitched cambered beam 	$x_m = \dfrac{l \times h_A}{2 \times h_1}$

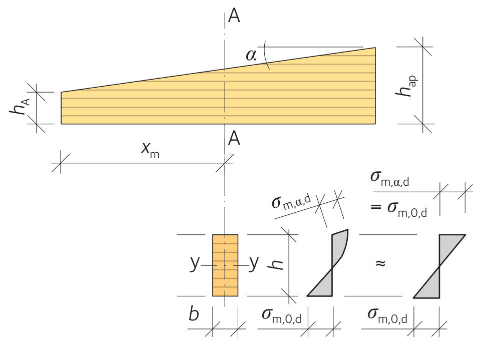
According to EKS 10, an unsymmetric snow load should be taken into account, whatever the pitch of the roof. As a safe approximation, one can assume a uniformly distributed snow load with the form factor μ_4 over the whole beam. The equations stated above then apply.

Table 10.3 Bending stress at $x = x_m$ according to EN 1995-1-1, section 6.4.2

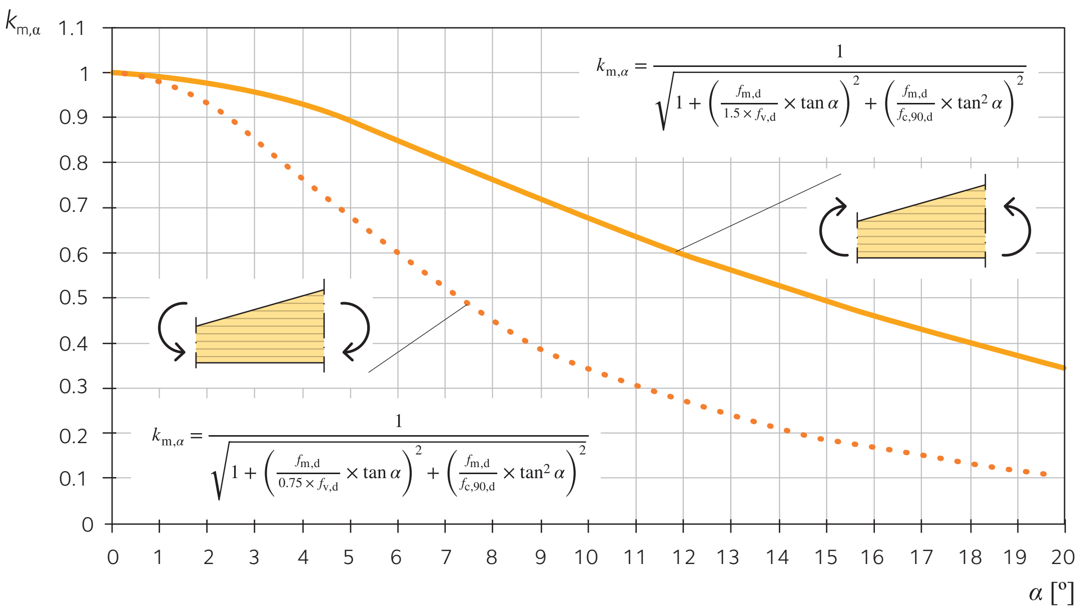
$$\sigma_{m,\alpha,d} = \sigma_{m,0,d} = \frac{6 \times M_d}{b \times h^2} \leq k_{m,\alpha} \times f_{m,d}$$

where:

$\sigma_{m,\alpha,d}$	is the design bending stress at an angle α to the grain.
$f_{m,d}$	is the design bending strength.
M_d	is the design bending moment at $x = x_m$.
b, h	are beam width and depth respectively at $x = x_m$.
$k_{m,\alpha}$	is a factor according to the figure below.



Factor $k_{m,\alpha}$ for strength class GL30c; service class 1, load duration: medium term.



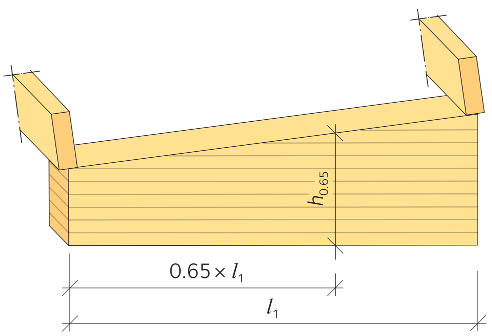


Figure 10.1 Location of cross section for stability check

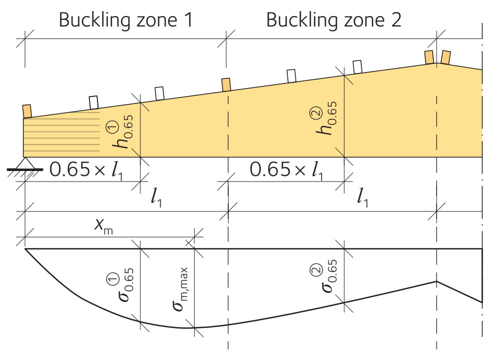
Lateral torsional buckling of beams with variable cross sectional depth

In beams with variable depth, there are two zones where lateral torsional buckling should be checked, namely:

- the zone with the greatest bending stress $\sigma_{m,d}$.
- the zone with the most unfavourable reduction factor for lateral torsional buckling k_{crit}.

According to Colling, F., Holzbau Grundlagen, Bemessungshilfen 2. überarbeitete Auflage, Vieweg+Teubner, Wiesbaden, 2008, the cross section for the stability check of a simply supported beam should be taken at $0.65 \times l_1$ from the lateral restrain closest to the support, where l_1 is the buckling length, *see figure 10.1*.

Table 10.4 Lateral torsional buckling of tapered beams according to Colling, F., Holzbau Grundlagen, Bemessungshilfen 2. überarbeitete Auflage, Vieweg+Teubner, Wiesbaden, 2008



Check for buckling in zone 1

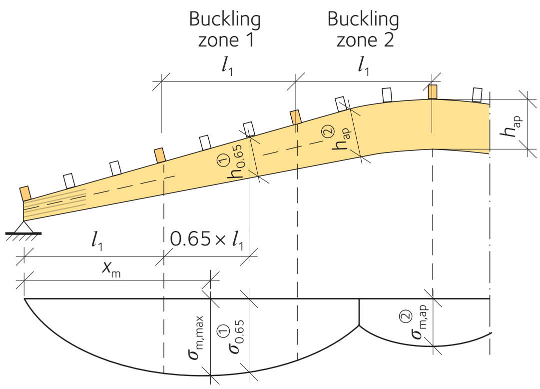
- Calculate $\sigma_{0.65}^{\textcircled{1}}$ (or, more conservatively: $\sigma_{m,max}$).
- Determine the reduction factor for lateral torsional buckling k_{crit}, *see table 9.7, page 34*, assuming:
  - buckling length $l = l_1$ and
  - cross section $b \times h_{0.65}^{\textcircled{1}}$.

Check for buckling in zone 2

- Calculate $\sigma_{0.65}^{\textcircled{2}}$ (= $M_{0.65}^{\textcircled{2}}$ / $W_{0.65}^{\textcircled{2}}$).
- Determine the reduction factor for lateral torsional buckling k_{crit}, *see table 9.7, page 34*, assuming:
  - buckling length $l = l_1$ and
  - cross section $b \times h_{0.65}^{\textcircled{2}}$.

Table 10.5 Lateral torsional buckling of curved beams according to Colling, F., Holzbau Grundlagen, Bemessungshilfen 2. überarbeitete Auflage, Vieweg+Teubner, Wiesbaden, 2008

1. Maximum bending stress $\sigma_{m,max}$ in the linear part of the beam



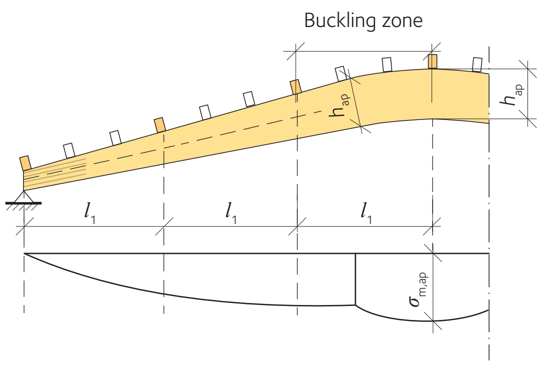
Check for buckling in zone 1

- Calculate $\sigma_{0.65}^{\textcircled{1}}$ (or, more conservatively: $\sigma_{m,max}$).
- Determine the reduction factor for lateral torsional buckling k_{crit}, assuming:
  - buckling length $l = l_1$ and
  - cross section $b \times h_{0.65}^{\textcircled{1}}$.

Check for buckling in zone 2

- Calculate $\sigma_{m,ap,d}$ (= $M_{ap,d}$ / W_{ap}).
- Determine the reduction factor for lateral torsional buckling k_{crit}, assuming:
  - buckling length $l = l_1$ and
  - cross section $b \times h_{ap}$.

2. Maximum bending stress $\sigma_{m,max}$ in the curved part of the beam

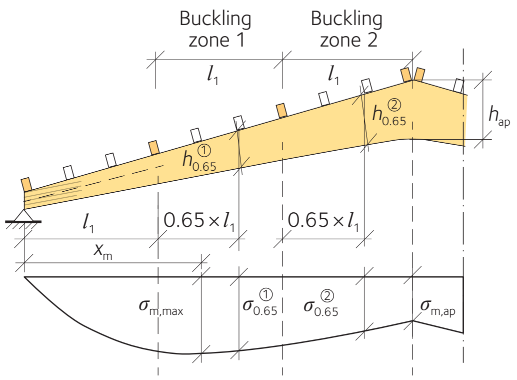


Check for buckling

- Calculate $\sigma_{m,ap,d}$ (= $M_{ap,d}$ / W_{ap}).
- Determine the reduction factor for lateral torsional buckling k_{crit}, assuming:
  - buckling length $l = l_1$ and
  - cross section $b \times h_{ap}$.

Table 10.6 Lateral torsional buckling of pitched cambered beams according to Colling, F., Holzbau Grundlagen, Bemessungshilfen 2. überarbeitete Auflage, Vieweg+Teubner, Wiesbaden, 2008

1. Maximum bending stress $\sigma_{m,max}$ in the linear part of the beam



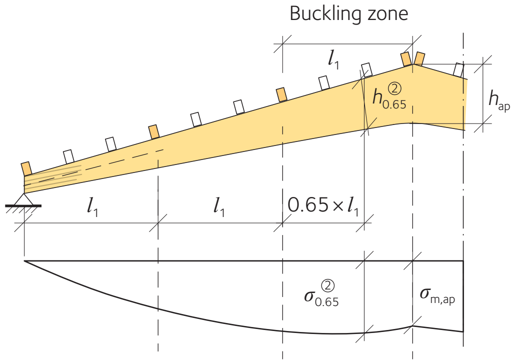
Check for buckling in zone 1

- Calculate $\sigma_{0.65}^{(1)}$ (or, more conservatively: $\sigma_{m,max}$).
- Determine the reduction factor for lateral torsional buckling k_{crit}, assuming:
  - buckling length $l = l_1$ and
  - cross section $b \times h_{0.65}^{(1)}$.

Check for buckling in zone 2

- Calculate $\sigma_{0.65}^{(2)}$ (= $M_{0.65}^{(2)}$ / $W_{0.65}^{(2)}$).
- Determine the reduction factor for lateral torsional buckling k_{crit}, assuming:
  - buckling length $l = l_1$ and
  - cross section $b \times h_{0.65}^{(2)}$ (or, more conservatively: $b \times h_{ap}$).

2. Maximum bending stress $\sigma_{m,max}$ in the curved part of the beam



Check for buckling

- Calculate max $\{\sigma_{m,ap,d}\,;\ \sigma_{0.65}^{(2)}\}$ i.e. max $\{M_{ap,d}/W_{ap\ m,ap,d};\ M_{0.65}^{(2)}/W_{0.65}^{(2)}\}$.
- Determine the reduction factor for lateral torsional buckling k_{crit}, assuming:
  - buckling length $l = l_1$ and
  - cross section $b \times h_{0.65}^{(2)}$ (or, more conservatively: $b \times h_{ap}$).

Table 10.7 Bending stress at the apex according to EN 1995-1-1, section 6.4.3

$$\sigma_{m,d} = k_l \times \frac{6 \times M_d}{b \times h_{ap}^2} \le k_r \times f_{m,d}$$

where:

$\sigma_{m,d}$	is the design bending stress at the apex.
$f_{m,d}$	is the design bending strength.
M_d	is the design bending moment at the apex.
b, h_{ap}	are beam width and depth respectively at the apex.
k_l	is a factor according to *table 10.8, page 47.*
k_r	$\begin{cases} 1 & \text{for } r_{in} \ge 240 \times t \\ 0.76 + \frac{r_{in}}{1000 \times t} & \text{for } r_{in} < 240 \times t \end{cases}$

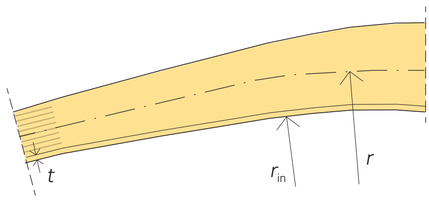


Table 10.8 k_l values for the calculation of the bending stress at the apex according to EN 1995-1-1, section 6.4.3

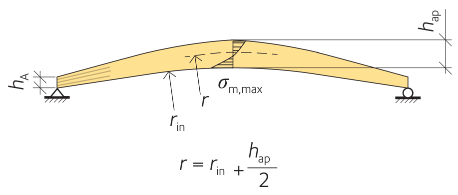
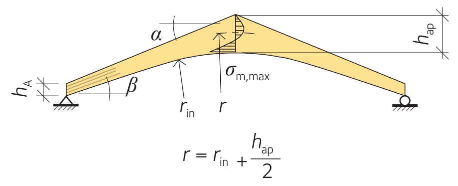
	$k_l = 1 + 0.35 \times \frac{h_{ap}}{r} + 0.6 \times \left(\frac{h_{ap}}{r}\right)^2$
	$k_l = k_1 + k_2 \times \left(\frac{h_{ap}}{r}\right) + k_3 \times \left(\frac{h_{ap}}{r}\right)^2 + k_4 \times \left(\frac{h_{ap}}{r}\right)^3$ $k_1 = 1 + 1.4 \times \tan\alpha + 5.4 \times \tan^2\alpha$ $k_2 = 0.35 - 8 \times \tan\alpha$ $k_3 = 0.6 + 8.3 \times \tan\alpha - 7.8 \times \tan^2\alpha$ $k_4 = 6 \times \tan^2\alpha$
α should be ≤ 15°	

Table 10.9 Tensile stress perpendicular to the grain according to EN 1995-1-1, section 6.4.3

$$\sigma_{t,90,d} = k_p \times \frac{6 \times M_d}{b \times h_{ap}^2} \le k_{dis} \times k_{vol} \times f_{t,90,d}$$

where:

$\sigma_{t,90,d}$	is the design tensile stress perpendicular to the grain at the apex.
$f_{m,d}$	is the design tensile strength perpendicular to the grain at the apex.
M_d	is the design bending moment at the apex.
k_{dis}	*see table 10.11, page 48.*
$k_{vol} = (V_0/V)^{0.2}$	is the ratio between the reference volume V_0 (for glulam $V_0 = 0.01\ m^3$) and the loaded volume at the apex, V in m^3, *see table 10.11, page 48.* V should not be taken greater than 2/3 of the total volume of the beam.
k_p	*see table 10.10.*

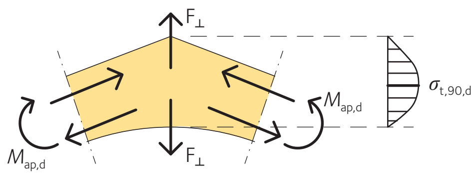


Table 10.10 k_p values for the calculation of tension perpendicular to grain stress at the apex according to EN 1995-1-1, section 6.4.3

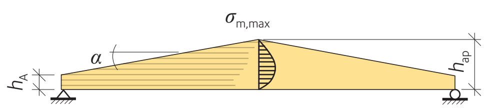
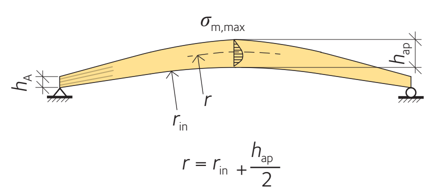
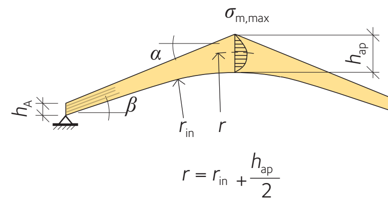
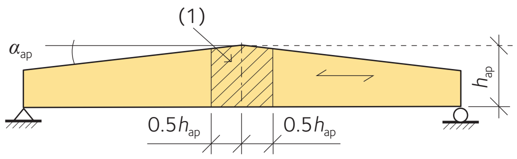
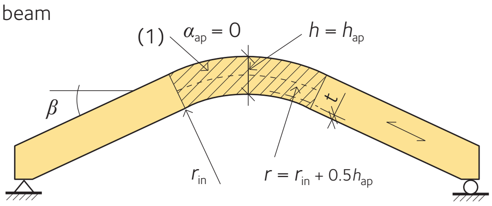
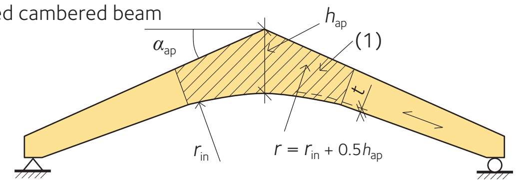
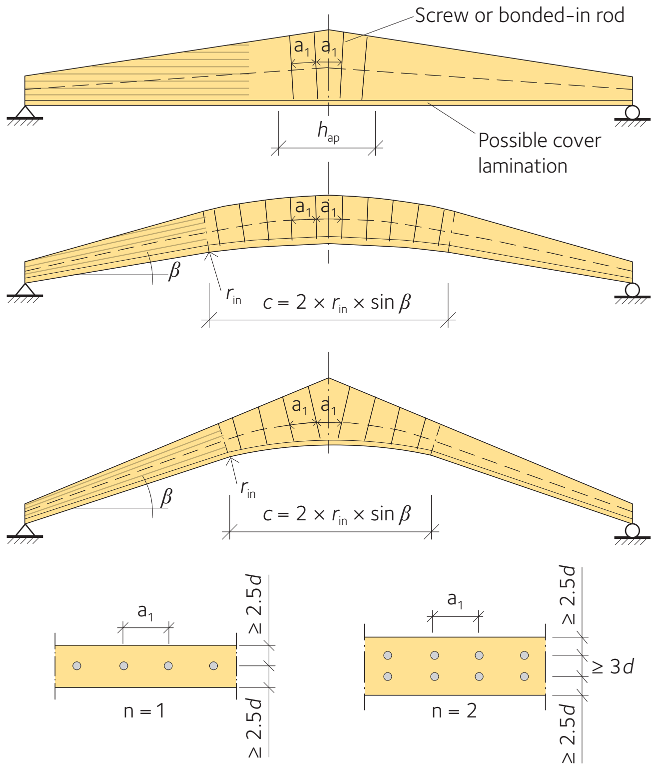
	$k_p = 0.2 \times \tan\alpha$ α should be ≤ 10°
	$k_p = 0.25 \times \frac{h_{ap}}{r}$
	$k_p = k_1 + k_2 \times \left(\frac{h_{ap}}{r}\right) + k_3 \times \left(\frac{h_{ap}}{r}\right)^2$ $k_1 = 0.2 \times \tan\alpha$ $k_2 = 0.25 - 1.5 \times \tan\alpha + 2.6 \times \tan^2\alpha$ $k_3 = 2.1 \times \tan\alpha - 4 \times \tan^2\alpha$

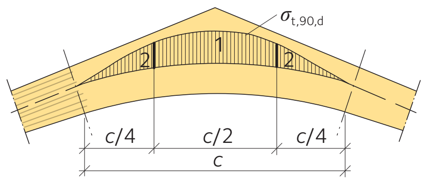
Table 10.11 Values of k_{dis} and V according to EN 1995-1-1, section 6.4.3 for typical beam types ("*b*" denotes the beam breadth)

Beam type	k_{dis}	V
Double tapered beam 	1.4	Volume of (1), see figure to the left $\sim b \times \left(h_{ap}\right)^2$
Curved beam 	1.4	Volume of the curved part (1) [1)] $\frac{\beta\pi}{180} b \left(h_{ap}^2 + 2h_{ap}r_{in}\right)$
Pitched cambered beam 	1.7	Volume of the curved part (1) [1)] $b\left(\sin\left(\alpha_{ap}\right)\cos\left(\alpha_{ap}\right)\left(r_{in}+h_{ap}\right)^2 - r_{in}^2\frac{\alpha_{ap}\pi}{180}\right)$

1) V needs, however, not be taken as more than 2/3 V_b, where V_b is the total volume of the beam.
Angles α and β in degrees. "*b*" is the width of the beam.

Table 10.12 Placement of the reinforcing screws or bonded-in rods according to DIN EN 1995-1-1/NA



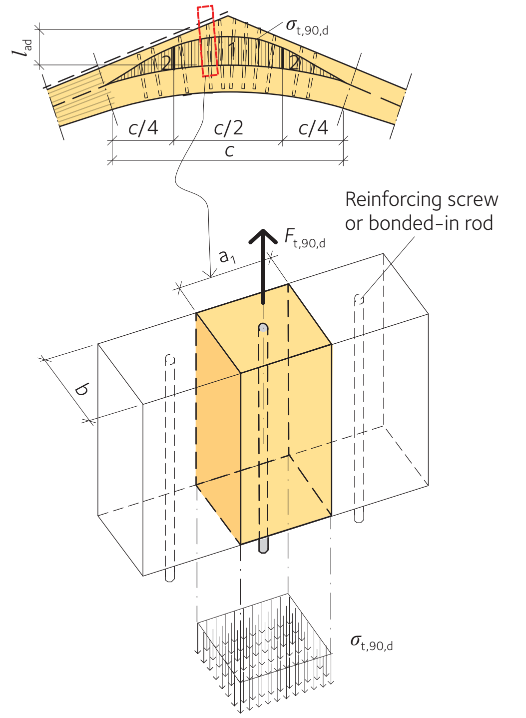


Zone 1: Denser distribution of screws/bars.
Zone 2: Sparser distribution of screws/bars.

a_1 is the spacing of reinforcement along the beam.
Recommended spacing: 250 mm ≤ a_1 ≤ 0.75 × h_{ap}, where h_{ap} is the depth of the beam at the apex.

Table 10.13 Design of the reinforcement against perpendicular to the grain tensile stress

Design tensile force perpendicular to the grain in the central part of the apex area: $F_{t,90,d} = \frac{\sigma_{t,90,d} \times b \times a_1}{n}$	
Design tensile force perpendicular to the grain in the outer quarters of the apex: $F_{t,90,d} = \frac{2}{3} \times \frac{\sigma_{t,90,d} \times b \times a_1}{n}$	
Check:	$F_{t,90,d} \leq R_{t,d}$
where:	
$\sigma_{t,90,d}$	is the design tensile stress perpendicular to grain.
b	is the beam width.
a_1	is the spacing of reinforcement along the beam. Recommended spacing: 250 mm ≤ a_1 ≤ 0.75 × h_{ap}, where h_{ap} is the depth of the beam at the apex, *see table 10.12, page 48*.
n	is the number of groups of reinforcement bars in the direction perpendicular to the longitudinal axis of the beam, *see table 10.12, page 48*.
R_{td}	is the design axial strength of the reinforcing screw/bonded-in bar, i.e. the minimum value between tensile and withdrawal strength. The bonding length in case of bonded-in rods or the penetration length in case of screws shall be taken equal to l_{ad}, see figure to the right.



11 Design for SLS

General

Calculation of deflections is usually based on mean values of stiffness properties. Time dependence may be considered by defining a final effective modulus of elasticity as

$$E_{\text{mean,fin}} = \frac{E_{\text{mean}}}{1 + k_{\text{def}}}$$

where k_{def} accounts for moisture effects on deformation according to *table 11.1*.

Table 11.1 Values of k_{def} for timber and wood-based materials according to EN 1995-1-1

Material	EN standard	Service class		
		1	2	3
Solid timber	EN 14081-1	0.60	0.80	2.00
Glulam	EN 14080	0.60	0.80	2.00
LVL	EN 14374	0.60	0.80	2.00
	EN 14279	0.60	0.80	2.00
Plywood	EN 636	0.80	1.00	2.50
OSB	EN 300			
	OSB/2	2.25	–	–
	OSB/3	1.50	2.25	–
	OSB/4	1.50	2.25	–

Table 11.2 Slip modulus K_{ser} per shear plane and per fastener in timber-to-timber and wood panel-to-timber connections
ρ_{m} is the mean density of the timber, in [kg/m³] and *d* is the fastener diameter, in [mm]. If the mean densities $\rho_{\text{m,1}}$ and $\rho_{\text{m,2}}$ of two jointed members are different, for ρ_{m} use a value that is $\rho_{\text{m}} = \sqrt{\rho_{\text{m,1}} \times \rho_{\text{m,2}}}$ where $\rho_{\text{m,1}}$ and $\times\, \rho_{\text{m,2}}$ are taken from *table 7.4* or *table 7.5, page 20*.

Fastener type	K_{ser} [N/mm]
Dowels	$\rho_{\text{m}}^{1.5} d / 23$
Bolts with or without clearance [1]	
Screws	
Nails (with predrilling)	
Nails (without pre-drilling)	$\rho_{\text{m}}^{1.5} d^{0.8} / 30$

[1] The clearance should be added separately to the slip of the fastener.

Deflections

Table 11.3 Calculation of deflections. The relevant load combinations can be taken from *table 5.3, page 11.* The values of k_{def} can be taken from *table 11.1, page 50.* The values of ψ_2 can be taken from *table 5.4, page 11.*

w_{inst}	Instantaneous deflection.
w_{creep}	Deflection due to creep: $w_{creep,p} = k_{def} \times w_{inst}$ due to permanent load. $w_{creep,v} = \psi_2 \times k_{def} \times w_{inst}$ due to variable load.
w_c	Possible precamber.
w_{fin}	Final deflection: $w_{fin} = w_{inst} + \Sigma\, w_{creep,i}$
$w_{net,fin}$	Total net deflection: $w_{net,fin} = w_{fin} - w_c$

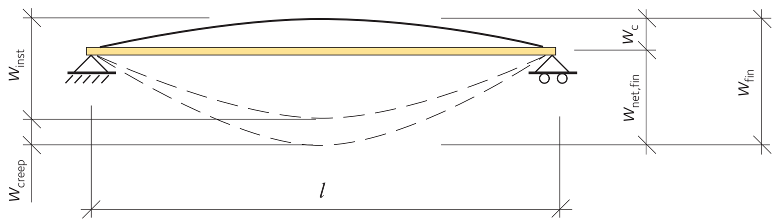


Table 11.4 Normally accepted limits for deformations in relation to the free span in the serviceability limit state
The table values are based on tried and tested methods and good engineering practice and have been converted to values in line with EN 1990, EN 1991, EN 1995 and the current EKS. They should be seen as industry recommendations to guide developers and their agents, as well as a basis for evaluating competing alternatives.

Application	Non-cambered structural elements		
	$u_{max,inst}$	$u_{max,frekv}$	$u_{max,fin}$
Roof beams			
Manufacturing	*L*/300	*L*/300	*L*/250
Schools, shops, etc.	*L*/375	*L*/375	*L*/300
Animal sheds	–	*L*/200	*L*/200 (maximum 30 mm)
Machine halls, barns, etc.	–	*L*/150	*L*/150 (maximum 40 mm)
Floor beams			
General [1)]	*L*/500	*L*/375	*L*/300
Storerooms and other premises with no public access	*L*/275	*L*/250	*L*/200
Animal sheds	–	*L*/200	*L*/200 (maximum 30 mm)
Barns, etc.	–	*L*/150	*L*/150 (maximum 40 mm)
Trusses			
Generally, without consideration of node deformations	*L*/625	*L*/500	*L*/400
In agricultural buildings, without considering nodal deformations	–	*L*/400	–
Cantilevers	*L*/250	*L*/250	*L*/200
Roof ridges			
Generally, without a separate ceiling	*L*/375	*L*/375	*L*/300
In agricultural buildings, without a separate ceiling	–	*L*/200	–
Generally, with a separate ceiling	*L*/200	*L*/200	*L*/150
In agricultural buildings with a separate ceiling	–	*L*/100	–

1) The stiffness of wooden floors should also be checked regarding sagging and vibrations.

L denotes the free span. For structural elements with camber, the table value/1.5 applies.
$u_{max,inst}$ is calculated according to EN 1990 (equation 6.14a), characteristic load combination and EN 1995 (equation 2.2.3 (2)).
$u_{max,fin}$ is calculated according to EN 1990 (equation 6.16a), quasi-permanent load combination and EN 1995 (equation 2.2.3 (3) and (5)).
There are no instructions for the frequent load combination according to EN 1990 (equation 6.15a) in EN 1995. The frequent load combination $u_{max,frekv}$ is calculated according to *equation 6.8, page 86.*

The table must also be supplemented with the constraints imposed by the conditions of the construction project in question. For example, the outer layer of the roof may require a maximum deformation of 30 mm for the characteristic load, to avoid damage where a shallow pitch causes a risk of standing water freezing into ice. There is also a risk of damage to ceramic floors and stone tiles, where a stiffness of at least around *L*/300 is reasonable for the characteristic load. Lintels over doors and windows are examples of instances where absolute measures of deformation must not exceed the space available. Floors must also not place a load on non-load-bearing internal walls. Glass roofs are highly sensitive to vertical and horizontal movements. The values for agricultural buildings were chosen in accordance with SIS-TS 37:2012.

12 Carpentry joints

Table 12.1 Strength of the joint according to the Swiss norm for timber structures SIA 265:2003

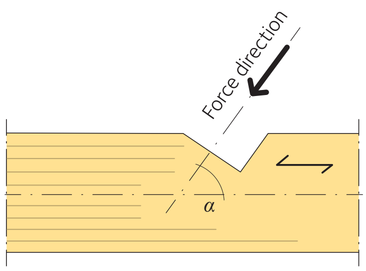
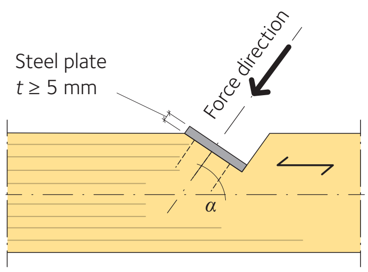
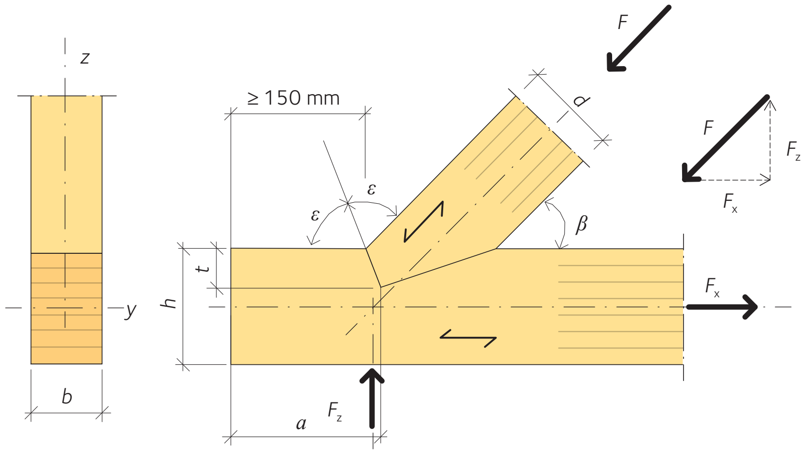
<table>
<tr><th>Without steel plate at the interface between diagonal member and horizontal member</th><th>With steel plate at the interface between diagonal member and horizontal member</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td colspan="2">Compression strength at an angle α to the grain</td></tr>
<tr><td>$f_{c,\alpha,d} \leq \dfrac{0.8 \times f_{c,0,d} \times f_{c,90,d}}{0.8 \times f_{c,0,d} \times \sin^2 \alpha + f_{c,90,d} \times \cos^2 \alpha}$</td><td>$f_{c,\alpha,d} \leq \dfrac{f_{c,0,d} \times f_{c,90,d}}{f_{c,0,d} \times \sin^2 \alpha + f_{c,90,d} \times \cos^2 \alpha}$</td></tr>
</table>

Table 12.2 Design of carpentry joints according to SIA 265:2003 with some modifications by the authors of *The Glulam Handbook Volume 3*



	Depth of the notch *t*	Length of chord toe *a*	Depth of the diagonal member *d*
Minimum values	$t \geq \dfrac{F_d \times \cos \beta}{b \times f_{c,\alpha,d}}$ [1]	$a \geq \dfrac{F_d \times \cos \beta}{b \times 0.5 \times f_{v,d}}$	$d \geq \dfrac{F_d}{b \times f_{c,\alpha,d}}$ [2]
Maximum values	$t \leq h/4$ for $\beta \leq 50°$ $t \leq h/6$ for $\beta \geq 60°$ [3]	$a \leq 8 \times t$ [4]	–

[1] For the determination of $f_{c,\alpha,d}$ use $\alpha = 0.5 \times \beta$.
[2] For the determination of $f_{c,\alpha,d}$, use $\alpha = \beta$.
[3] For $50° < \beta < 60°$ linear interpolation of t can be used.
[4] A larger chord toe length (a) than that prescribed in the table can be adopted.
When calculating the load-carrying capacity, use the maximum shear length $a = 8 \times t$.

13 Connections with metal fasteners

The rules for the design of connections with metal fasteners treated in this section are basically those of the EC5 approach. However, it is always recommendable to design connections with ductile properties. In *tables 13.3 – 13.11, pages 56 – 64*, therefore, the suggested material thicknesses for a given fastener diameter with corresponding load bearing capacities relates only to ductile failure modes of the connection and excludes therefore brittle failure modes. In particular, the presented approach aims at ensuring the formation of one or two plastic hinges in the fasteners which correspond to failure modes f and k for timber-to-timber connections and failure modes b, e, h and m for steel-to-timber connections, *see figure 13.1*. These failure modes can be ensured by requiring either sufficient thicknesses of the timber parts or smaller fastener diameter.

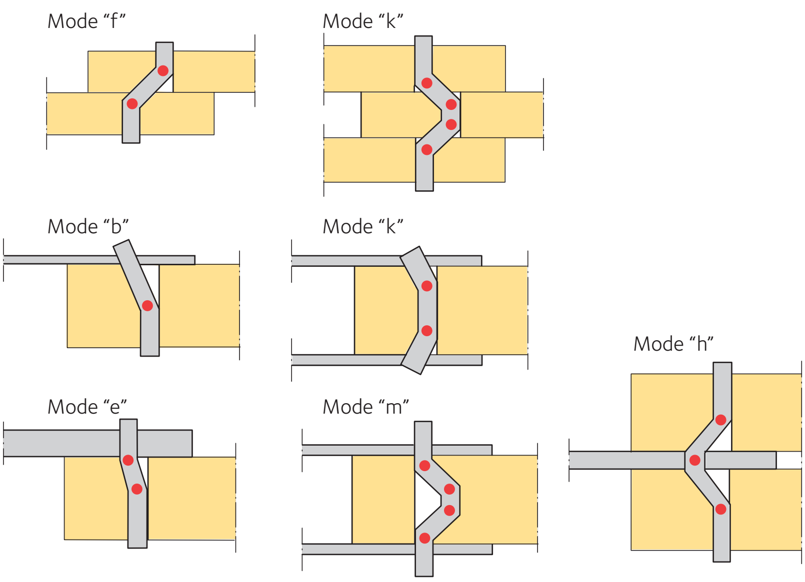


Figure 13.1 Ductile failure of connections

If thinner timber parts or larger fastener diameter than those suggested in *tables 13.3 – 13.11, pages 56 – 64* are adopted, the failure mode will become brittle and the equations of EN 1995-1-1, section 8.2 should be used for the determination of the load bearing capacity of the connection. However, for a quick (and conservative) estimation of the load bearing capacity of a connection in case of brittle failure modes, the approach given in DIN EN 1995 1-1/NA:2010-12 can be used. According to this approach, the load-carrying capacity related to the ductile mode shall be decreased linearly with the thickness of the timber member, giving capacities slightly smaller than EN 1995-1-1, EC5. The discrepancies between the DIN and EN 1995-1-1, EC5, approaches are illustrated in *figure 13.2, page 54*, for a steel-to-timber connection with a thick steel plate in single shear.

Parameters used in design of timber connections

$F_{v,Rk}$	is the characteristic capacity per shear plane, per fastener.
$f_{h,i,k}$	is the characteristic embedment strength of member "i".
t_i	is the thickness of the timber member (or penetration depth for nails and screws).
d	is the diameter of the fastener.
$M_{y,Rk}$	is the characteristic yield moment in the fastener.
$\beta = f_{h,2,k} / f_{h,1,k}$	is the ratio between embedment strengths of members "1" and "2". For single shear timber-to-timber connections the designation "1" or "2" can be given arbitrarily. However, for double shear timber-to-timber connections designation "2" shall always be given to the member in the middle.

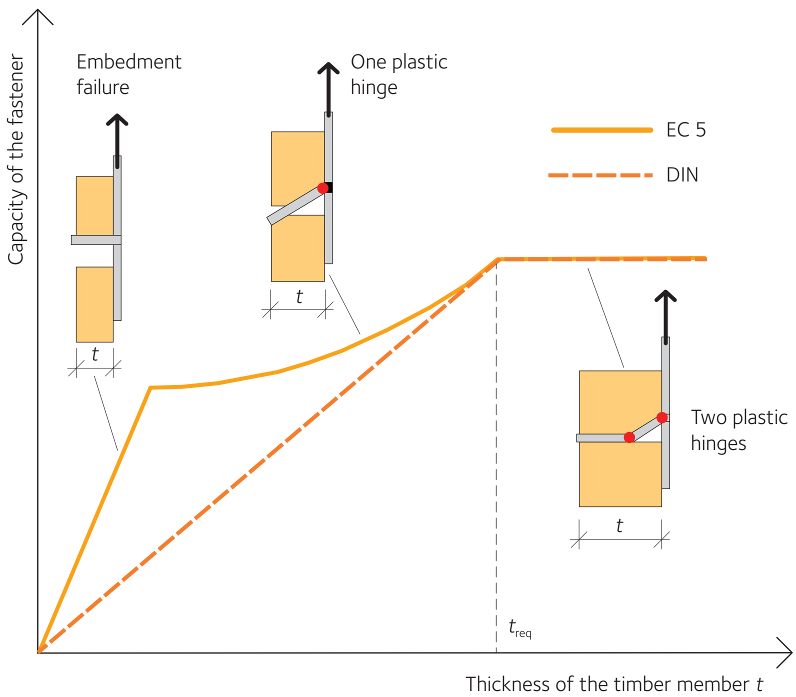
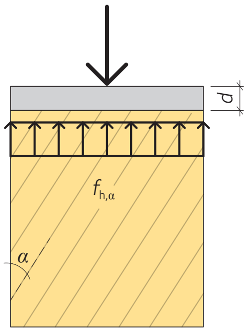


Figure 13.2 **Steel-to-timber connection with a thick steel plate in single shear – discrepancies between EN 1995-1-1 approach and DIN approach**

Table 13.1 **Characteristic embedment strength for glulam in strength class GL30c (ρ_k = 390 kg/m³)**

Embedment strength



Fasteners with predrilled holes:

$$f_{h,\alpha,k} = \frac{0.082 \times (1 - 0.01 \times d) \times \rho_k}{(1.35 + 0.015 \times d) \times \sin^2 \alpha + \cos^2 \alpha}$$

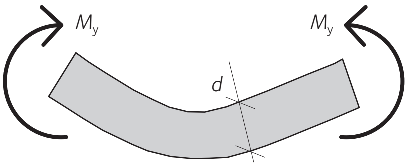
Nails with non-predrilled holes ($d \leq 6$ mm):

$$f_{h,k} = 0.082 \times d^{-0.3} \times \rho_k$$

Fastener diameter *d* [mm]			4	8	10	12	16	20	24	30
$f_{h,\alpha,k}$ [MPa] for strength class GL30c	Angle to the grain	0°	21.1	29.4	28.8	28.1	26.9	25.6	24.3	22.4
		20°	21.1	27.9	27.2	26.5	25.1	23.8	22.4	20.5
		40°	21.1	24.6	23.9	23.1	21.6	20.2	18.8	16.8
		60°	21.1	21.8	20.9	20.1	18.6	17.2	15.9	14.0
		80°	21.1	20.2	19.4	18.6	17.1	15.7	14.4	12.6
		90°	21.1	20.0	19.2	18.4	16.9	15.5	14.2	12.4

Table 13.2 **Characteristic yield moment for bolts and dowels of some typical steel grades**

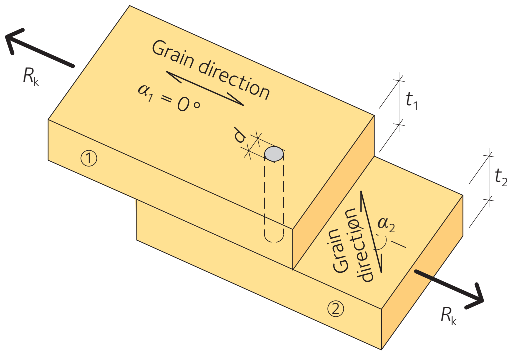
Yield moment



$$M_{y,k} = 0.3 \times f_{u,k} \times d^{2.6}$$

Fastener diameter *d* [mm]			4	8	10	12	16	20	24	30
$M_{y,k}$ [Nmm]	Strength class/ steel grade for bolt/ nail	S355	-	34,098	60,910	**97,850**	206,730	369,292	593,254	1,059 758
		4.6/4.8	-	26,743	47,773	**76,745**	162,141	289,640	465,297	831,183
		5.6/5.8	-	33,429	59,716	**95,932**	202,676	362,051	581,622	1,038 978
		f_u = 600	**6,617**	-	-	-	-	-	-	-
		8.8	-	53,487	95,546	**153,491**	324,282	579,281	930,594	1,662 365

Table 13.3 **Single shear timber-to-timber connection. Minimum required thickness of the timber for the formation of two plastic hinges in the fastener and corresponding shear capacity of the connection.** Timber parts: Glulam in strength class GL30c. Fastener: steel dowel or bolt of structural steel grade S355. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod} /γ_M), where γ_M = 1.3.



$$R_k = 1.15 \times \sqrt{\frac{2 \times \beta}{1+\beta}} \times \sqrt{2 \times M_{y,k} \times f_{h,1,k} \times d} \quad ^{1)}$$

$$t_1 \geq t_{1,req} = 1.15 \times \left(2 \times \sqrt{\frac{\beta}{1+\beta}} + 2\right) \times \sqrt{\frac{M_{y,k}}{f_{h,1,k} \times d}}$$

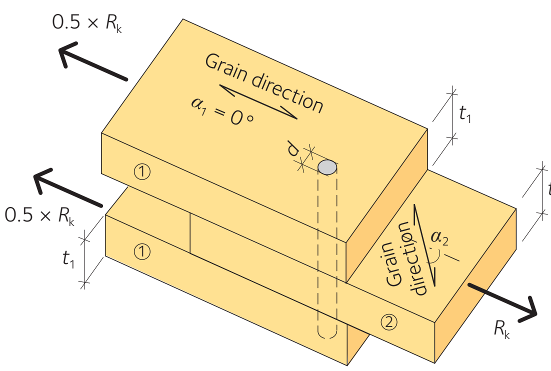
$$t_2 \geq t_{2,req} = 1.15 \times \left(2 \times \sqrt{\frac{1}{1+\beta}} + 2\right) \times \sqrt{\frac{M_{y,k}}{f_{h,2,k} \times d}}$$

d = 8 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	47 mm	0°	47 mm	0°	46 mm	0°	45 mm
	0°	47 mm	30°	51 mm	60°	57 mm	90°	59 mm
	R_k = 4.6 kN		**R_k = 4.5 kN**		**R_k = 4.2 kN**		**R_k = 4.1 kN**	
d = 10 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	57 mm	0°	56 mm	0°	55 mm	0°	55 mm
	0°	57 mm	30°	61 mm	60°	69 mm	90°	73 mm
	R_k = 6.8 kN		**R_k = 6.6 kN**		**R_k = 6.2 kN**		**R_k = 6.1 kN**	
d = 12 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	67 mm	0°	65 mm	0°	64 mm	0°	63 mm
	0°	67 mm	30°	67 mm	60°	72 mm	90°	86 mm
	R_k = 9.3 kN		**R_k = 9.1 kN**		**R_k = 8.6 kN**		**R_k = 8.3 kN**	
d = 16 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	86 mm	0°	84 mm	0°	83 mm	0°	82 mm
	0°	86 mm	30°	97 mm	60°	107 mm	90°	113 mm
	R_k = 15.3 kN		**R_k = 14.5 kN**		**R_k = 13.9 kN**		**R_k = 13.5 kN**	
d = 20 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	105 mm	0°	105 mm	0°	101 mm	0°	100 mm
	0°	105 mm	30°	111 mm	60°	134 mm	90°	142 mm
	R_k = 22.4 kN		**R_k = 21.9 kN**		**R_k = 20.0 kN**		**R_k = 19.4 kN**	
d = 24 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	125 mm	0°	123 mm	0°	119 mm	0°	118 mm
	0°	125 mm	30°	138 mm	60°	161 mm	90°	172 mm
	R_k = 30.3 kN		**R_k = 29 kN**		**R_k = 26.9 kN**		**R_k = 26.0 kN**	

1) If $t_1 < t_{1,req}$ or $t_2 < t_{2,req}$, the load bearing capacity can be calculated as:

$$R_{k,red} = R_k \times \min\left\{\frac{t_1}{t_{1,req}}; \frac{t_2}{t_{2,req}}\right\}$$

Table 13.4 Double shear timber-to-timber connection. Minimum required thickness of the timber for the formation of two plastic hinges in the fastener and corresponding shear capacity of the connection. Timber parts: Glulam in strength class GL30c. Fastener: steel dowel or bolt of structural steel grade S355. Member "2" is the central member. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod}/γ_M), where $\gamma_M = 1.3$.



$$R_k = 2 \times \left(1.15 \times \sqrt{\frac{2 \times \beta}{1+\beta}} \times \sqrt{2 \times M_{y,k} \times f_{h,1,k} \times d} \right)^{1)}$$

$$t_1 \geq t_{1,req} = 1.15 \times \left(2 \times \sqrt{\frac{\beta}{1+\beta}} + 2 \right) \times \sqrt{\frac{M_{y,k}}{f_{h,1,k} \times d}}$$

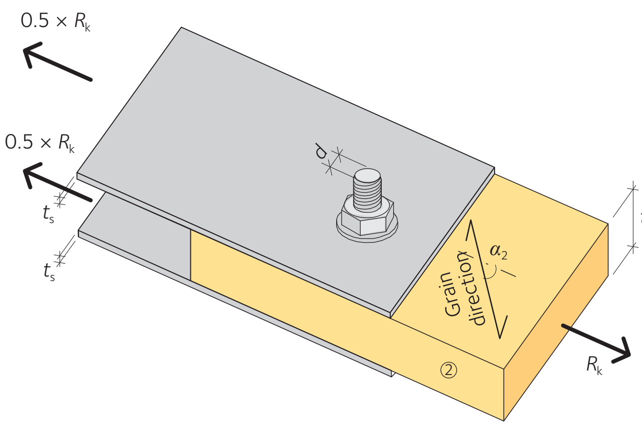
$$t_2 \geq t_{2,req} = 1.15 \times \left(4 \times \sqrt{\frac{1}{1+\beta}} \right) \times \sqrt{\frac{M_{y,k}}{f_{h,2,k} \times d}}$$

d = 8 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	47 mm	0°	47 mm	0°	46 mm	0°	45 mm
	0°	39 mm	30°	43 mm	60°	49 mm	90°	52 mm
	R_k = 9.2 kN		**R_k = 9.0 kN**		**R_k = 8.5 kN**		**R_k = 8.3 kN**	
d = 10 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	57 mm	0°	56 mm	0°	55 mm	0°	55 mm
	0°	47 mm	30°	52 mm	60°	60 mm	90°	63 mm
	R_k = 13.6 kN		**R_k = 13.2 kN**		**R_k = 12.5 kN**		**R_k = 12.2 kN**	
d = 12 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	67 mm	0°	65 mm	0°	64 mm	0°	63 mm
	0°	55 mm	30°	61 mm	60°	71 mm	90°	76 mm
	R_k = 18.7 kN		**R_k = 18.2 kN**		**R_k = 17.2kN**		**R_k = 16.7 kN**	
d = 16 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	86 mm	0°	84 mm	0°	83 mm	0°	82 mm
	0°	71 mm	30°	83 mm	60°	93 mm	90°	100 mm
	R_k = 30.7 kN		**R_k = 29.1 kN**		**R_k = 27.7 kN**		**R_k = 26.9 kN**	
d = 20 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	105 mm	0°	105 mm	0°	101 mm	0°	100 mm
	0°	87 mm	30°	93 mm	60°	117 mm	90°	125 mm
	R_k = 44.7 kN		**R_k = 43.7 kN**		**R_k = 40.1 kN**		**R_k = 38.8 kN**	
d = 24 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	125 mm	0°	123 mm	0°	119 mm	0°	118 mm
	0°	104 mm	30°	117 mm	60°	141 mm	90°	152 mm
	R_k = 60.5 kN		**R_k = 58.0 kN**		**R_k = 53.8 kN**		**R_k = 52.0 kN**	

1) If $t_1 < t_{1,req}$ or $t_2 < t_{2,req}$, the load bearing capacity can be calculated as:

$$R_{k,red} = R_k \times \min \left\{ \frac{t_1}{t_{1,req}}; \frac{t_2}{t_{2,req}} \right\}$$

Table 13.5 Double shear steel-to-timber connection, with thin steel plates. Minimum required thickness of the timber for the formation of one plastic hinge in the fastener and corresponding shear capacity of the connection. Timber parts: Glulam in strength class GL30c. Fastener: steel bolt, strength class 4.8. If the thickness of the plates is greater than 0.4 × d, the capacity of the connection may be calculated taking into account the rope effect, $R_{k,tot} = R_k + \Delta R_k$. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod} / γ_M), where $\gamma_M = 1.3$.



$$R_k = 2 \times 1.15 \times \sqrt{2 \times M_{y,k} \times f_{h,2,k} \times d} \quad ^{1)}$$

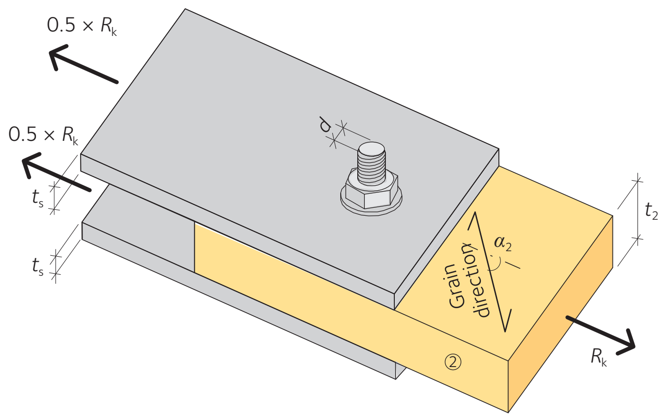
$$0.4 \times d < t_s \leq 0.5 \times d$$

$$t_2 \geq t_{2,req} = 1.15 \times \left(2 \times \sqrt{2}\right) \times \sqrt{\frac{M_{y,k}}{f_{h,2,k} \times d}}$$

d = 8 mm ($\Delta R_k = F_{ax}/4$ = 3.0 kN)	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	35 mm	30°	37 mm	60°	40 mm	90°	42 mm
	R_k = 8.2 kN		**R_k = 7.7 kN**		**R_k = 7.0 kN**		**R_k = 6.7 kN**	
d = 10 mm ($\Delta R_k = F_{ax}/4$ = 4.7 kN)	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	42 mm	30°	44 mm	60°	49 mm	90°	51 mm
	R_k = 12.1 kN		**R_k = 11.4 kN**		**R_k = 10.3 kN**		**R_k = 9.8 kN**	
d = 12 mm (/4 = 6.9 kN)	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	49 mm	30°	52 mm	60°	58 mm	90°	61 mm
	R_k = 16.6 kN		**R_k = 15.6 kN**		**R_k = 14.0 kN**		**R_k = 13.4 kN**	
d = 16 mm ($\Delta R_k = F_{ax}/4$ = 12.8 kN)	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	63 mm	30°	70 mm	60°	76 mm	90°	80 mm
	R_k = 27.2 kN		**R_k = 24.6 kN**		**R_k = 22.6 kN**		**R_k = 21.5 kN**	
d = 20 mm ($\Delta R_k = F_{ax}/4$ = 20.0 kN)	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	77 mm	30°	81 mm	60°	94 mm	90°	99 mm
	R_k = 39.6 kN		**R_k = 37.9 kN**		**R_k = 32.5 kN**		**R_k = 30.8kN**	
d = 24 mm ($\Delta R_k = F_{ax}/4$= 28.8 kN)	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	92 mm	30°	100 mm	60°	114 mm	90°	120 mm
	R_k = 53.6 kN		**R_k = 49.4 kN**		**R_k = 43.3 kN**		**R_k = 41 kN**	
d = 30 mm ($\Delta R_k = F_{ax}/4$= 45.9 kN)	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
	0°	114 mm	30°	125 mm	60°	145 mm	90°	154 mm
	R_k = 76.8 kN		**R_k = 70.2 kN**		**R_k = 60.8 kN**		**R_k = 57.3 kN**	

1) If $t_2 < t_{2,req}$, the load bearing capacity can be calculated as: $R_{k,red} = R_k \times t_2 / t_{2,req}$.

Table 13.6 Double shear steel-to-timber connection, with thick steel plates. Minimum required thickness of the timber for the formation of three plastic hinges in the fastener and corresponding shear capacity of the connection. Timber parts: Glulam in strength class GL30c. Fastener: steel bolt, strength class 4.8. If the thickness of the plates is greater than 0.4 × d, the capacity of the connection may be calculated taking into account the rope effect, $R_{k,tot} = R_k + \Delta R_k$. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod}/γ_M), where γ_M = 1.3.



$$R_k = 2 \times 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,2,k} \times d} \quad ^{1)}$$

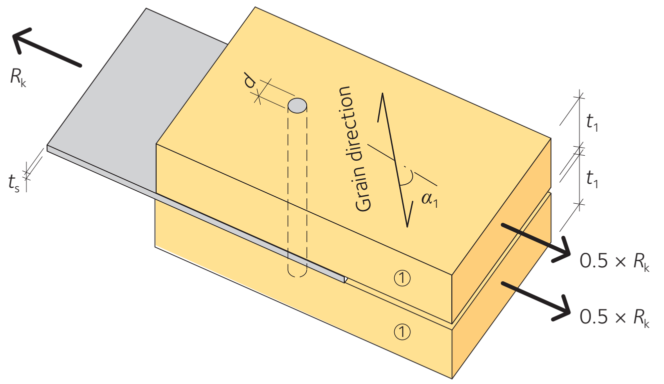
$$t_s \geq d$$

$$t_2 \geq t_{2,req} = 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,2,k} \times d}}$$

	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$	α_2	$t_{2,req}$
d = 8 mm ($\Delta R_k = F_{ax}/4$ = 3.0 kN)	0°	49 mm	30°	52 mm	60°	57 mm	90°	59 mm
	R_k = 11.5 kN		**R_k = 10.9 kN**		**R_k = 9.9 kN**		**R_k = 9.5 kN**	
d = 10 mm ($\Delta R_k = F_{ax}/4$ = 4.7 kN)	0°	59 mm	30°	63 mm	60°	69 mm	90°	73 mm
	R_k = 17.1 kN		**R_k = 16.1 kN**		**R_k = 14.5 kN**		**R_k = 13.9 kN**	
d = 12 mm (/4 = 6.9 kN)	0°	69 mm	30°	74 mm	60°	82 mm	90°	86 mm
	R_k = 23.4 kN		**R_k = 22.0 kN**		**R_k = 19.8 kN**		**R_k = 18.9 kN**	
d = 16 mm ($\Delta R_k = F_{ax}/4$ = 12.8 kN)	0°	89 mm	30°	99 mm	60°	107 mm	90°	113 mm
	R_k = 38.4 kN		**R_k = 34.8 kN**		**R_k = 32.0 kN**		**R_k = 30.5 kN**	
d = 20 mm ($\Delta R_k = F_{ax}/4$ = 20.0 kN)	0°	109 mm	30°	114 mm	60°	133 mm	90°	141 mm
	R_k = 56.0 kN		**R_k = 53.6 kN**		**R_k = 45.9 kN**		**R_k = 43.6 kN**	
d = 24 mm ($\Delta R_k = F_{ax}/4$= 28.8 kN)	0°	130 mm	30°	141 mm	60°	161 mm	90°	170 mm
	R_k = 75.8 kN		**R_k = 69.8 kN**		**R_k = 61.2 kN**		**R_k = 58.0 kN**	
d = 30 mm ($\Delta R_k = F_{ax}/4$= 45.9 kN)	0°	162 mm	30°	177 mm	60°	205 mm	90°	217 mm
	R_k = 108.7 kN		**R_k = 99.2 kN**		**R_k = 85.9 kN**		**R_k = 81.0 kN**	

1) If $t_2 < t_{2,req}$, the load bearing capacity can be calculated as: $R_{k,red} = R_k \times t_2 / t_{2,req}$.

Table 13.7 Steel-to-timber connection with one slotted-in steel plate. Minimum required thickness of the timber for the formation of three plastic hinges in the fastener and corresponding shear capacity of the connection. Timber parts: Glulam in strength class GL30c. Fastener: dowel of structural steel grade S355. The thickness of the steel plate is assumed equal to d/1.5 where d is the diameter of the fastener. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod} /γ_M), where γ_M = 1.3.



$$R_k = 2 \times 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,1,k} \times d} \quad ^{1)}$$

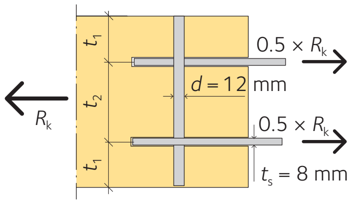
$$t_s \geq 0.5 \times d$$

$$t_1 \geq t_{1,req} = 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,1,k} \times d}} - 0.5 \times t_s$$

d = 8 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	0°	57 mm	30°	60 mm	60°	66 mm	90°	69 mm
	R_k = 12.2 kN		**R_k = 11.5 kN**		**R_k = 10.5 kN**		**R_k = 10 kN**	
d = 10 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	0°	68 mm	30°	73 mm	60°	81 mm	90°	84 mm
	R_k = 18.0 kN		R_k = 17.0 kN		R_k = 15.3 kN		R_k = 14.7 kN	
d = 12 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	0°	80 mm	30°	85 mm	60°	95 mm	90°	100 mm
	R_k = 24.7kN		**R_k = 23.2 kN**		**R_k = 20.9 kN**		**R_k = 20.0 kN**	
d = 16 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	0°	103 mm	30°	110 mm	60°	124 mm	90°	131 mm
	R_k = 40.5 kN		**R_k = 37.8 kN**		**R_k = 33.7 kN**		**R_k = 32.1 kN**	
d = 20 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	0°	126 mm	30°	136 mm	60°	155 mm	90°	163 mm
	R_k = 59.0 kN		**R_k = 54.8 kN**		**R_k = 48.4 kN**		**R_k = 46.0 kN**	
d = 24 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	0°	149 mm	30°	162 mm	60°	186 mm	90°	197 mm
	R_k = 79.9 kN		**R_k = 73.6 kN**		**R_k = 64.5 kN**		**R_k = 61.1 kN**	
d = 30 mm	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$	α_1	$t_{1,req}$
	0°	186 mm	30°	190 mm	60°	221 mm	90°	235 mm
	R_k = 114.6 kN		**R_k = 104.6 kN**		**R_k = 90.6 kN**		**R_k = 85.4 kN**	

1) If $t_1 < t_{1,req}$, the load bearing capacity can be calculated as: $R_{k,red} = R_k \times t_1/t_{1,req}$.

Table 13.8 Steel-to-timber connection with two slotted-in steel plates. The thickness of each timber central member is the minimum required for the formation of three plastic hinges in the fastener. The thickness of each outer timber member is the minimum required for the formation of one plastic hinge in the fastener, at each outer member. The thickness of the steel plates is t_s = 8 mm. The width of the slots is t_{sl} = 10 mm. The diameter of the dowel is d = 12 mm, structural steel grade S355. Timber material: Glulam in strength class GL30c. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod} /γ_M), where γ_M = 1.3.



$$R_k = R_{k,centre} + R_{k,lateral}$$

$$R_{k,centre} = 2 \times 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,k} \times d}$$

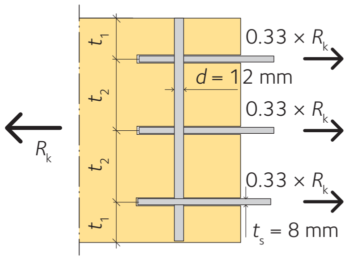
$$R_{k,lateral} = 2 \times \begin{cases} f_{h,k} \times d \times t_1 \left(\sqrt{2 + \dfrac{4 \times M_{y,k}}{f_{h,k} \times d \times t_1^2}} - 1 \right) & \text{if} \quad \sqrt{2} \times \sqrt{\dfrac{M_{y,k}}{f_{h,k} \times d}} < t_1 \leq 1.15 \times 4 \times \sqrt{\dfrac{M_{y,k}}{f_{h,k} \times d}} \\ 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,k} \times d} & \text{if} \quad t_1 \geq 1.15 \times 4 \times \sqrt{\dfrac{M_{y,k}}{f_{h,k} \times d}} \end{cases}$$

$$t_1 > \sqrt{2} \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}}$$

$$t_2 \geq t_{2,req} = 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}}$$

α = 0°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	29 mm	88 mm	146 mm	≥ 83 mm	88 mm	≥ 255 mm
	R_k = 43 kN			**R_k = 53 kN**		
α = 30°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	31 mm	93 mm	154 mm	≥ 88 mm	93 mm	≥ 270 mm
	R_k = 40 kN			**R_k = 50 kN**		
α = 60°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	33 mm	103 mm	170 mm	≥ 98 mm	103 mm	≥ 298 mm
	R_k = 36 kN			**R_k = 45 kN**		
α = 90°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	35 mm	107 mm	176 mm	≥ 102 mm	107 mm	≥ 311 mm
	R_k = 34 kN			**R_k = 43 kN**		

Table 13.9 Steel-to-timber connection with three slotted-in steel plates. The thickness of each timber central member is the minimum required for the formation of three plastic hinges in the fastener. The thickness of each outer timber member is the minimum required for the formation of one plastic hinge in the fastener, at each outer member. The thickness of the steel plates is t_s = 8 mm. The width of the slots is t_{sl} = 10 mm. The diameter of the dowel is d = 12 mm, structural steel grade S355. Timber material: Glulam in strength class GL30c. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod} /γ_M), where γ_M = 1.3.



$$R_k = R_{k,centre} + R_{k,lateral}$$

$$R_{k,centre} = 2 \times \left(2 \times 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,k} \times d}\right)$$

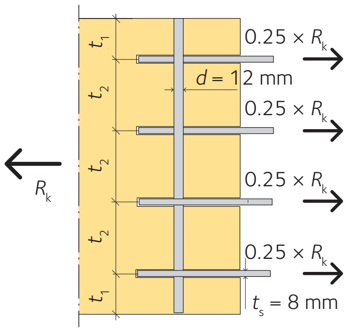
$$R_{k,lateral} = 2 \times \begin{cases} f_{h,k} \times d \times t_1 \left(\sqrt{2 + \frac{4 \times M_{y,k}}{f_{h,k} \times d \times t_1^2}} - 1\right) & \text{if} \quad \sqrt{2} \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}} < t_1 \leq 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}} \\ 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,k} \times d} & \text{if} \quad t_1 \geq 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}} \end{cases}$$

$$t_1 > \sqrt{2} \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}}$$

$$t_2 \geq t_{2,req} = 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}}$$

α = 0°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	29 mm	88 mm	235 mm	≥ 83 mm	88 mm	≥ 343 mm
	R_k = 69 kN			**R_k = 79 kN**		
α = 30°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	31 mm	93 mm	248 mm	≥ 88 mm	93 mm	≥ 363 mm
	R_k = 65 kN			**R_k = 74 kN**		
α = 60°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	33 mm	103 mm	272 mm	≥ 98 mm	1033 mm	≥ 400 mm
	R_k = 58 kN			**R_k = 67 kN**		
α = 90°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	35 mm	107 mm	283 mm	≥ 102 mm	107 mm	≥ 417 mm
	R_k = 60 kN			**R_k = 64 kN**		

Table 13.10 Steel-to-timber connection with four slotted-in steel plates. The thickness of each timber central member is the minimum required for the formation of three plastic hinges in the fastener. The thickness of each outer timber member is the minimum required for the formation of one plastic hinge in the fastener, at each outer member. The thickness of the steel plates is t_s = 8 mm. The width of the slots is t_{sl} = 10 mm. The diameter of the dowel is d = 12 mm, structural steel grade S355. Timber material: Glulam in strength class GL30c. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod} /γ_M), where γ_M = 1.3.



$$R_k = R_{k,centre} + R_{k,lateral}$$

$$R_{k,centre} = 3 \times \left(2 \times 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,k} \times d}\right)$$

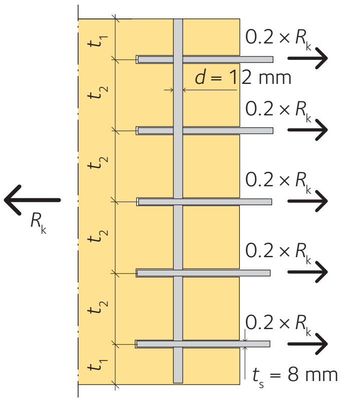
$$R_{k,lateral} = 2 \times \begin{cases} f_{h,k} \times d \times t_1 \left(\sqrt{2 + \frac{4 \times M_{y,k}}{f_{h,k} \times d \times t_1^2}} - 1\right) & \text{if} \quad \sqrt{2} \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}} < t_1 \leq 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}} \\ 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,k} \times d} & \text{if} \quad t_1 \geq 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}} \end{cases}$$

$$t_1 > \sqrt{2} \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}}$$

$$t_2 \geq t_{2,req} = 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}}$$

α = 0°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	29 mm	88 mm	323 mm	≥ 83 mm	88 mm	≥ 432 mm
	R_k = 96 kN			**R_k = 106 kN**		
α = 30°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	31 mm	93 mm	341 mm	≥ 88 mm	93 mm	≥ 457 mm
	R_k = 90 kN			**R_k = 99 kN**		
α = 60°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	33 mm	103 mm	375 mm	≥ 98 mm	103 mm	≥ 503 mm
	R_k = 81 kN			**R_k = 90 kN**		
α = 90°	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	35 mm	107 mm	390 mm	≥ 102 mm	107 mm	≥ 524 mm
	R_k = 77 kN			**R_k = 85 kN**		

Table 13.11 Steel-to-timber connection with five slotted-in steel plates. The thickness of each timber central member is the minimum required for the formation of three plastic hinges in the fastener. The thickness of each outer timber member is the minimum required for the formation of one plastic hinge in the fastener, at each outer member. The thickness of the steel plates is t_s = 8 mm. The width of the slots is t_{sl} = 10 mm. The diameter of the dowel is d = 12 mm, structural steel grade S355. Timber material: Glulam in strength class GL30c. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod} /γ_M), where γ_M = 1.3.



$$R_k = R_{k,centre} + R_{k,lateral}$$

$$R_{k,centre} = 4 \times \left(2 \times 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,k} \times d}\right)$$

$$R_{k,lateral} = 2 \times \begin{cases} f_{h,k} \times d \times t_1 \left(\sqrt{2 + \dfrac{4 \times M_{y,k}}{f_{h,k} \times d \times t_1^2}} - 1\right) & \text{if} \quad \sqrt{2} \times \sqrt{\dfrac{M_{y,k}}{f_{h,k} \times d}} < t_1 \leq 1.15 \times 4 \times \sqrt{\dfrac{M_{y,k}}{f_{h,k} \times d}} \\ 1.15 \times \sqrt{2} \times \sqrt{2 \times M_{y,k} \times f_{h,k} \times d} & \text{if} \quad t_1 \geq 1.15 \times 4 \times \sqrt{\dfrac{M_{y,k}}{f_{h,k} \times d}} \end{cases}$$

$$t_1 > \sqrt{2} \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}}$$

$$t_2 \geq t_{2,req} = 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}}$$

$\alpha = 0°$	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	29 mm	88 mm	411 mm	≥ 83 mm	88 mm	≥ 520 mm
	R_k = 122 kN			**R_k = 132 kN**		
$\alpha = 30°$	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	31 mm	93 mm	435 mm	≥ 88 mm	93 mm	≥ 550 mm
	R_k = 115 kN			**R_k = 124 kN**		
$\alpha = 60°$	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	33 mm	103 mm	477 mm	≥ 98 mm	103 mm	≥ 605 mm
	R_k = 103 kN			**R_k = 112 kN**		
$\alpha = 90°$	$t_{1,min}$	$t_2 = t_{2,req}$	t_{TOT}	t_1	$t_2 = t_{2,req}$	t_{TOT}
	35 mm	107 mm	497 mm	≥ 102 mm	107 mm	≥ 631 mm
	R_k = 99 kN			**R_k = 107 kN**		

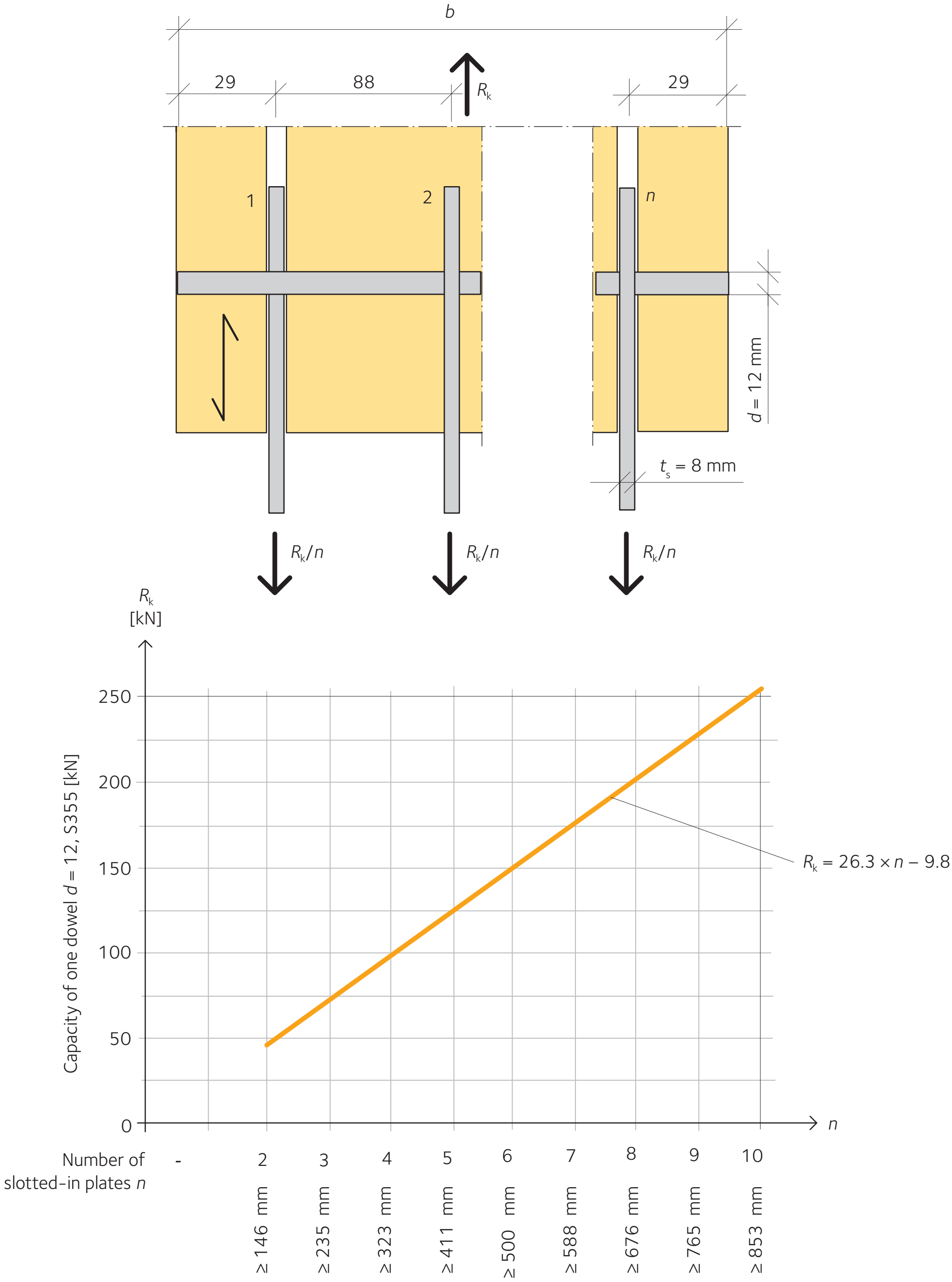
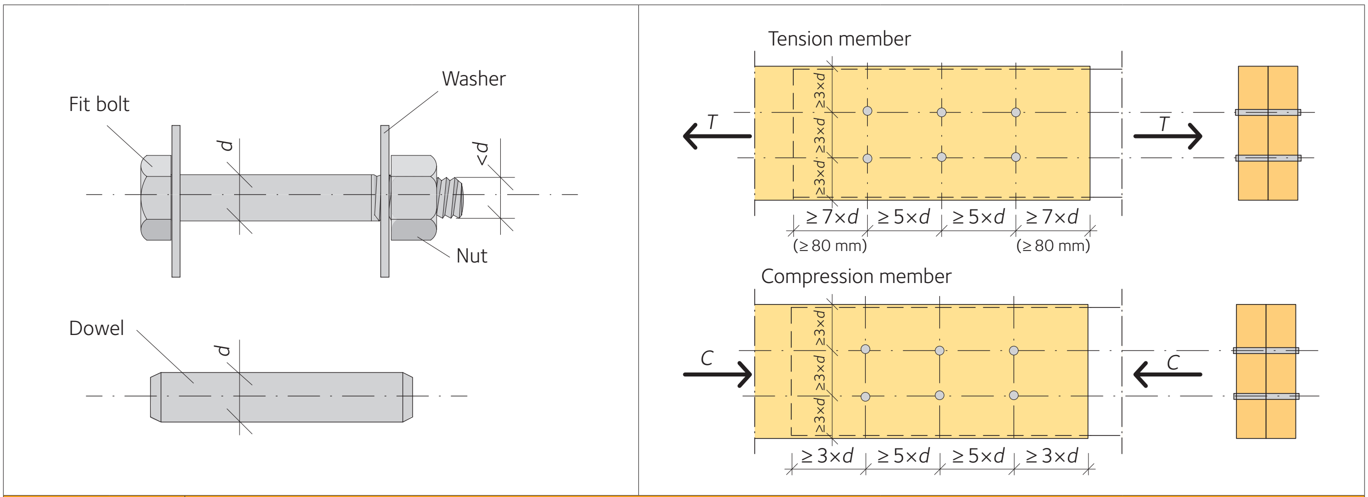


Figure 13.3 Steel-to-timber connection with *n* slotted-in steel plates loaded in the direction parallel to the grain ($n \geq 2$)
The thickness of the central members is 78 mm, which means that the centre-to-centre distance between slotted in plates is 88 mm. The thickness of the outer members is 24 mm. The thickness of the steel plates is t_s = 8 mm. The width of the slots is t_{sl} = 10 mm. The diameter of the dowel is d =12 mm, structural steel grade S355. Timber material: Glulam in strength class GL30c. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod}/γ_M), where γ_M = 1.3.

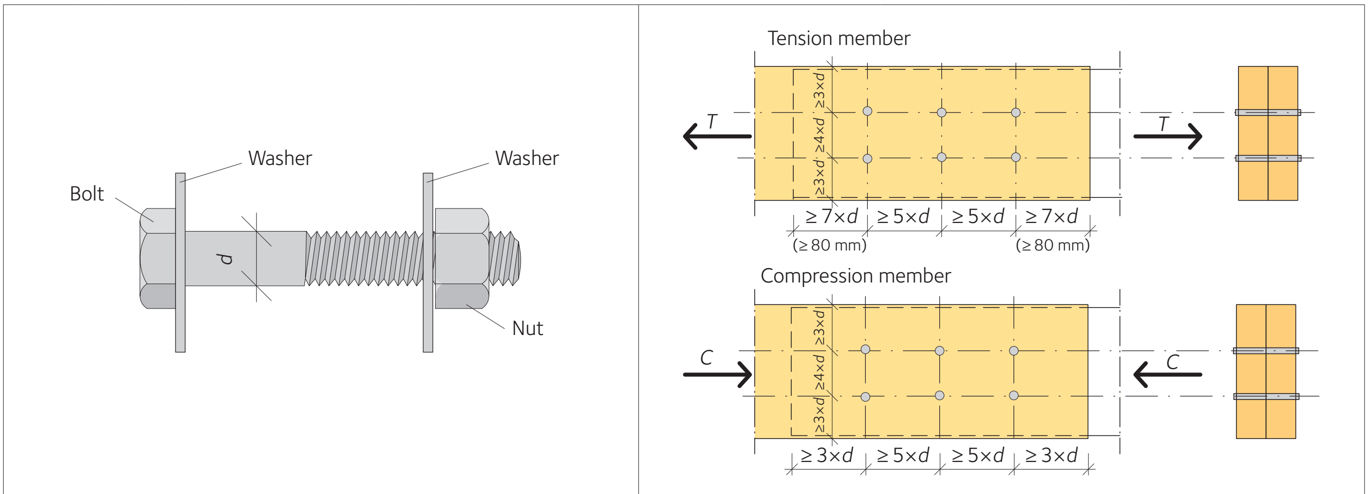
Table 13.12 **Minimum spacings, edge and end distances for dowels and fit bolts in members loaded parallel to the grain**



Fastener diameter [mm]	Distances/spacings				
	between fasteners		to loaded end	to unloaded end	to the edge
	$a_{\perp}$	$a_{//}$	$a_{//}$	$a_{//}$	$a_{\perp}$
	$3 \times d$	$5 \times d$	$7 \times d$	$3 \times d$	$3 \times d$
d = 8	24 mm	40 mm	56 mm	24 mm	24 mm
d = 12	36 mm	60 mm	84 mm	36 mm	36 mm
d = 16	48 mm	80 mm	112 mm	48 mm	48 mm
d = 20	60 mm	100 mm	140 mm	60 mm	60 mm
d = 24	72 mm	120 mm	168 mm	72 mm	72 mm

// = parallel to the grain, ⊥ = perpendicular to the grain.

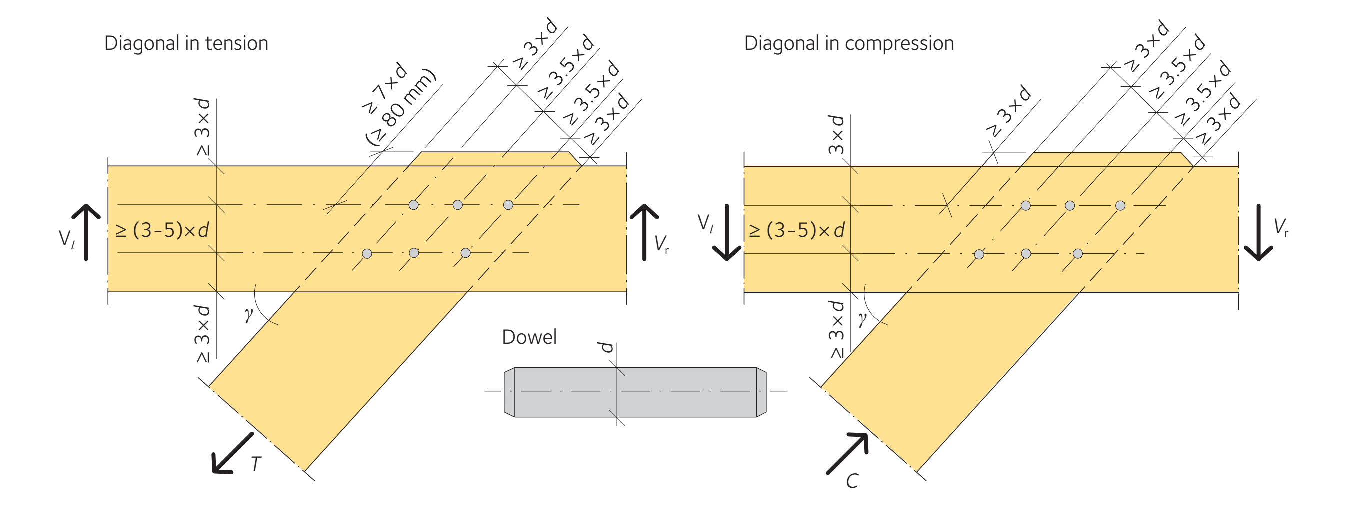
Table 13.13 **Minimum spacings, edge and end distances for bolts in members loaded parallel to the grain**



Fastener diameter [mm]	Distances/spacings				
	between fasteners		to loaded end	to unloaded end	to the edge
	$a_{\perp}$	$a_{//}$	$a_{//}$	$a_{//}$	$a_{\perp}$
	$4 \times d$	$5 \times d$	min{$7 \times d$; 80}	$3 \times d$	$4 \times d$
d = 8	32 mm	40 mm	80 mm	24 mm	32 mm
d = 12	48 mm	60 mm	84 mm	36 mm	48 mm
d = 16	64 mm	80 mm	112 mm	48 mm	64 mm
d = 20	80 mm	100 mm	140 mm	60 mm	80 mm
d = 24	96 mm	120 mm	168 mm	72 mm	96 mm

// = parallel to the grain, ⊥ = perpendicular to the grain.

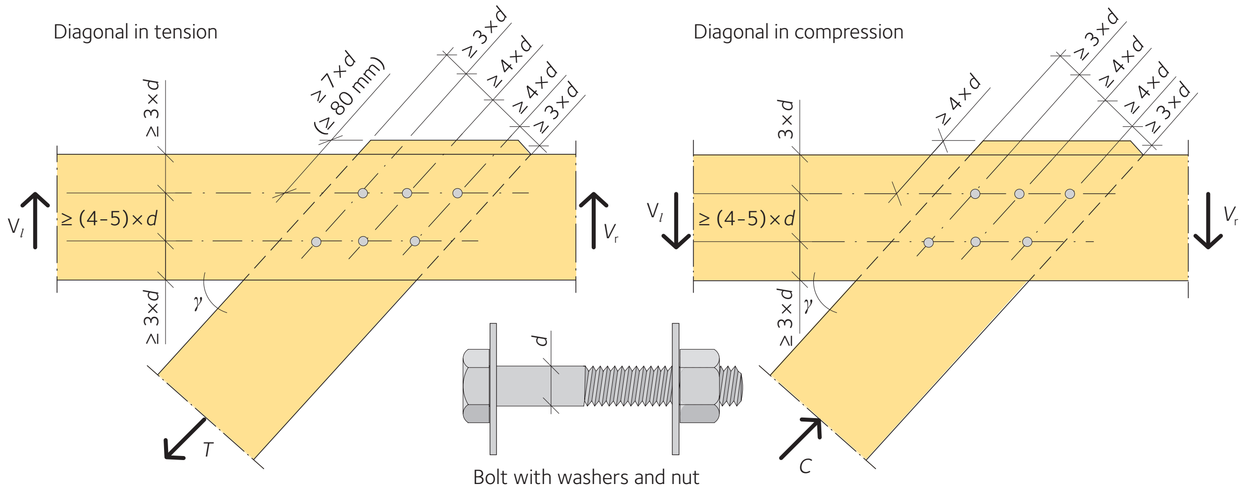
Table 13.14 **Minimum spacings, edge and end distances for dowels and fit bolts in members loaded at an angle to the grain**



Fastener diameter [mm]		Distances/spacings				
		between fasteners		to loaded end	to unloaded end	to the edge
		$a_{\perp}$	$a_{//}$	$a_{//}$	$a_{//}$	$a_{\perp}$
		3.5 × *d* (incl.) (3-5) × *d* (horiz.)	governed by $a_{\perp}$ in the horizontal member	max{7 × *d*; 80} (incl.)	3 × *d*	3 × *d*
Spacing in inclined member	8	28 mm	–	80 mm	24 mm	24 mm
	10	35 mm	–	80 mm	30 mm	30 mm
	12	42 mm	–	84 mm	36 mm	36 mm
	16	56 mm	–	112 mm	48 mm	48 mm
	20	70 mm	–	140 mm	60 mm	60 mm
	24	84 mm	–	168 mm	72 mm	72 mm
Spacing in horizontal member	8	26 mm ($\gamma \leq 40°$)	–	–	–	24 mm
		35 mm ($40° < \gamma \leq 60°$)				
		40 mm ($60° < \gamma \leq 90°$)				
	10	33 mm ($\gamma \leq 40°$)	–	–	–	30 mm
		44 mm ($40° < \gamma \leq 60°$)				
		50 mm ($60° < \gamma \leq 90°$)				
	12	39 mm ($\gamma \leq 40°$)	–	–	–	36 mm
		52 mm ($40° < \gamma \leq 60°$)				
		60 mm ($60° < \gamma \leq 90°$)				
	16	52 mm ($\gamma \leq 40°$)	–	–	–	48 mm
		70 mm ($40° < \gamma \leq 60°$)				
		80 mm ($60° < \gamma \leq 90°$)				
	20	65 mm ($\gamma \leq 40°$)	–	–	–	60 mm
		87 mm ($40° < \gamma \leq 60°$)				
		100 mm ($60° < \gamma \leq 90°$)				
	24	78 mm ($\gamma \leq 40°$)	–	–	–	72 mm
		104 mm ($40° < \gamma \leq 60°$)				
		120 mm ($60° < \gamma \leq 90°$)				

// = parallel to the grain, ⊥ = perpendicular to the grain.

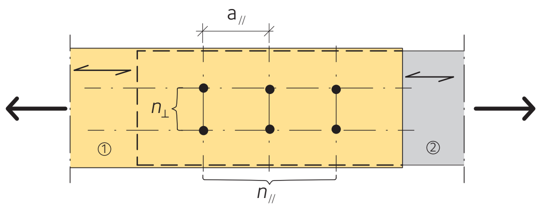
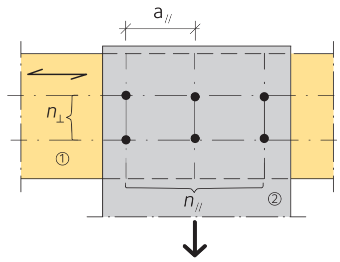
Table 13.15 **Minimum spacings, edge and end distances for bolts in members loaded at an angle to the grain**



Fastener diameter [mm]		Distances/spacings				
		between fasteners		to loaded end	to unloaded end	to the edge
		$a_\perp$	$a_{//}$	$a_{//}$	$a_{//}$	$a_\perp$
		$4 \times d$ (incl.) $(4-5) \times d$ (horiz.)	governed by $a_\perp$ in the horizontal member	max{$7 \times d$; 80} (incl.)	$4 \times d$	$3 \times d$
Spacing in inclined member	8	32 mm	–	80 mm	32 mm	24 mm
	10	40 mm	–	80 mm	40 mm	30 mm
	12	48 mm	–	84 mm	48 mm	36 mm
	16	64 mm	–	112 mm	64 mm	48 mm
	20	80 mm	–	140 mm	80 mm	60 mm
	24	96 mm	–	168 mm	96 mm	72 mm
Spacing in horizontal member	8	26 mm ($\gamma \leq 40°$)	–	–	–	24 mm
		35 mm ($40° < \gamma \leq 60°$)				
		40 mm ($60° < \gamma \leq 90°$)				
	10	33 mm ($\gamma \leq 40°$)	–	–	–	30 mm
		44 mm ($40° < \gamma \leq 60°$)				
		50 mm ($60° < \gamma \leq 90°$)				
	12	39 mm ($\gamma \leq 40°$)	–	–	–	36 mm
		52 mm ($40° < \gamma \leq 60°$)				
		60 mm ($60° < \gamma \leq 90°$)				
	16	52 mm ($\gamma \leq 40°$)	–	–	–	48 mm
		70 mm ($40° < \gamma \leq 60°$)				
		80 mm ($60° < \gamma \leq 90°$)				
	20	65 mm ($\gamma \leq 40°$)	–	–	–	60 mm
		87 mm ($40° < \gamma \leq 60°$)				
		100 mm ($60° < \gamma \leq 90°$)				
	24	78 mm ($\gamma \leq 40°$)	–	–	–	72 mm
		104 mm ($40° < \gamma \leq 60°$)				
		120 mm ($60° < \gamma \leq 90°$)				

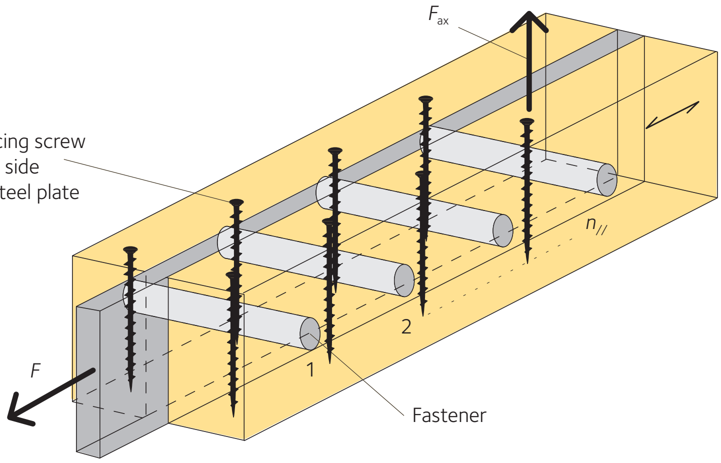
// = parallel to the grain, ⊥ = perpendicular to the grain.

Table 13.16 Connections with several dowels or bolts in a row: Effective number of bolts n_{ef} according to EN 1995-1-1, section 8.5.1

Load // grain	Load ⊥ grain
	
$n_{ef\parallel} = \min \begin{cases} n_{\parallel} \\ n_{\parallel}^{0.9} \times 4\sqrt{\dfrac{a_{\parallel}}{13 \times d}} \end{cases}$	$n_{ef\parallel} = n_{\parallel}$
$n_{ef\perp} = n_{\perp}$	$n_{ef\perp} = n_{\perp}$
$n_{ef,tot} = n_{ef\parallel} \times n_{\perp}$	$n_{ef,tot} = n_{\parallel} \times n_{\perp}$

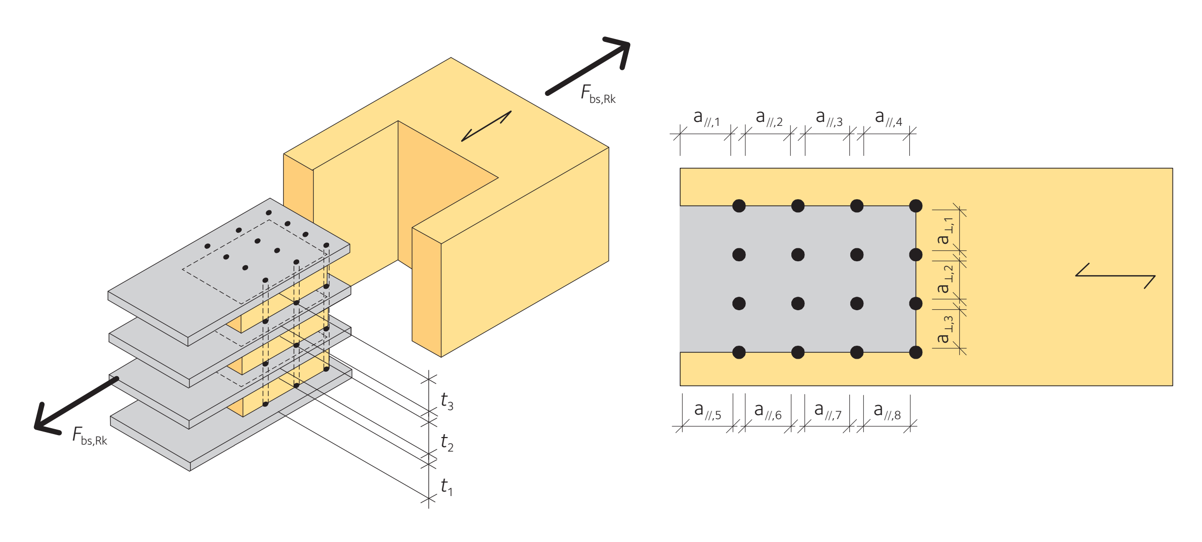
// = parallel to the grain, ⊥ = perpendicular to the grain.

Table 13.17 Connections with several dowels or bolts in a row strengthened by means of reinforcing screws. Effective number of bolts n_{ef}



Effective number of connectors in the direction of the load	$n_{ef\parallel} = n_{\parallel}$
The reinforcing screw shall be designed to resist an axial force:	$F_{ax} = 0.3 \times \dfrac{F}{n_{\parallel} \times n_{tm}}$
	where: $n_{//}$ is the number of fasteners (4 in the figure above). n_{tm} is the number of timber members (2 in the figure above).

Table 13.18 Block shear failure at multiple dowel-type steel-to-timber connections according to EN 1995-1-1, Annex A. Simplifications recommended by the authors of *The Glulam Handbook Volume 3* have been included. As a rule of thumb, in order to reduce the risk for block shear failure, not more than five dowels in a row in the direction of the grain should be used.



Characteristic load carrying capacity of fracture along the perimeter of the fastener area	$F_{\text{bs,Rk}} = \max \begin{cases} 1.5 A_{\text{net,t}} f_{\text{t,0,k}} \\ 0.7 A_{\text{net,v}} f_{\text{v,k}} \end{cases}$
Tensile net area	$A_{\text{net,t}} = \sum_i \text{a}_{\perp,\text{i}} \times \sum_j t_{\text{j}}$
Shear area	$A_{\text{net,v}} = \sum_i \text{a}_{\parallel,\text{i}} \times \sum_j t_{\text{j}}$
Characteristic tensile strength // grain	$f_{\text{t,0,k}}$
Characteristic shear strength	$f_{\text{v,k}}$

// = parallel to the grain, ⊥ = perpendicular to the grain.

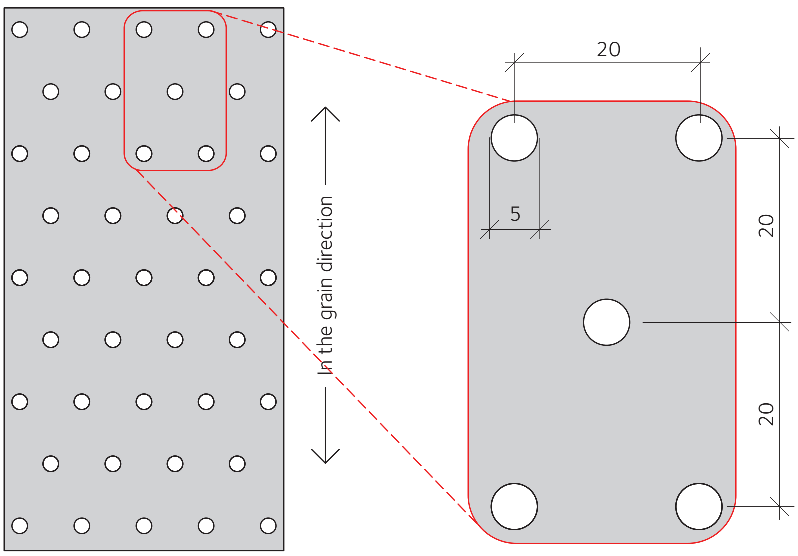
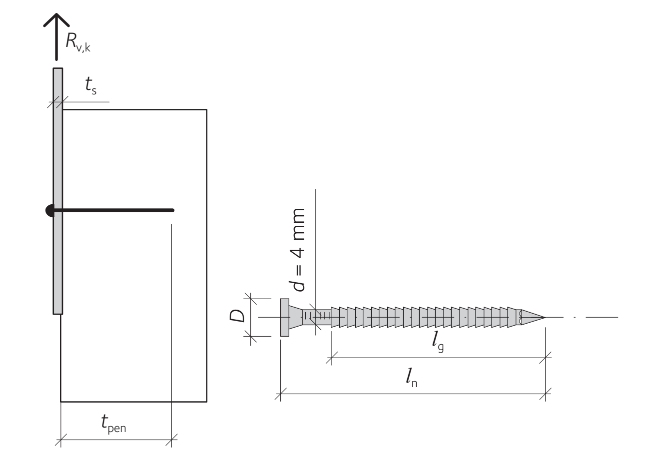


Figure 13.4 Connections with nailing plates. Steel nailing plate with typical hole pattern for anchor nails with nominal diameter d = 4 mm. The thickness of the plate varies normally within the range t_s 2 – 6 mm.

Table 13.19 Nailing plate connection. The thickness of the steel plate varies between 2 – 6 mm. The diameter of the nail is d = 4 mm, $f_{u,k} \geq 600$ MPa. Timber material: Glulam in strength class GL30c. Rope effect not taken into account. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod}/γ_M), where $\gamma_M = 1.3$.



$$R_k = \begin{cases} 1.15 \times \sqrt{2} \times \sqrt{M_{y,k} \times f_{h,k} \times d} & \text{if} \quad t_s \leq \frac{d}{2} \\ 1.15 \times 2 \times \sqrt{M_{y,k} \times f_{h,k} \times d} & \text{if} \quad t \geq d \end{cases}$$

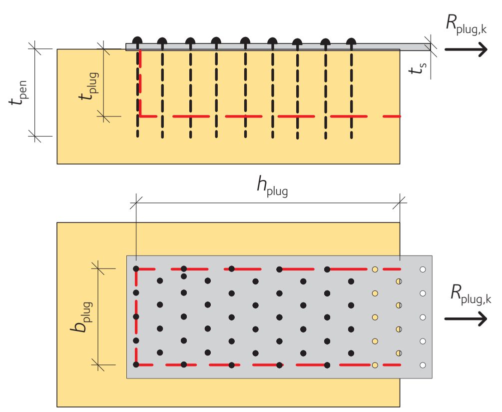
$$t_{pen,req} \geq 1.15 \times \left(\sqrt{2} + 2\right) \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}} \quad \text{if} \quad t_s \leq \frac{d}{2}$$

$$t_{pen,req} \geq 1.15 \times 4 \times \sqrt{\frac{M_{y,k}}{f_{h,k} \times d}} \quad \text{if} \quad t \geq d$$

Plate thickness t_s	Minimum penetration length $t_{pen,req}$	Recommended nail length l_n	Capacity R_k 1)
2 mm	35 mm	40 – 50 mm	1.22 kN
2.5 mm	36 mm	40 – 50 mm	1.34 kN
3 mm	38 mm	50 mm	1.47 kN
4 – 6 mm	41 mm	50 – 60 mm	1.72 kN

1) If $t_{pen} < t_{pen,req}$, the load bearing capacity can be calculated as: $R_{k,red} = R_k \times t_{pen} / t_{pen,req}$.

Table 13.20 Plug shear capacity of nailed plate connection. The thickness of the steel plate varies between 2 – 6 mm. The diameter of the nail is d = 4 mm, $f_{u,k} \geq 600$ MPa. Timber material: Glulam in strength class GL30c. Corresponding design values can be obtained by multiplying the characteristic values by (k_{mod}/γ), where $\gamma = 1.3$. Approach according to EN 1995-1-1, Annex A. Simplifications recommended by the authors of *The Glulam Handbook Volume 3* have been included.



$$R_{plug,k} = \max \begin{cases} 1.5 \times b_{plug} \times t_{plug} \times f_{t,0,k} \\ 0.7 \times \left(2 \times h_{plug} \times t_{plug} + h_{plug} \times b_{plug}\right) \times f_{v,k} \end{cases}$$

$$f_{t,0,k} = 20 \text{ MPa}$$

$$f_v = 3.5 \text{ MPa}$$

Plate thickness t_s	Nail length l_n	Plug depth t_{plug}	Capacity $R_{plug,k}$ 1)	
			$\frac{h_{plug}}{b_{plug}} \leq 4.1$	$\frac{h_{plug}}{b_{plug}} > 4.1$
2 mm	40 – 50 mm	17 mm	510 × b_{plug} 1)	83 × h_{plug} + 2.4 × b_{plug} × h_{plug} 1)
2.5 mm	40 – 50 mm	18 mm	540 × b_{plug} 1)	88 × h_{plug} + 2.4 × b_{plug} × h_{plug} 1)
3 mm	50 mm	19 mm	570 × b_{plug} 1)	93 × h_{plug} + 2.4 × b_{plug} × h_{plug} 1)
4 – 6 mm	50 – 60 mm	20 mm	600 × b_{plug} 1)	98 × h_{plug} + 2.4 × b_{plug} × h_{plug} 1)

1) b_{plug} and h_{plug} in [mm]. $R_{plug,k}$ in [N].

Table 13.21 Connections with several nails in a row. Effective number of nails n_{ef}, according to EN 1995-1-1, section 8.3.1

<table>
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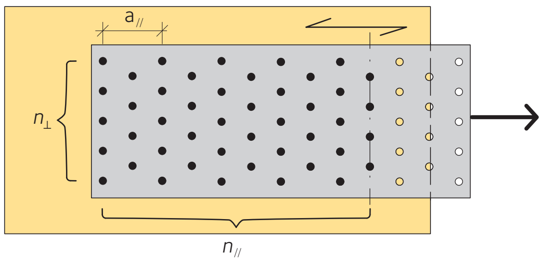
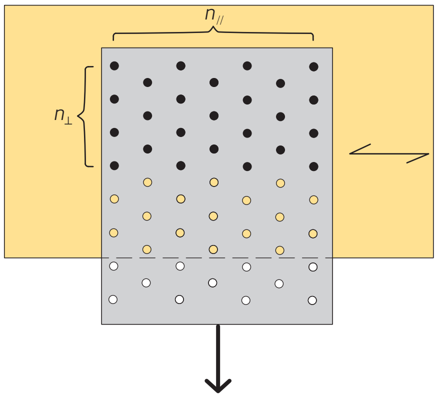
<tr><td colspan="2">
</td><td>
</td></tr>
<tr><td>$n_{ef\parallel} = n_{\parallel}^{k_{ef}}$</td><td><table>
<tr><th>Spacing</th><th>k_{ef}</th></tr>
<tr><td>$a_{//} \geq 14 \times d$</td><td>1.0</td></tr>
<tr><td>$a_{//} \geq 10 \times d$</td><td>0.85</td></tr>
<tr><td>$a_{//} \geq 7 \times d$</td><td>0.7</td></tr>
<tr><td colspan="2">For intermediate spacing, linear interpolation of k_{ef} can be used.</td></tr>
</table></td><td>$n_{ef\parallel} = n_{\parallel}$</td></tr>
<tr><td colspan="2">$n_{ef\perp} = n_{\perp}$</td><td>$n_{ef\perp} = n_{\perp}$</td></tr>
<tr><td colspan="2">$n_{ef,tot} = n_{ef\parallel} \times n_{\perp}$</td><td>$n_{ef,tot} = n_{\parallel} \times n_{\perp}$</td></tr>
</table>

// = parallel to the grain, ⊥ = perpendicular to the grain.

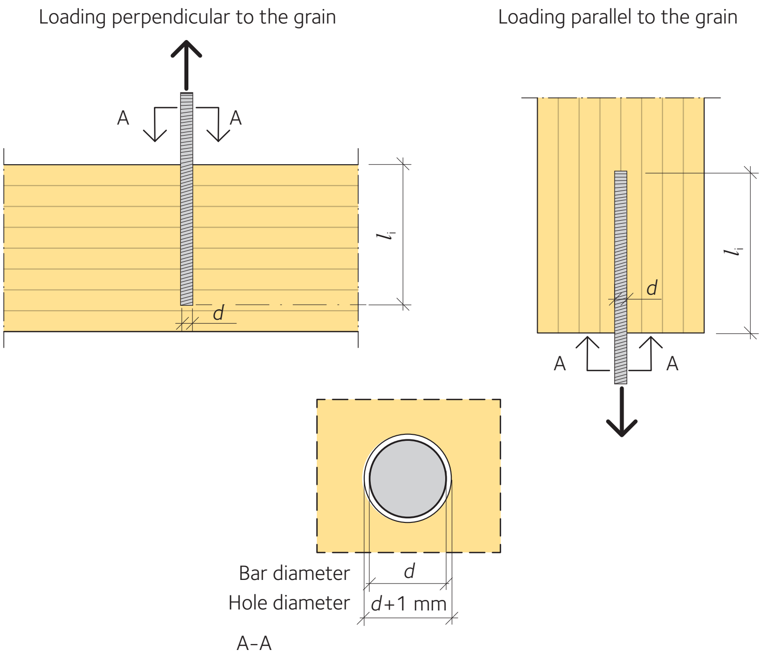
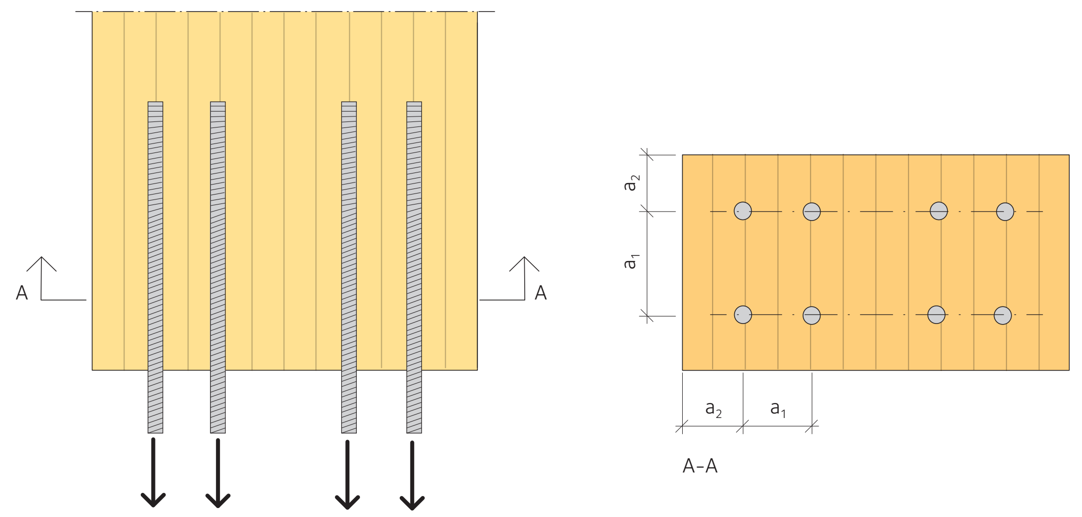


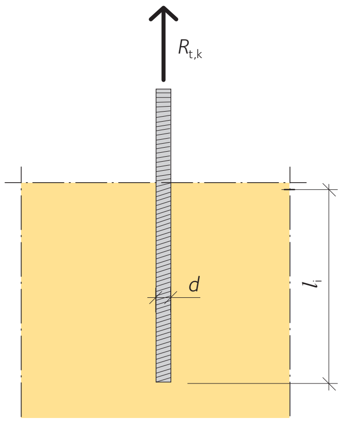
Figure 13.5 Connections with bonded-in rods. Connections with bonded-in rods loaded in the direction perpendicular (left) and parallel (right) to the grain respectively. Typically the nominal diameter of the bar is d = 10 – 20 mm. The steel grade is normally 4.8 – 8.8. The diameter of the drilled hole is 1 mm greater than the nominal diameter of the bar. Common adhesives are PUR (polyurethane) and EPX (epoxy).

Table 13.22 Recommended distances between rods and edge distances



Bar type	Recommended distance between rods a_1 (= 4 × *d*)	Recommended end distance a_2 (= 2.5 × *d*)
M10	40 mm	25 mm
M12	48 mm	30 mm
M16	64 mm	40 mm
M20	80 mm	50 mm

Table 13.23 Capacity of axial loaded bonded in rods according to the Swedish national technical approval 1396/78, issued by SP Technical Research Institute of Sweden, valid for service class 1. The bonded-in length is 350 mm for all the cases. The nominal diameter of the bonded-in rod varies in the range *d* 10 – 20 mm. Timber material: Glulam in strength class GL30c. Corresponding design values for the tensile capacity of the steel rod $(R_{t,d})_{rod}$ can be obtained by multiplying the characteristic values by (1/1.2). Corresponding design values for the withdrawal $(R_{t,d})_{timber}$ can be obtained by multiplying the characteristic values by (k_{mod}/γ_M), where γ_M = 1.3. Threaded bonded-in rods must not be used in service class 3.



$$(R_{t,k})_{rod} = 0.6 \times f_{ub} \times A_s$$

$$(R_{t,k})_{timber} = \pi \times (d + 1\text{ mm}) \times l_i \times f_{ax,k} \times k_1 \times \kappa_1$$

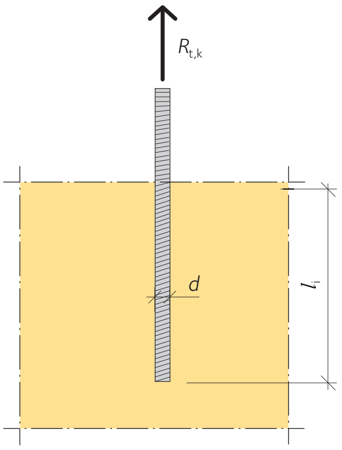
$$f_{ax,k} = 5.5\text{ MPa}$$

$$\kappa_1 = \begin{cases} 1 \text{ for service class 1} \\ 0.85 \text{ for service class 2} \end{cases} \qquad k_1 = \begin{cases} 0.55 \text{ for M10} \\ 0.59 \text{ for M12} \\ 0.64 \text{ for M16} \\ 0.69 \text{ for M20} \end{cases}$$

Steel grade	Bar type	Tensile capacity of the steel rod $(R_{t,k})_{rod}$	Withdrawal capacity [1)] $(R_{t,k})_{timber}$
4.8	M10	13.9 kN	36.8 kN
	M12	20.2 kN	46.3 kN
	M16	37.7 kN	66.3 kN
	M20	58.8 kN	87.3 kN
5.8	M10	17.4 kN	36.8 kN
	M12	25.3 kN	46.3 kN
	M16	47.1 kN	66.3 kN
	M20	73.5 kN	87.3 kN
8.8	M10	27.8 kN	36.8 kN
	M12	40.5 kN	46.3 kN
	M16	75.4 kN	66.3 kN
	M20	117.6 kN	87.3 kN

[1)] Valid for service class 1. For service class 2, multiply $(R_{t,d})_{timber}$ by a factor κ_1 = 0.85.

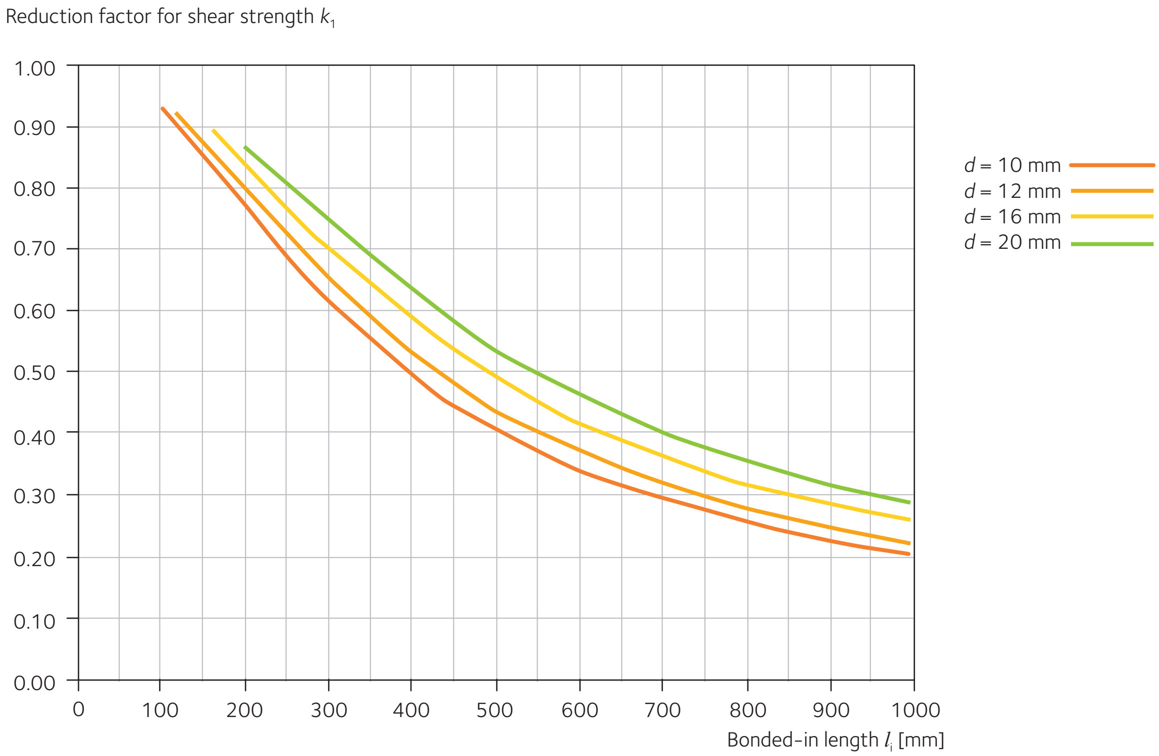
Table 13.24 Reduction factor for shear strength k_1 as function of the bonded-in length l_i. Timber material: Glulam in strength class GL30c. Threaded bonded-in rods must not be used in service class 3.



$$(R_{t,k})_{timber} = \pi \times (d + 1\ \mathrm{mm}) \times l_i \times f_{ax,k} \times k_1 \times \kappa_1$$

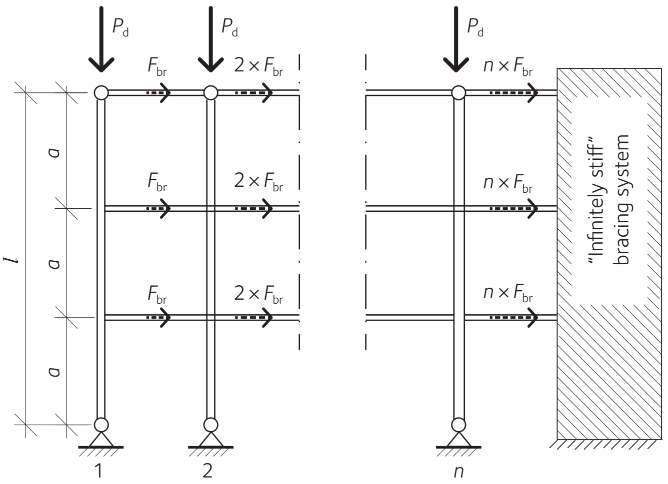
$$f_{ax,k} = 5.5\ \mathrm{MPa}$$

$$\kappa_1 = \begin{cases} 1 \text{ for service class 1} \\ 0.85 \text{ for service class 2} \end{cases}$$



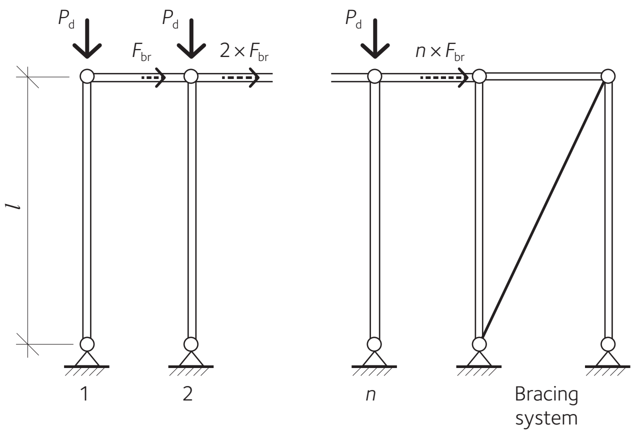
14 Bracing of columns

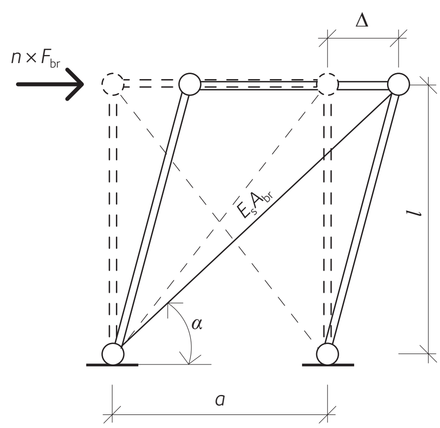
Table 14.1 **Bracing forces in a system of columns stabilised by means of an infinitely stiff bracing system bracing**



Bracing forces
$F_{br} = 0.01 \times P_d$

Table 14.2 **Bracing forces in a system of columns stabilised by means of a braced-bay system with threaded steel diagonals**





Bracing forces
$F_{br} = 0.01 \times P_d$

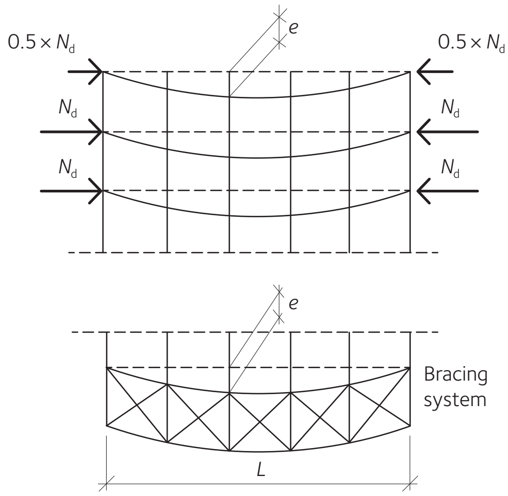
Required stiffness of the braced bay system

$$k_{br} = \frac{E_s \times A_{br} \times \cos^3 \alpha}{a} \geq 2 \times \left(\frac{n \times P_d}{l} \right)$$

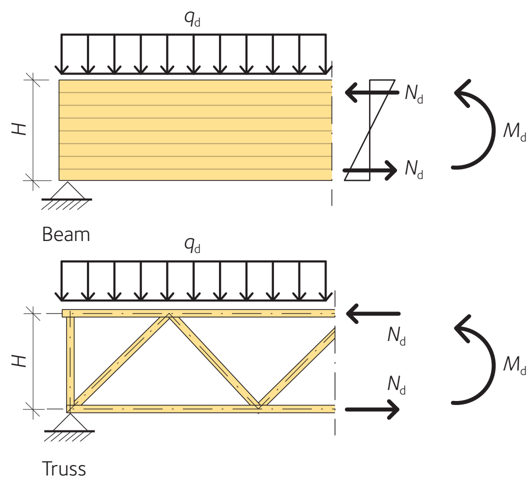
15 Bracing of beams

Table 15.1 Equivalent compression forces in simply supported beams and trusses due to bending moment generated by gravity loads and out-of-plane imperfection bounds

Plane view (with bow imperfections)



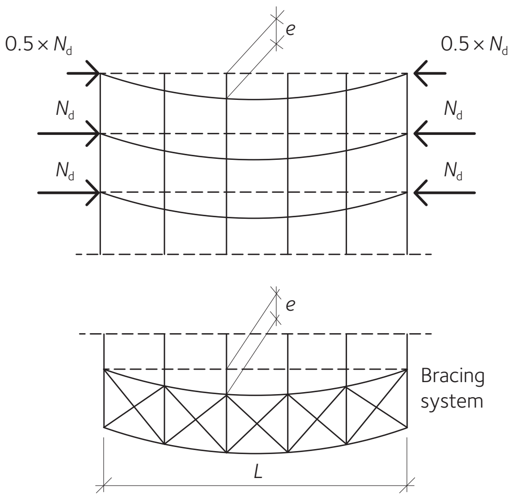
Elevation

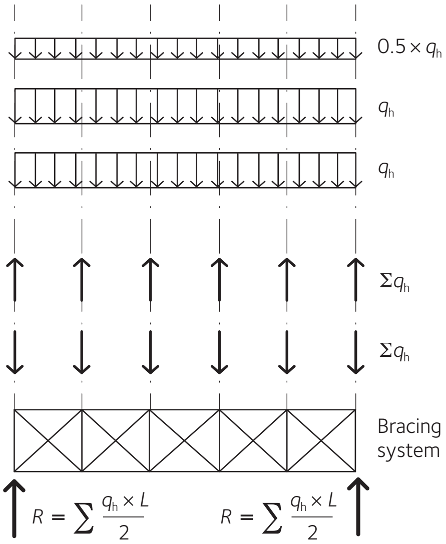


Bending moment	Equivalent compression force N_d		Imperfections	
	Beam	Truss	Initial	Final
$M_d = \frac{Q_d \times L^2}{8}$	$N_d = \frac{3 \times M_d}{2 \times H}$	$N_d = \frac{M_d}{H}$	$e_{init} = \frac{L}{500}$	$e_{fin} = \frac{L}{250}$

Table 15.2 Equivalent lateral destabilising loads in simply supported beams and trusses. These loads must be resisted by the bracing system.

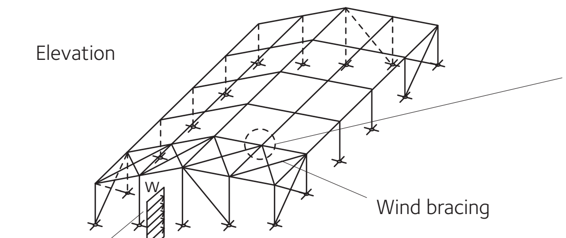
Plane view (with bow imperfections)	**Equivalent lateral destabilising loads**





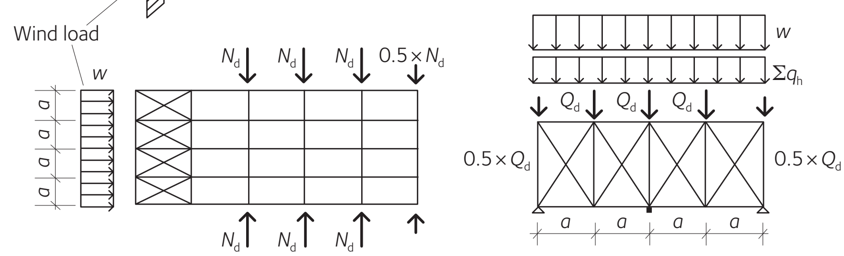
Equivalent lateral destabilizing load q_h	
Beam	**Truss**
$q_h = \frac{M_d}{20 \times L \times H}$	$q_h = \frac{M_d}{30 \times L \times H}$

Table 15.3 Example. Design case where "equivalent" lateral destabilizing loads and wind loads act simultaneously
Two loads combinations are normally checked, namely: 1) Snow load as leading load (gives largest destabilising load) and 2) Wind load as leading load (gives smallest destabilising load).



	Equivalent compression force N_d
Beam	$N_d = \frac{3 \times M_d}{2 \times H}$
Truss	$N_d = \frac{M_d}{H}$

Plan



Destabilising load		**Equivalent forces acting on the bracing system**
Beam	**Truss**	
$q_h = \frac{M_d}{20 \times L \times H}$	$q_h = \frac{M_d}{30 \times L \times H}$	$Q_d = \left(w + \sum q_h\right) \times a$

16 Design values for glulam

Table 16.1 Design strength and stiffness of combined glulam in strength class GL30c for different load duration classes according to EN 14080 and to EKS 10. The values applied in service classes 1 and 2. For service class 3 the strength values may be obtained by multiplication of the values in the table with 0.78. For service class 3 the stiffness values are the same as for service classes 1 and 2. Properties in [MPa].

Property	Load with shortest duration in the load combination				
	P (Permanent) e.g. self-weight	**L (Long duration)** e.g. storage	**M (Medium duration)** e.g. snow load	**S (Short duration)** e.g. wind load	**I (Instantaneous)** e.g. accidental load
Bending // f_m	14.4	16.8	19.2	21.6	26.4
Tension // $f_{t,0}$	9.4	10.9	12.5	14.0	17.2
Tension ⊥ $f_{t,90}$	0.2	0.3	0.3	0.4	0.4
Compression // $f_{c,0,g,k}$	11.8	13.7	15.7	17.6	21.6
Compression ⊥ $f_{c,90}$ [1)]	2.5 (1.2)	2.5 (1.4)	2.5 (1.6)	2.5 (1.8)	2.5 (2.2)
Shear f_v	1.7	2.0	2.2	2.5	3.1
Rolling shear f_r	0.6	0.7	0.8	0.9	1.1
E-modulus // deflections analysis	13,000	13,000	13,000	13,000	13,000
E-modulus // buckling analysis	10,800	10,800	10,800	10,800	10,800
E-modulus // II order analysis	10,400	10,400	10,400	10,400	10,400
Shear modulus deflections analysis	650	650	650	650	650
Shear modulus buckling analysis	542	542	542	542	542

1) The values in brackets apply for $g_k/q_k > 0.4$, see more in *The Glulam Handbook Volume 2, section 4.1.4, page 58.*
// = parallel to the grain, ⊥ = perpendicular to the grain.

Table 16.2 Design strength and stiffness of homogeneous glulam in strength class GL30h for different load duration classes according to EN 14080 and EKS 10. The values applied in service classes 1 and 2. For service class 3 the strength values may be obtained by multiplication of the values in the table with 0.78. For service class 3 the stiffness values are the same as for service classes 1 and 2. Properties in [MPa].

Property	Load with shortest duration in the load combination				
	P (Permanent) e.g. self-weight	**L (Long duration)** e.g. storage	**M (Medium duration)** e.g. snow load	**S (Short duration)** e.g. wind load	**I (Instantaneous)** e.g. accidental load
Bending // f_m	14.4	16.8	19.2	21.6	26.4
Tension // $f_{t,0}$	11.5	13.4	15.4	17.3	21.1
Tension ⊥ $f_{t,90}$	0.2	0.3	0.3	0.4	0.4
Compression // $f_{c,0,g,k}$	14.4	16.8	19.2	21.6	26.4
Compression ⊥ $f_{c,90}$ [1)]	2.5 (1.2)	2.5 (1.4)	2.5 (1.6)	2.5 (1.8)	2.5 (2.2)
Shear f_v	1.7	2.0	2.2	2.5	3.1
Rolling shear f_r	0.6	0.7	0.8	0.9	1.1
E-modulus // deflections analysis	13,600	13,600	13,600	13,600	13,600
E-modulus // buckling analysis	11,300	11,300	11,300	11,300	11,300
E-modulus // II order analysis	10,880	10,880	10,880	10,880	10,880
Shear modulus deflections analysis	650	650	650	650	650
Shear modulus buckling analysis	542	542	542	542	542

1) The values in brackets apply for $g_k/q_k > 0.4$, see more in *The Glulam Handbook Volume 2, section 4.1.4, page 58.*
// = parallel to the grain, ⊥ = perpendicular to the grain.

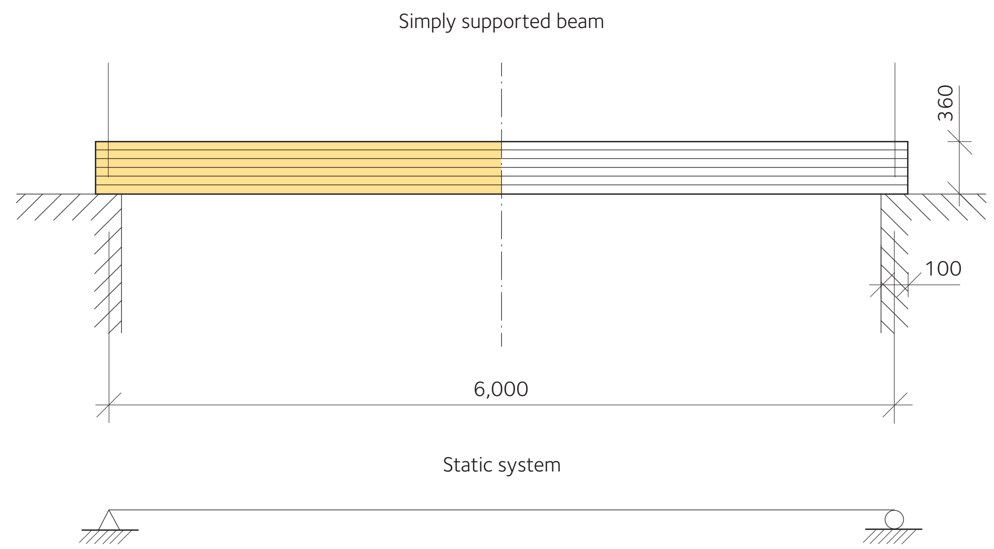
Riding school, Bökeberg, Sweden.

Design examples

Example 1: Design of a simply supported beam

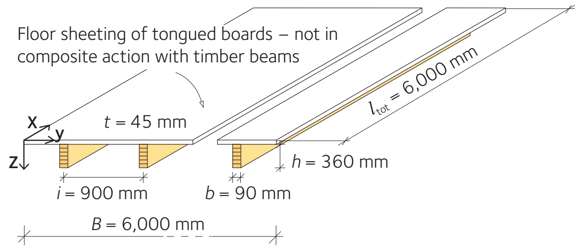
1.1 System, dimensions and design parameters

Design and verify the beam below.



The cross-section dimensions are determined using the preliminary design method shown in *section 1.4, page 82*.

The beam is made of glulam, strength class	GL30c
The floor sheeting is made of tongued boards, strength class	C24
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_q = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$



1.2 Loads

The loads considered in the design are:

Structural

$$g_{k,1} = 0.2 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.5 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i = 0.5 \times 0.9 = 0.5 \text{ kN/m}$

Variable load

$Q_k = 2 \text{ kN/m}^2$ $\quad q_k = Q_k \times i = 2 \times 0.9 = 1.8 \text{ kN/m}$

1.3 Load combinations

Two different load combinations are considered (*EN 1995-1-1, clause 6.4.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1.0 \times 1.2 \times \left(0.2 + 0.5\right) = 0.8 \text{ kN/m}$$

Combination 2 (self-load leading + variable-load leading, medium term load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_q + q_k\right] = 1.0 \times \left[1.2 \times \left(0.2 + 0.5\right) + 1.5 \times 1.8\right] = 3.5 \text{ kN/m}$$

Select the critical combination in the ultimate limit state:

$$\frac{q_{dI}}{k_{mod,1}} = \frac{0.8}{0.6} = 1.3 \quad < \quad \frac{q_{dII}}{k_{mod,2}} = \frac{3.5}{0.8} = 4.4$$

Thus combination 2 is leading.

1.4 Preliminary design

$$b = 90 \text{ mm}$$

$$h = \frac{l_{tot}}{17} = \frac{6 \times 10^3}{17} = 353 \text{ mm} \quad \rightarrow \quad h = 360 \text{ mm}$$

1.5 ULS verifications

a) Compression perpendicular to the grain

$$N_{\mathrm{Ed}} = q_{\mathrm{dII}} \times \frac{l_{\mathrm{tot}}}{2} = 3.48 \times \frac{6}{2} = 10.44 \text{ kN}$$

$$\sigma_{\mathrm{c,90,d}} = \frac{N_{\mathrm{Ed}}}{b \times l_{\mathrm{support}}} = \frac{10.44 \times 10^3}{90 \times (100 + 30)} = 0.89 \text{ MPa}$$

Verify the figure for compression perpendicular to the grain (*EN 1995-1-1, equation 6.3*):

$$\frac{\sigma_{\mathrm{c,90,d}}}{f_{\mathrm{c,90,d}} \times k_{\mathrm{c,90}}} = \frac{0.89}{2.5 \times 1.75} = 0.20 \quad < 1 \quad \textbf{OK}$$

b) Shear

The design shear stress τ_{d} is determined using the reduced value of the shear force at the support V_{red}, *see table 8.5, page 22*:

$$V_{\mathrm{Ed}} = q_{\mathrm{dII}} \times \frac{l_{\mathrm{tot}}}{2} = 3.48 \times \frac{6}{2} = 10.44 \text{ kN}$$

$$V_{\mathrm{red}} = \frac{2 \times V_{\mathrm{Ed}}}{l_{\mathrm{tot}}} \times \left(\frac{l_{\mathrm{tot}}}{2} - \frac{b_{\mathrm{support}}}{2} - h\right) = \frac{2 \times 10.44}{6} \times \left(\frac{6}{2} - \frac{0.1}{2} - 0.36\right) = 9.01 \text{ kN}$$

$$\tau = \frac{3 \times V_{\mathrm{red}}}{2 \times b \times h} = \frac{3 \times 9.01 \times 10^3}{2 \times 90 \times 360} = 0.42 \text{ MPa}$$

Verify the figure for shear stress (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau}{f_{\mathrm{v,d}} \times k_{\mathrm{cr}}} = \frac{0.42}{2.24 \times 0.86} = 0.22 \quad < 1 \quad \textbf{OK}$$

c) Bending moment

Lateral torsional buckling is prevented by the slab:

$$M_{\mathrm{Ed}} = q_{\mathrm{dII}} \times \frac{{l_{\mathrm{tot}}}^2}{8} = 3.48 \times \frac{6^2}{8} = 15.66 \text{ kNm}$$

$$\sigma_{\mathrm{m,d}} = \frac{6 \times M_{\mathrm{Ed}}}{b \times h^2} = \frac{6 \times 15.66 \times 10^6}{90 \times 360^2} = 8.06 \text{ MPa}$$

Verify the figure for bending stress (*EN 1995-1-1, equation 6.11*):

$$\frac{\sigma_{\mathrm{m,d}}}{f_{\mathrm{m,d}} \times k_{\mathrm{h}}} = \frac{8.06}{19.2 \times 1.05} = 0.4 \quad < 1 \quad \textbf{OK}$$

1.6 SLS verifications

a) Deflection

Two different load combinations are considered:

Combination SLS 1 (permanent loads):

$$q_{\text{sls},1} = \left(g_{\text{k},1} + g_{\text{k},2}\right) = 0.7 \text{ kN/m}$$

Combination SLS 2 (variable load):

$$q_{\text{sls},2} = q_{\text{k}} = 1.8 \text{ kN/m}$$

Instantaneous deflection at the mid-span, *see The Glulam Handbook Volume 2, section 6.2.6, page 90*, The deflection is calculated for a uniformly distributed unit load:

$$w_1 = \frac{5}{384} \times \frac{{l_{\text{tot}}}^4}{E_{0,\text{mean}} \times \frac{b \times h^3}{12}} = \frac{5}{384} \times \frac{6000^4}{13000 \times \frac{90 \times 360^3}{12}} = 3.7 \text{ mm}$$

The shear-deformation contribution is neglected.

Instantaneous deflection due to permanent load:

$$w_{\text{inst,permanent}} = w_1 \times q_{\text{sls},1} = 3.7 \times 0.7 = 2.4 \text{ mm}$$

Instantaneous deflection due to variable load:

$$w_{\text{inst,variable}} = w_1 \times q_{\text{sls},2} = 3.7 \times 1.8 = 6.7 \text{ mm}$$

Verify the figure for instantaneous deflection, *see table 11.4, page 51*:

$$w_{\text{inst,permanent}} + w_{\text{inst,variable}} = 9.1 \text{ mm} \quad < \quad \frac{l_{\text{tot}}}{500} = 12 \text{ mm} \quad \textbf{OK}$$

Final deflection due to permanent load:

$$w_{\text{final,perm}} = w_{\text{inst,permanent}} \times \left(1 + k_{\text{def}}\right) = 2.4 \times \left(1 + 0.6\right) = 3.9 \text{ mm}$$

Final deflection due to variable load:

$$w_{\text{final,variable}} = w_{\text{inst,variable}} \times \left(1 + \psi_2 \times k_{\text{def}}\right) = 6.7 \times \left(1 + 0.3 \times 0.6\right) = 7.9 \text{ mm}$$

Total final deflection:

$$w_{\text{final,tot}} = w_{\text{final,variable}} + w_{\text{final,perm}} = 7.9 + 3.9 = 11.7 \text{ mm}$$

Verify the figure for total final deflection, *see table 11.4, page 51*:

$$w_{\text{final,tot}} = 11.7 \text{ mm} \quad < \quad \frac{l_{\text{tot}}}{300} = 20 \text{ mm} \quad \textbf{OK}$$

b) Vibration

The equivalent plate bending stiffness of the floor about the y-axis:

$$EI_l = E_{0,\text{mean}} \times \frac{b \times h^3}{12} \times \frac{1}{i} = 13000 \times 10^6 \times \frac{0.09 \times 0.36^3}{12} \times \frac{1}{0.9} = 5054400\ \text{N} \times \frac{\text{m}^2}{\text{m}}$$

Equivalent plate bending stiffness of the floor about the x-axis:

$$EI_b = E_{0,\text{mean,floor}} \times \frac{t^3}{12} = 11000 \times 10^6 \times \frac{0.045^3}{12} = 83531.25\ \text{N} \times \frac{\text{m}^2}{\text{m}}$$

where t is the slab thickness.

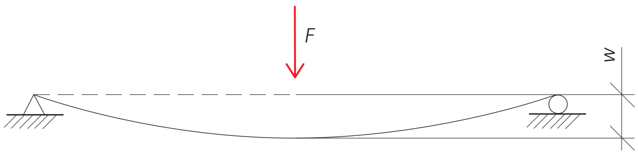
Mass per unit area:

$$m = 72\ \text{kg/m}^2$$

Fundamental frequency (*EN 1995-1-1, section* 7.3):

$$f_1 = \frac{\pi}{2 \times l_{tot}^2} \times \sqrt{\frac{EI_l}{m}} = \frac{3.14}{2 \times 6^2} \times \sqrt{\frac{5054400}{72}} = 11.55\ \text{Hz}$$

The first fundamental frequency of the floor is $f_1 > 8$ Hz. Therefore, the verification procedure according to *EN 1995-1-1, section 7.3.3* should be adopted.



Static deflection under a concentrated load at the mid-span:

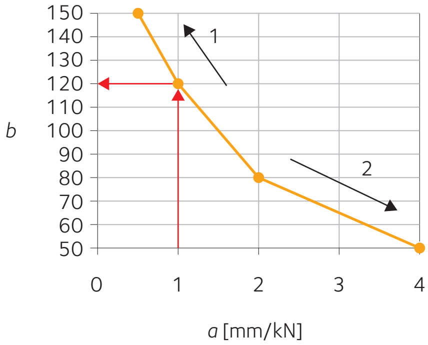
$$\frac{w}{F} = \frac{\frac{12 \times F \times 6000^3}{48 \times 13000 \times 90 \times 360^3} \times 10^3}{F} = 0.989 \frac{\text{mm}}{\text{kN}}$$

The parameter a shall be chosen greater than w/F but smaller than 1.5 mm/kN:

Choose $a = 1$

$$\frac{w}{F} = 0.989 \quad < \quad a = 1 \quad < \quad 1.55 \frac{\text{mm}}{\text{kN}}$$

According to *EN 1995-1-1, figure 7.2*:



$$a = 1 \frac{\text{mm}}{\text{kN}} \quad \rightarrow \quad b = 120$$

Number of first-order modes of vibration with natural frequency up to 40 Hz:

$$\eta_{40} = \left[\left[\left(\frac{40}{f_1}\right)^2 - 1\right] \times \left(\frac{B}{l_{tot}}\right)^4 \times \frac{EI_l}{EI_B}\right]^{0.25} = \left[\left[\left(\frac{40}{11.55}\right)^2 - 1\right] \times \left(\frac{6}{6}\right)^4 \times \frac{5.05 \times 10^6}{8.35 \times 10^4}\right]^{0.25} = 5.08$$

Impulse velocity response:

$$v = \frac{4 \times (0.4 + 0.6 \times \eta_{40})}{m \times B \times l_{tot} + 200} = \frac{4 \times (0.4 + 0.6 \times 5.08)}{72 \times 6 \times 6 + 200} = 0.0049 \frac{N}{m \times s^2}$$

Verify the figure for vibrations (*EN 1995-1-1, equation 7.4*):

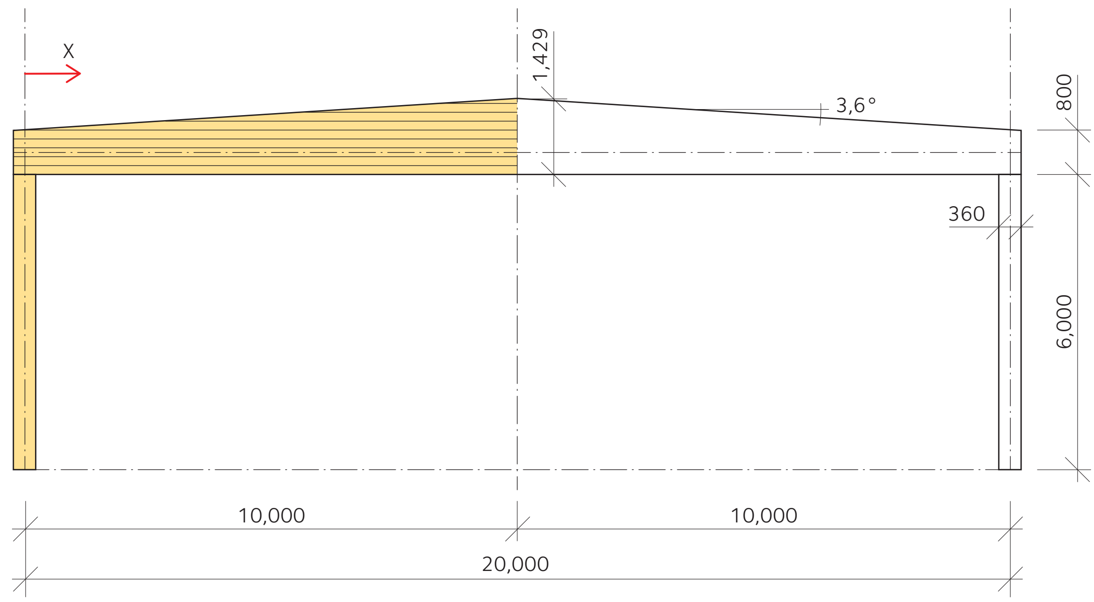
$$v = 0.0003 \quad < \quad b^{(f_1 \times \zeta - 1)} = 120^{(11.55 \times 0.01 - 1)} = 0.014 \frac{N}{m \times s^2} \quad \textbf{OK}$$

Example 2: Design of a tapered beam

2.1 System, dimensions and design parameters

Design and verify the tapered beam below.

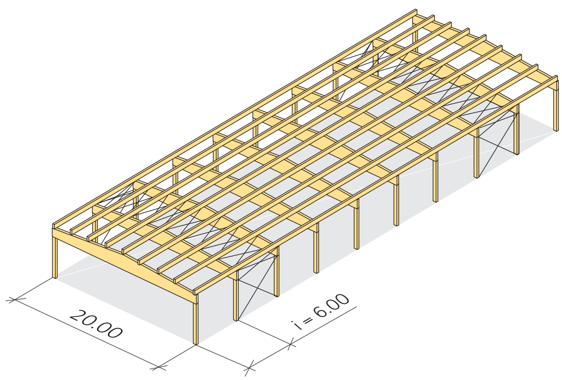
Tapered beam



Static system

The cross-section dimensions are determined using the preliminary design method shown in *section 2.4, page 88*.

The beam is made of glulam, strength class	GL30c
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for snow load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$



2.2 Loads

The loads considered in the design of the beam are:

Structural

$$g_{k,1} = 1.1 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.6 \text{ kN/m}^2$ $\qquad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.6 \times 6 \times 1.1 = 4 \text{ kN/m}$

Snow load

$S_k = 1.5 \text{ kN/m}^2$ $\qquad s_k = S_k \times i \times \mu \times 1.1 = 1.5 \times 6 \times 0.854 \times 1.1 = 8.46 \text{ kN/m}$

Factor 1.1 used in the equations above accounts for the continuity of purlins over trusses.

2.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times (1.1 + 4) = 6.1 \text{ kN/m}$$

Combination 2 (self-load leading + snow load, medium term load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_k\right] = 1 \times \left[1.2 \times (1.1 + 4) + 1.5 \times 8.46\right] = 18.8 \text{ kN/m}$$

Select the critical combination in the the ultimate limit state:

$$\frac{q_{dI}}{k_{mod,1}} = \frac{6.1}{0.6} = 10.1 \quad < \quad \frac{q_{dII}}{k_{mod,2}} = \frac{18.8}{0.8} = 22.5$$

Thus combination 2 is leading.

2.4 Preliminary design

The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, section 7.3.2, page 109*:

Beam:

$$b = \frac{l_{tot}}{110} = \frac{20 \times 10^3}{110} = 182 \text{ mm} \quad \rightarrow \quad b = 190 \text{ mm}$$

$$h_0 = \frac{l_{tot}}{4} \times \left(3 \times \sqrt{\frac{q_{dII}}{b \times 0.9 \times f_{m,d}}} - \tan(\alpha)\right) = \frac{20 \times 10^3}{4} \times \left(3 \times \sqrt{\frac{18.8}{190 \times 0.9 \times 19.2}} - \tan\left(3.6°\right)\right) = 820.6 \text{ mm} \quad \rightarrow \quad h_0 = 800 \text{ mm}$$

$$h_{apex} = \frac{l_{tot}}{4} \times \left(3 \times \sqrt{\frac{q_{dII}}{b \times 0.9 \times f_{m,d}}} + \tan(\alpha)\right) = \frac{20 \times 10^3}{4} \times \left(3 \times \sqrt{\frac{18.8}{190 \times 0.9 \times 19.2}} + \tan\left(3.6°\right)\right) = 1449.7 \text{ mm} \quad \rightarrow \quad h_{apex} = 1429 \text{ mm}$$

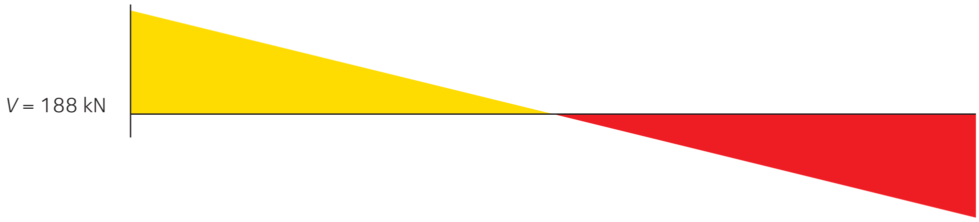
Column:

$$h_{col,min} = \frac{q_{dII} \times \frac{l_{tot}}{2}}{b \times k_{c,90} \times f_{c,90,d}} = \frac{18.8 \times \frac{20 \times 10^3}{2}}{190 \times 1.75 \times 1.6} = 353.4 \text{ mm} \quad \rightarrow \quad h_{col} = 360 \text{ mm}$$

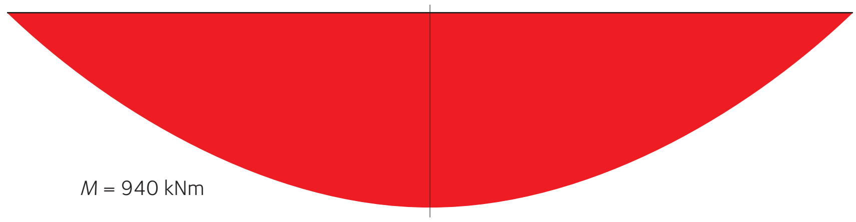
The depth of the column is determined on the basis of the compression strength perpendicular to the grain of the beam. Column buckling verification is shown in *example 6, page 117*.

2.5 Internal forces and moments

Shear:



Bending moment:



2.6 ULS verifications

a) Shear

The design shear stress τ_d is determined using the reduced value of the shear force at the support V_{red}, *see table 8.5, page 22*:

$$V_{red} = \frac{2 \times V_{Ed}}{l_{tot}} \times \left(\frac{l_{tot}}{2} - \frac{h_{col}}{2} - h_0 \right) = 169.6 \text{ kN}$$

$$\tau_d = \frac{3 \times V_{red}}{2 \times b \times h_0} = \frac{3 \times 169.6 \times 10^3}{2 \times 190 \times 800} = 1.67 \text{ MPa}$$

Shear verification (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau_d}{f_{v,d} \times k_{cr}} = \frac{1.67}{2.24 \times 0.86} = 0.87 \quad < 1 \quad \textbf{OK}$$

b) Compression perpendicular to the grain at the support

$$\sigma_{c,90,d} = \frac{N_{Ed}}{b \times (h_{col} + 30)} = \frac{188.02 \times 10^3}{190 \times (360 + 30)} = 2.54 \text{ MPa}$$

Compression perpendicular to the grain verification (*EN 1995-1-1, equation 6.3*):

$$\frac{\sigma_{c,90,d}}{k_{c,90} \times f_{c,90,d}} = \frac{2.54}{1.75 \times 1.6} = 0.91 \quad < 1 \quad \textbf{OK}$$

$f_{c,90,d}$ can not be replaced by $f_{c,90,k}$ due to the fact that $g_k/s_k = 0.60 > 0.4$, *see table 8.11, page 25, 8.12, page 25* and *8.13, page 26*.

c) Bending stress at the most stressed cross-section

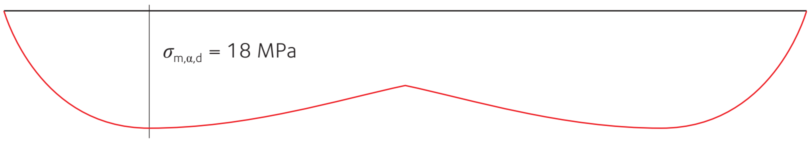
$$x_{max} = \frac{l_{tot} \times h_0}{2 \times h_{apex}} = \frac{20 \times 10^3 \times 800}{2 \times 1429} = 5598.3 \text{ mm}$$

$$M_{x,max} = \frac{q_{dII} \times x_{max}}{2} \times \left(l_{tot} - x_{max}\right) = \frac{18.8 \times 5.6}{2} \times \left(\frac{20 \times 10^3}{10^3} - 5.6\right) = 758 \text{ kNm}$$

$$h_{x,max} = h_0 + x_{max} \times \tan(\alpha) = 800 + 5598.3 \times \tan\left(3.6°\right) = 1152.2 \text{ mm}$$

$$\sigma_{m,\alpha,d} = \frac{6M_{x,max}}{b \times {h_{x,max}}^2} = \frac{6 \times 758 \times 10^6}{190 \times 1152.2^2} = 18 \text{ MPa}$$

Bending stress diagram:



The design bending strength shall be decreased by the factor $k_{m,\alpha}$, which accounts for the simultaneous action of bending stress, shear stress and compression stress:

$$k_{m,\alpha} = \frac{1}{\sqrt{1 + \left(\frac{f_{m,d}}{1.5 \times f_{v,d}} \times \tan(\alpha)\right)^2 + \left[\frac{f_{m,d}}{f_{c,90,d}} \times \left(\tan(\alpha)\right)^2\right]^2}} = \frac{1}{\sqrt{1 + \left(\frac{19.2}{1.5 \times 2.24} \times \tan\left(3.6°\right)\right)^2 + \left(\frac{19.2}{1.6} \times \tan\left(3.6°\right)^2\right)^2}} = 0.94$$

Verify the figure for bending stress (*EN 1995-1-1, equation 6.38*):

$$\frac{\sigma_{m,\alpha,d}}{k_{m,\alpha} \times f_{m,d}} = \frac{18}{0.94 \times 19.2} = 0.997 \quad < 1 \quad \textbf{OK}$$

d) Bending stress at the apex

$$\sigma_{m,d} = k_l \times \frac{6 \times M_{max}}{b \times {h_{apex}}^2} = 1.11 \times \frac{6 \times 940 \times 10^6}{190 \times 1429^2} = 16.1 \text{ MPa}$$

The bending stress at the apex is increased by k_l, which takes into account that the neutral axis is not at the mid-depth of cross-section (*EN 1995-1-1, equation 6.43*):

$$k_l = 1 + 1.4 \times \tan(\alpha) + 5.4 \times \left(\tan(\alpha)\right)^2 = 1 + 1.4 \times \tan\left(3.6°\right) + 5.4 \times \tan\left(3.6°\right)^2 = 1.11$$

Verify the figure for bending stress (*EN 1995-1-1, equation 6.41*):

$$\frac{\sigma_{m,d}}{f_{m,d}} = \frac{16.1}{19.2} = 0.84 \quad < 1 \quad \textbf{OK}$$

e) Tensile stress perpendicular to the grain at the apex

$$k_p = 0.2 \times \tan(\alpha) = 0.2 \times \tan\left(3.6°\right) = 0.013$$

$$\sigma_{t,90,d} = k_p \times \frac{6 \times M_{max}}{b \times h_{apex}{}^2} - 0.6 \times \frac{q_{dII}}{b} = 0.01 \times \frac{6 \times 940 \times 10^6}{190 \times 1429^2} - 0.6 \times \frac{18.8}{190} = 0.086 \text{ MPa}$$

$$Vol \cong b \times h_{apex}{}^2 = 0.19 \times 1.429^2 = 0.388 \text{ m}^3 \qquad k_{vol} = \left(\frac{V_0}{Vol}\right)^{0.2} = \left(\frac{0.01}{0.388}\right)^{0.2} = 0.481$$

Tensile stress perpendicular to the grain verification (*EN 1995-1-1, equation 6.50*):

$$\frac{\sigma_{t,90,d}}{k_{dis} \times k_{vol} \times f_{t,90,d}} = \frac{0.086}{1.4 \times 0.48 \times 0.32} = 0.40 \quad < 1 \quad \textbf{OK}$$

f) Stability check for lateral torsional buckling

The beam is laterally stiffened by means of a bracing system; braced points are 1.80 m apart. Between two purlins the cross-section depth is considered constant. The verification is performed at the location where the bending moment is maximum, i.e. at $x = x_{max}$:

$$\sigma_{m,\alpha,d} = \frac{6M_{x,max}}{b \times h_{x,max}{}^2} = \frac{6 \times 758 \times 10^6}{190 \times 1152.2^2} = 18 \text{ MPa}$$

Effective buckling length:

$$l_{0,z} = 1.80 \text{ m}$$

Critical bending stress:

$$\sigma_{cr,m} = \frac{\pi}{l_{0,z} \times W_y} \times \sqrt{E_{0.05} \times I_z \times G_{05} \times I_{tor}} = \frac{\pi}{1.8 \times 10^3 \times \frac{1152.2^2 \times 190}{6}} \times \sqrt{10800 \times \frac{1152.2 \times 190^3}{12} \times 542 \times \frac{190^3 \times 1152.2}{3}} = 132.3 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{cr,m}}} = \sqrt{\frac{30}{132.3}} = 0.5$$

Critical factor for lateral torsional buckling:

$$k_{crit} = 1$$

The lateral torsional buckling critical factor is equal to 1. Hence, no need for lateral torsional buckling verification.

2.7 SLS verifications

Two different load combinations are considered:

Combination SLS 1 (permanent loads):

$$q_{\text{sls},1} = \left(g_{\text{k},1} + g_{\text{k},2}\right) = 5.1 \text{ kN/m}$$

Combination SLS 2 (snow load):

$$q_{\text{sls},2} = s_{\text{k}} = 8.46 \text{ kN/m}$$

Select the critical combination in the the ultimate limit state:

$$h_{\text{e}} = h_0 + 0.33 \times l_{\text{tot}} \times \tan(\alpha) = 1215.2 \text{ mm}$$

$$w_1 = \frac{5}{384} \times \frac{q_1 l_{\text{tot}}^{\ 4}}{E_{0,\text{mean}} \times \frac{b \times h_{\text{e}}^{\ 3}}{12}} + 0.35 \times \frac{q_1 l_{\text{tot}}^{\ 2}}{G_{\text{mean}} \times b \times \left(h_{\text{apex}} + h_0\right)} = \frac{5}{384} \times \frac{1 \times 20000^4}{13000 \times \frac{190 \times 1215.24^3}{12}} + 0.35 \times \frac{1 \times 20000^2}{650 \times 190 \times (1429 + 800)} = 6.15 \text{ mm}$$

Where the deflection due to the shear force is:

$$w_{\text{shear}} = 0.35 \times \frac{l_{\text{tot}}^{\ 2}}{G_{\text{mean}} \times b \times \left(h_{\text{apex}} + h_0\right)} = 0.51 \qquad \frac{w_{\text{shear}}}{w_1} = 8.3\ \%$$

the deflection due to the bending moment is:

$$w_{\text{bending}} = \frac{5}{384} \times \frac{l_{\text{tot}}^{\ 4}}{E_{0,\text{mean}} \times \frac{b \times h_{\text{e}}^{\ 3}}{12}} = 5.64 \qquad \frac{w_{\text{bending}}}{w_1} = 91.7\ \%$$

Instantaneous deflection due to permanent load:

$$w_{\text{inst,permanent}} = w_1 \times q_{\text{sls},1} = 6.1 \times 5.1 = 31.1 \text{ mm}$$

Instantaneous deflection due to snow load:

$$w_{\text{inst,snow}} = w_1 \times q_{\text{sls},2} = 6.1 \times 8.46 = 52.0 \text{ mm}$$

Instantaneous deflection verification, *see table 11.4, page 51*:

$$w_{\text{inst,permanent}} + w_{\text{inst,snow}} = 83.1 \text{ mm} \quad > \quad \frac{l_{\text{tot}}}{300} = 67 \text{ mm}$$ **NOT OK**

Final deflection due to permanent load:

$$w_{\text{final,perm}} = w_{\text{inst,permanent}} \times \left(1 + k_{\text{def}}\right) = 31.1 \times (1 + 0.6) = 49.8 \text{ mm}$$

Final deflection due to snow load:

$$w_{\text{final,snow}} = w_{\text{inst,snow}} \times \left(1 + \psi_{2,\text{snow}} \times k_{\text{def}}\right) = 52.0 \times (1 + 0.1 \times 0.6) = 55.1 \text{ mm}$$

Total final deflection:

$$w_{\text{final,tot}} = w_{\text{final,snow}} + w_{\text{final,perm}} = 55.1 + 49.8 = 104.9 \text{ mm}$$

Total deflection verification, *see table 11.4, page 51*:

$$w_{\text{final,tot}} = 104.9 \text{ mm} \quad > \frac{l_{\text{tot}}}{250} = 80 \text{ mm} \quad \textbf{NOT OK}$$

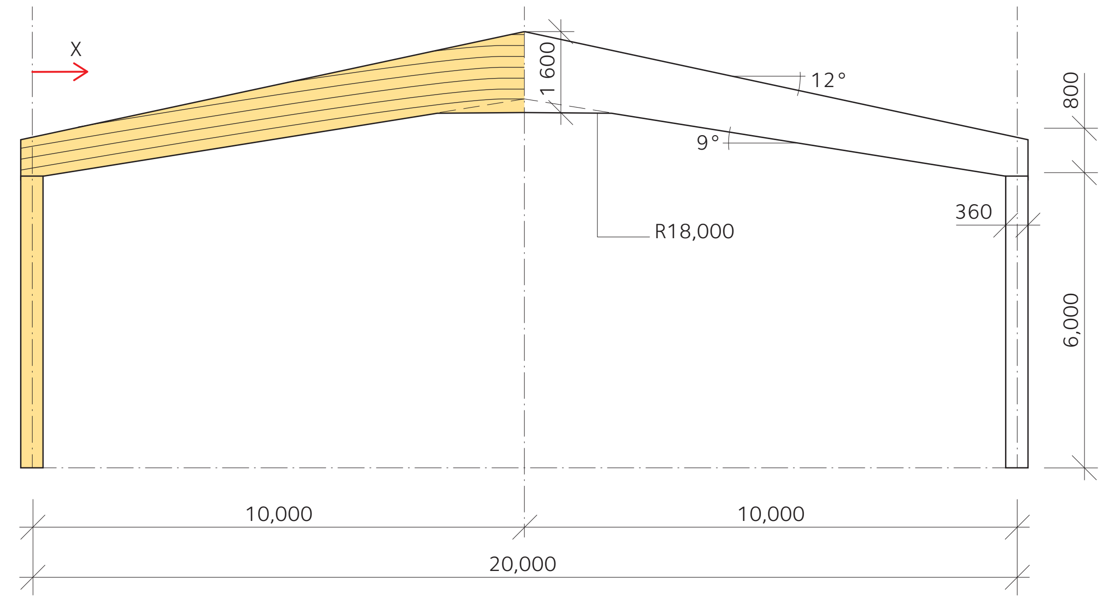
The verification for instantaneous deflection is not satisfied. Therefore, the depth of the cross-section may be increased, e.g. by means of one or more laminations. It is also possible to precamber the beam.

Example 3: Design of a pitched cambered beam

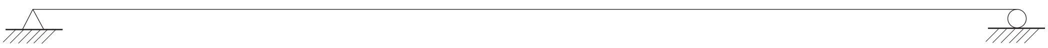
3.1 System, dimensions and design parameters

Design and verify the pitched cambered beam below.

Pitched cambered beam

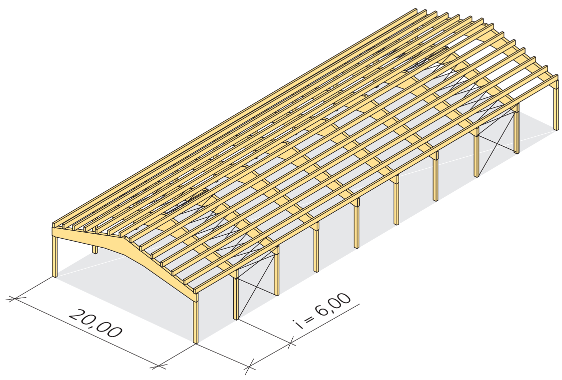


Static system



The cross-section dimensions are determined using the preliminary design method shown in *section 3.4, page 95*.

The beam is made of glulam, strength class	GL30c
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for snow load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$



3.2 Loads

The loads considered in the design are:

Structural

$$g_{k,1} = 1.2 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.60 \text{ kN/m}^2$ $\qquad$ $g_{k,2} = G_{k,2} \times i \times 1.1 = 0.60 \times 6 \times 1.1 = 4 \text{ kN/m}$

Snow load

$S_k = 1.5 \text{ kN/m}^2$ $\qquad$ $s_k = S_k \times i \times \mu \times 1.1 = 1.5 \times 6 \times 0.98 \times 1.1 = 9.7 \text{ kN/m}$

Factor 1.1 used in the equations above accounts for the continuity of purlins over beams.

3.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times (1.2 + 4) = 6.2 \text{ kN/m}$$

Combination 2 (self-load leading + snow load, medium term load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_k\right] = 1 \times \left[1.2 \times (1.2 + 4) + 1.5 \times 9.7\right] = 20.8 \text{ kN/m}$$

Select the critical combination in the the ultimate limit state:

$$\frac{q_{dI}}{k_{mod,1}} = \frac{6.2}{0.6} = 10.3 \quad < \quad \frac{q_{dII}}{k_{mod,2}} = \frac{20.8}{0.8} = 26.0$$

Thus combination 2 is leading.

3.4 Preliminary design

Beam:

$$r \geq 10 \text{ m} \quad \rightarrow \quad r = 18 \text{ m}$$

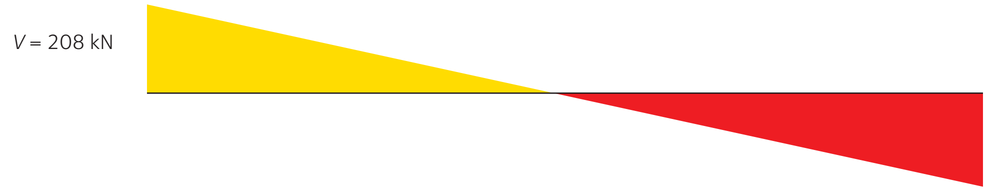
$$b = \frac{l_{tot}}{100} = \frac{20 \times 10^3}{100} = 200 \text{ mm} \quad \rightarrow \quad b = 215 \text{ mm}$$

$$h_{apex} = \frac{l_{tot}}{13} = \frac{20 \times 10^3}{13} = 1538 \text{ mm} \quad \rightarrow \quad h_{apex} = 1600 \text{ mm}$$

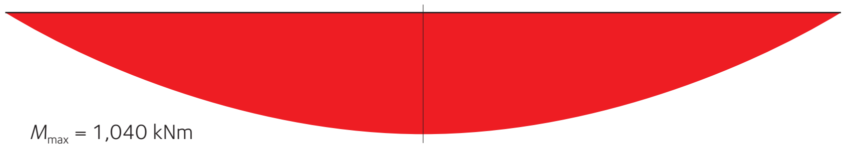
$$h_0 = \frac{l_{tot}}{30} = \frac{20 \times 10^3}{30} = 667 \text{ mm} \quad \rightarrow \quad h_0 = 800 \text{ mm}$$

3.5 Internal forces and moments

Shear:



Bending moment:



3.6 ULS verifications

a) Shear

The design shear stress τ_d is determined using the reduced value of the shear force at the support V_{red}, *see table 8.5, page 22*:

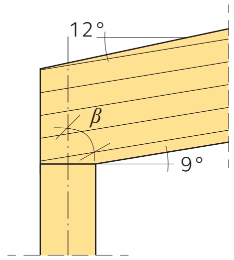
$$V_{red} = \frac{2 \times V_{Ed}}{l_{tot}} \times \left(\frac{l_{tot}}{2} - \frac{h_{col}}{2} - h_0\right) = \frac{2 \times 208}{20 \times 10^3} \times \left(\frac{20 \times 10^3}{2} - \frac{360}{2} - 800\right) = 188 \text{ kN}$$

$$\tau_d = \frac{3 \times V_{red}}{2 \times b \times h_0} = \frac{3 \times 188 \times 10^3}{2 \times 215 \times 800} = 1.64 \text{ MPa}$$

Shear verification (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau_d}{f_{v,d} \times k_{cr}} = \frac{1.64}{2.24 \times 0.86} = 0.85 \quad < 1 \quad \textbf{OK}$$

b) Compression at an angle β to the grain at the support



In this example a column with a depth of 360 mm is considered:

$$\beta = 90° - \alpha_{int} = 90° - 9° = 81°$$

The compression stress at the support is:

$$\sigma_{c,\beta,d} = \frac{q_{dII} \times l_{tot}}{2 \times b_{col} \times \left(h_{col} + 30 \times \cos(9°)\right)} = \frac{20.8 \times 20 \times 10^3}{2 \times 215 \times \left(360 + 30 \times \cos(9°)\right)} = 2.48 \text{ MPa}$$

Compression strength at an angle β to the grain:

$$f_{c,\beta,d} = \frac{f_{c,0,d}}{\frac{f_{c,0,d}}{\left(k_{c,90} \times f_{c,90,k}\right)} \times \left(\sin(\beta)\right)^2 + \left(\cos(\beta)\right)^2} = \frac{15.68}{\frac{15.68}{1.75 \times 1.6} \times \sin(81°)^2 + \cos(81°)^2} = 2.86 \text{ MPa}$$

$f_{c,90,d}$ can not be replaced by $f_{c,90,k}$ due to the fact that $g_k/s_k = 0.54 > 0.4$, *see table 8.11, page 25, 8.12, page 25* and *8.13, page 26*.

Compression at an angle β to the grain verification (*EN 1995-1-1, equation 6.16*):

$$\frac{\sigma_{c,\beta,d}}{f_{c,\beta,d}} = \frac{2.48}{2.86} = 0.87 \quad < 1 \quad \textbf{OK}$$

c) Bending stress at the most stressed cross-section

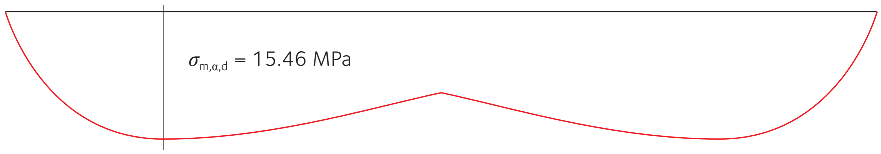
$$x_{\max} = \frac{l_{\text{tot}} \times h_0}{2 \times h_{\text{apex}}} = \frac{20 \times 10^3 \times 800}{2 \times 1600} = 5000 \text{ mm}$$

$$M_{\text{x,max}} = \frac{q_{\text{dII}} \times x_{\max}}{2} \times \left(l_{\text{tot}} - x_{\max}\right) = \frac{20.8 \times 5}{2} \times (20 - 5) = 780 \text{ kNm}$$

$$h_{\text{x,max}} = 848 + \left(x_{\max} - \frac{h_{\text{col}}}{2}\right) \times \left(\tan\left(\alpha_{\text{ext}}\right) - \tan\left(\alpha_{\text{int}}\right)\right) \times \cos\left(\alpha_{\text{int}}\right) = 848 + \left(5000 - \frac{360}{2}\right) \times \left(\tan(12°) - \tan(9°)\right) \times \cos(9°) = 1105.9 \text{ mm}$$

$$\sigma_{\text{m},\alpha,\text{d}} = \frac{6M_{\text{x,max}}}{b \times h_{\text{x,max}}^{\;2}} = \frac{6 \times 780 \times 10^6}{215 \times 1105.9^2} = 17.8 \text{ MPa}$$

Bending stress diagram:



The design bending strength shall be decreased by the factor $k_{\text{m},\alpha}$, which accounts for the simultaneous action of bending stress, shear stress and compression stress:

$$k_{\text{m},\alpha} = \frac{1}{\sqrt{1 + \left(\frac{f_{\text{m,d}}}{1.5 \times f_{\text{v,d}}} \times \tan\left(\Delta_\alpha\right)\right)^2 + \left[\frac{f_{\text{m,d}}}{f_{\text{c,90,d}}} \times \left(\tan\left(\Delta_\alpha\right)\right)^2\right]^2}} = \frac{1}{\sqrt{1 + \left(\frac{19.2}{1.5 \times 2.24} \times \tan(12° - 9°)\right)^2 + \left[\frac{19.2}{1.6} \times \left(\tan(12° - 9°)\right)^2\right]^2}} = 0.96$$

Bending stress verification (*EN 1995-1-1, equation 6.38*):

$$\frac{\sigma_{\text{m},\alpha,\text{d}}}{k_{\text{m},\alpha} \times f_{\text{m,d}}} = \frac{17.8}{0.96 \times 19.2} = 0.97 \quad < 1 \quad \textbf{OK}$$

d) Bending stress at the apex

The bending stress at the apex is increased by k_l, which takes into account that the neutral axis is not at the mid-depth of cross-section (*EN 1995-1-1, equation 6.43*):

$$k_1 = 1 + 1.4 \times \tan\left(\alpha_{\text{ext}}\right) + 5.4 \times \left(\tan\left(\alpha_{\text{ext}}\right)\right)^2 = 1 + 1.4 \times \tan(12°) + 5.4 \times \tan(12°)^2 = 1.54$$

$$k_2 = 0.35 - 8 \times \tan\left(\alpha_{\text{ext}}\right) = 0.35 - 8 \times \tan(12°) = -1.35$$

$$k_3 = 0.6 + 8.3 \times \tan\left(\alpha_{\text{ext}}\right) - 7.8 \times \left(\tan\left(\alpha_{\text{ext}}\right)\right)^2 = 0.6 + 8.3 \times \tan(12°) - 7.8 \times \tan(12°)^2 = 2.01$$

$$k_4 = 6 \times \left(\tan\left(\alpha_{\text{ext}}\right)\right)^2 = 6 \times \tan(12°)^2 = 0.27$$

$$R = R_{\text{int}} + 0.5 \times h_{\text{apex}} = 18 \times 10^3 + 0.5 \times 1.6 \times 10^3 = 1.88 \times 10^4 \text{ mm}$$

$$k_l = k_1 + k_2 \times \left(\frac{h_{\text{apex}}}{R}\right) + k_3 \times \left(\frac{h_{\text{apex}}}{R}\right)^2 + k_4 \times \left(\frac{h_{\text{apex}}}{R}\right)^3 = 1.54 + -1.35 \times \frac{1.6 \times 10^3}{1.88 \times 10^4} + 2.01 \times \left(\frac{1.6 \times 10^3}{1.88 \times 10^4}\right)^2 + 0.27 \times \left(\frac{1.6 \times 10^3}{1.88 \times 10^4}\right)^3 = 1.44$$

Bending stress:

$$\sigma_{\text{m,d}} = k_l \times \frac{6 \times M_{\max}}{b \times h_{\text{apex}}^{\;2}} = 1.44 \times \frac{6 \times 1040 \times 10^6}{215 \times 1600^2} = 16.3 \text{ MPa}$$

The tensile strength perpendicular to the grain shall be modified by factor k_r (*EN 1995-1-1, equation 6.49*):

$$\frac{R_{int}}{45} = \frac{18 \times 10^3}{45} = 400$$

$$k_r = 1.0$$

Bending stress verification (*EN 1995-1-1, equation 6.41*):

$$\frac{\sigma_{m,d}}{k_r \times f_{m,d}} = \frac{16.3}{1 \times 19.2} = 0.85 \quad < 1 \quad \textbf{OK}$$

e) Tensile stress perpendicular to the grain at the apex

Factor k_p (*EN 1995-1-1, equation 6.56*):

$$k_5 = 0.2 \times \tan\left(\alpha_{ext}\right) = 0.2 \times \tan(12°) = 0.04$$

$$k_6 = 0.25 - 1.5 \times \tan\left(\alpha_{ext}\right) + 2.6 \times \left(\tan\left(\alpha_{ext}\right)\right)^2 = 0.25 - 1.5 \times \tan(12°) + 2.6 \times \tan(12°)^2 = 0.05$$

$$k_7 = 2.1 \times \tan\left(\alpha_{ext}\right) - 4 \times \left(\tan\left(\alpha_{ext}\right)\right)^2 = 2.1 \times \tan(12°) - 4 \times \tan(12°)^2 = 0.27$$

$$k_p = k_5 + k_6 \times \left(\frac{h_{apex}}{R}\right) + k_7 \times \left(\frac{h_{apex}}{R}\right)^2 = 0.04 + 0.05 \times \frac{1.6 \times 10^3}{1.88 \times 10^4} + 0.27 \times \left(\frac{1.6 \times 10^3}{1.88 \times 10^4}\right)^2 = 0.05$$

Tensile stress (*EN 1995-1-1, equation 6.54*):

$$\sigma_{t,90,d} = k_p \times \frac{6 \times M_{max}}{b \times h_{apex}^{\ 2}} - 0.6 \times \frac{q_{dII}}{b} = 0.05 \times \frac{6 \times 1040 \times 10^6}{215 \times 1600^2} - 0.6 \times \frac{20.8}{215} = 0.51 \text{ MPa}$$

Tensile strength perpendicular to the grain shall be modified by factors k_{vol} and k_{dis} (*EN 1995-1-1, equation 6.51 and 6.52*):

$$Vol = b \times \left[\left(R_{int} + h_{apex}\right)^2 \times \sin\left(\alpha_{int}\right) \times \left(\cos\left(\alpha_{int}\right) - \sin\left(\alpha_{int}\right) \times \tan\left(\alpha_{ext} - \alpha_{int}\right)\right) - R_{int}^{\ 2} \times \frac{\alpha_{int} \times \pi}{180°}\right] = 1.713 \text{ m}^3$$

$$k_{dis} = 1.7$$

$$k_{vol} = \left(\frac{V_0}{Vol}\right)^{0.2} = \left(\frac{0.01}{1.713}\right)^{0.2} = 0.357$$

Tensile stress perpendicular to the grain verification (*EN 1995-1-1, equation 6.50*):

$$\frac{\sigma_{t,90,d}}{k_{dis} \times k_{vol} \times f_{t,90,d}} = \frac{0.51}{1.7 \times 0.36 \times 0.32} = 2.6 \quad > 1 \quad \textbf{NOT OK}$$

The beam needs to be reinforced for tensile stress perpendicular to the grain at the apex zone; the design of the reinforcement is shown in *example 16, page 189*.

f) Stability check for lateral torsional buckling

The beam is laterally stiffened by means of a bracing system; braced points are 1.80 m apart. Between two purlins the cross-section depth is considered constant. The verification is performed at the location where the bending moment is maximum, i.e. at $x = x_{max}$:

$$\sigma_{m,\alpha,d} = \frac{6 \times M_{x,max}}{b \times h_{x,max}{}^2} = \frac{6 \times 780 \times 10^6}{215 \times 1105.89^2} = 17.8 \text{ MPa}$$

Effective buckling length:

$$l_{0,z} = 1.80 \text{ m}$$

Critical bending stress:

$$\sigma_{cr,m} = \frac{\pi}{l_{0,z} \times W_y} \times \sqrt{E_{0.05} \times I_z \times G_{05} \times I_{tor}} = \frac{\pi}{1.8 \times 10^3 \times \frac{1105.9^2 \times 215}{6}} \times \sqrt{10800 \times \frac{1105.9 \times 215^3}{12} \times 542 \times \frac{215^3 \times 1105.9}{3}} = 176.5 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{cr,m}}} = \sqrt{\frac{30}{176.5}} = 0.4$$

Critical factor for lateral torsional buckling:

$$k_{crit} = 1$$

The lateral torsional buckling critical factor is equal to 1. Hence, no need for lateral torsional buckling verification.

3.7 SLS verifications

Two different load combinations are considered:

Combination SLS 1 (permanent loads):

$$q_{sls,1} = g_{k,1} + g_{k,2} = 5.2 \text{ kN/m}$$

Combination SLS 2 (snow load):

$$q_{sls,2} = s_k = 9.7 \text{ kN/m}$$

The deflection is calculated for a uniformly distributed unit load q_1, *see The Glulam Handbook Volume 2, section 6.2.6, page 90*:

$$k_I = 0.15 + 0.85 \times \frac{h_0}{h_{apex}} = 0.58$$

$$w_{1,bending} = \frac{5}{384} \times \frac{l_{tot}^4}{E_{0,mean} \times \frac{b \times h_{apex}{}^3}{12}} \times \frac{1}{k_I} \times \frac{1}{\cos\left(\frac{\alpha_{ext} + \alpha_{int}}{2}\right)} = \frac{5}{384} \times \frac{20000^4}{13000 \times \frac{215 \times 1600^3}{12}} \times \frac{1}{0.6} \times \frac{1}{\cos\left(\frac{12° + 9°}{2}\right)} = 3.9 \text{ mm}$$

$$w_{1,shear} = 1.2 \times \frac{l_{tot}^2}{8 \times G_{mean} \times b \times h_0} \times \frac{2 \times h_0^{\frac{2}{3}}}{h_0^{\frac{2}{3}} + h_{apex}^{\frac{2}{3}}} = 0.4 \text{ mm}$$

The additional deflection of the beam due to shear deformation is neglected:

$$w_1 = w_{1,\text{bending}} = 3.9 \text{ mm}$$

Instantaneous deflection due to permanent load:

$$w_{\text{inst,permanent}} = w_1 \times q_{\text{sls},1} = 3.9 \times 5.2 = 19.9 \text{ mm}$$

Instantaneous deflection due to snow load:

$$w_{\text{inst,snow}} = w_1 \times q_{\text{sls},2} = 3.9 \times 9.7 = 37.8 \text{ mm}$$

Instantaneous deflection verification, *see table 11.4, page 51*:

$$w_{\text{inst,permanent}} + w_{\text{inst,snow}} = 57.7 \text{ mm} \quad < \quad \frac{l_{\text{tot}}}{300/1.5} = 100 \text{ mm} \quad \textbf{OK}$$

Final deflection due to permanent load:

$$w_{\text{final,perm}} = w_{\text{inst,permanent}} \times \left(1 + k_{\text{def}}\right) = 19.9 \times \left(1 + 0.6\right) = 31.9 \text{ mm}$$

Final deflection due to snow load:

$$w_{\text{final,snow}} = w_{\text{inst,snow}} \times \left(1 + \psi_{2,\text{snow}} \times k_{\text{def}}\right) = 37.8 \times \left(1 + 0.1 \times 0.6\right) = 40.1 \text{ mm}$$

Total final deflection:

$$w_{\text{final,tot}} = w_{\text{final,snow}} + w_{\text{final,perm}} = 40.1 + 31.9 = 72.0 \text{ mm}$$

Total deflection verification, *see table 11.4, page 51*:

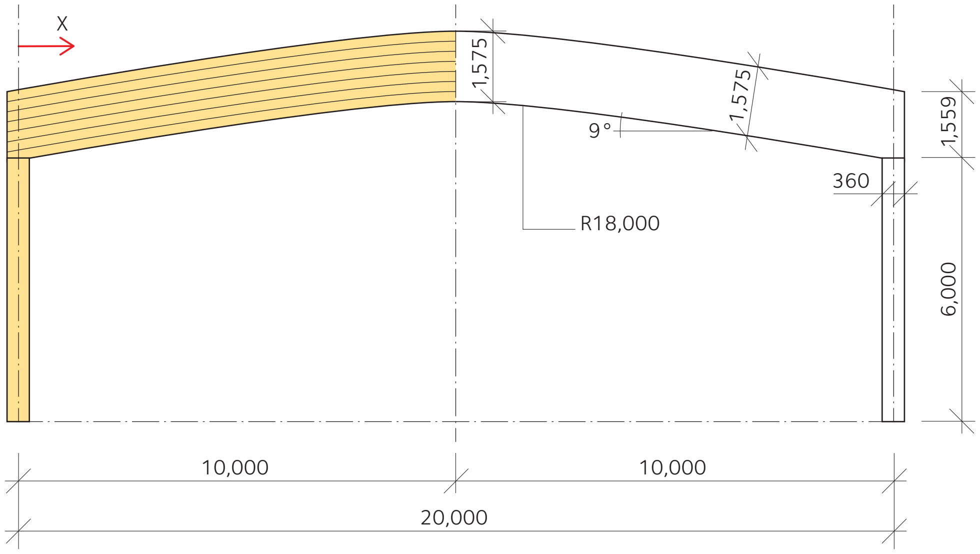
$$w_{\text{final,tot}} = 72.0 \text{ mm} \quad < \quad \frac{l_{\text{tot}}}{250/1.5} = 120 \text{ mm} \quad \textbf{OK}$$

Example 4: Design of a curved beam

4.1 System, dimensions and design parameters

Design and verify the curved beam below.

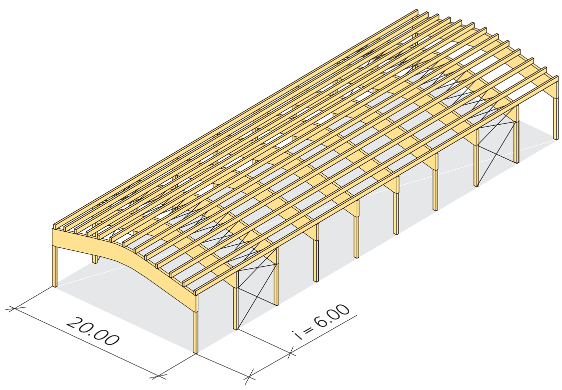
Curved beam



Static system

The cross-section dimensions are determined using the preliminary design method shown in *section 4.4, page 102*.

The beam is made of glulam, strength class	GL30c
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for snow load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$



4.2 Loads

The loads considered in the design are:

Structural

$$g_{k,1} = 1 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.60 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.6 \times 6 \times 1.1 = 4 \text{ kN/m}$

Snow load

$S_k = 1.5 \text{ kN/m}^2$ $\quad s_k = S_k \times i \times \mu \times 1.1 = 1.5 \times 6 \times 0.935 \times 1.1 = 9.3 \text{ kN/m}$

Factor 1.1 used in the equations above accounts for the continuity of purlins over trusses.

4.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times (1 + 4) = 6 \text{ kN/m}$$

Combination 2 (self-load leading + snow load, medium term symmetric load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_k\right] = 1 \times \left[1.2 \times (1 + 4) + 1.5 \times 9.3\right] = 19.8 \text{ kN/m}$$

Leading design combinations at ULS:

$$\frac{q_{dI}}{k_{mod,1}} = \frac{6}{0.6} = 9.9 \quad < \quad \frac{q_{dII}}{k_{mod,2}} = \frac{19.8}{0.8} = 24.8$$

Thus combination 2 is leading.

4.4 Preliminary design

The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, section 7.3.4, page 114*:

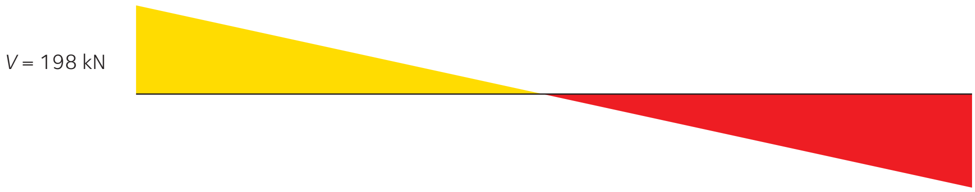
A beam with a constant cross-section is chosen:

$$b = \frac{l_{tot}}{120} = \frac{20 \times 10^3}{120} = 167 \text{ mm} \quad \rightarrow \quad b = 165 \text{ mm}$$

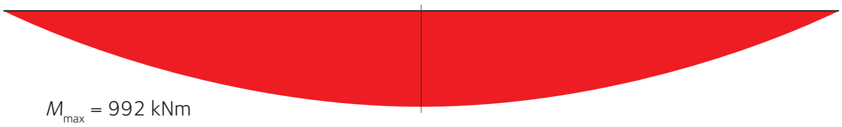
$$h_{apex} = \frac{l_{tot}}{13} = \frac{20 \times 10^3}{13} = 1538 \text{ mm} \quad \rightarrow \quad h_{apex} = 1575 \text{ mm}$$

4.5 Internal forces and moments

Shear:



Bending moment:



4.6 ULS verifications

a) Shear

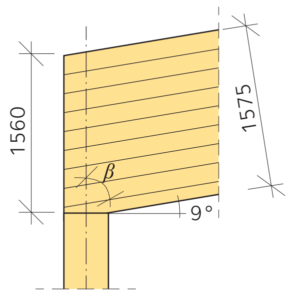
The design shear stress τ_d is determined using the reduced value of the shear force at the support V_{red}, *see table 8.5, page 22*:

$$V_{red} = \frac{2 \times V_{Ed}}{l_{tot}} \times \left(\frac{l_{tot}}{2} - \frac{h_{col}}{2} - h_0\right) = \frac{2 \times V}{20 \times 10^3} \times \left(\frac{20 \times 10^3}{2} - \frac{360}{2} - 1559\right) = 163.87$$

$$\tau_d = \frac{3 \times V_{red}}{2 \times b \times h_0} = \frac{3 \times 163.87 \times 10^3}{2 \times 165 \times 1559} = 0.96 \text{ MPa}$$

Shear verification (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau_d}{f_{v,d} \times k_{cr}} = \frac{0.96}{2.24 \times 0.86} = 0.50 \quad < 1 \quad \textbf{OK}$$



b) Compression at an angle β to the grain at the support

In this example a column with a depth of 360 mm is considered:

$$\beta = 90° - \alpha = 90° - 9° = 81°$$

The compression stress at the support is:

$$\sigma_{c,\beta,d} = \frac{q_{dII} \times l_{tot}}{2 \times b_{col} \times \left(h_{col} + 30 \times \cos(9°)\right)} = \frac{19.8 \times 20 \times 10^3}{2 \times 165 \times \left(360 + 30 \times \cos(9°)\right)} = 3.09 \text{ MPa}$$

Compression strength at an angle β to the grain (*EN 1995-1-1, equation 6.16*):

$$f_{c,\beta,d} = \frac{f_{c,0,d}}{\frac{f_{c,0,d}}{\left(k_{c,90} \times f_{c,90,d}\right)} \times \left(\sin(\beta)^2 + \cos(\beta)^2\right)} = \frac{15.68}{\frac{15.68}{1.75 \times 1.6} \times \left(\sin(81°)^2 + \cos(81°)^2\right)} = 2.86 \text{ MPa}$$

Compression at an angle β to the grain verification (*EN 1995-1-1, equation 6.16*):

ex4-6-07 $$\frac{\sigma_{c,\beta,d}}{f_{c,\beta,d}} = \frac{3.09}{2.86} = 1.08 \quad > 1 \quad \textbf{NOT OK}$$

The length of the support should be increased, or the support should be reinforced, for example with wood screws.

c) Bending stress

The bending stress at the apex is increased by k_l, which takes into account that the neutral axis is not at the mid-depth of cross-section (*EN 1995-1-1, equation 6.43*):

$$k_1 = 1 + 1.4 \times \tan\left(\alpha_{apex}\right) + 5.4 \times \left(\tan\left(\alpha_{apex}\right)\right)^2 = 1 + 1.4 \times \tan(0) + 5.4 \times \tan(0)^2 = 1$$

$$k_2 = 0.35 - 8 \times \tan\left(\alpha_{apex}\right) = 0.35 - 8 \times \tan(0) = 0.35$$

$$k_3 = 0.6 + 8.3 \times \tan\left(\alpha_{apex}\right) - 7.8 \times \left(\tan\left(\alpha_{apex}\right)\right)^2 = 0.6 + 8.3 \times \tan(0) - 7.8 \times \tan(0)^2 = 0.6$$

$$k_4 = 6 \times \left(\tan\left(\alpha_{apex}\right)\right)^2 = 6 \times \tan(0)^2 = 0$$

$$R = R_{int} + 0.5 \times h_{apex} = 18 \times 10^3 + 0.5 \times 1575 = 18788 \text{ mm}$$

$$k_l = k_1 + k_2 \times \left(\frac{h_{apex}}{R}\right) + k_3 \times \left(\frac{h_{apex}}{R}\right)^2 + k_4 \times \left(\frac{h_{apex}}{R}\right)^3 = 1 + 0.35 \times \frac{1.58 \times 10^3}{1.88 \times 10^4} + 0.6 \times \left(\frac{1.58 \times 10^3}{1.88 \times 10^4}\right)^2 + 0 \times \left(\frac{1.58 \times 10^3}{1.88 \times 10^4}\right)^3 = 1.03$$

$$\sigma_{m,d} = k_l \times \frac{6 \times M_{max}}{b \times {h_{apex}}^2} = 1.03 \times \frac{6 \times 992 \times 10^6}{165 \times 1575^2} = 14.98 \text{ MPa}$$

The tensile strength perpendicular to the grain shall be modified by factor k_r (*EN 1995-1-1, equation 6.49*):

$$\frac{R_{int}}{45} = \frac{18 \times 10^3}{45} = 400$$

$$k_r = 1.0$$

Bending stress verification (*EN 1995-1-1, equation 6.41*):

$$\frac{\sigma_{m,d}}{k_r \times f_{m,d}} = \frac{14.98}{1 \times 19.2} = 0.78 \quad < 1 \quad \textbf{OK}$$

d) Tensile stress perpendicular to the grain at the apex

Factor k_p (*EN 1995-1-1, equation 6.56*):

$$k_5 = 0.2 \times \tan\left(\alpha_{apex}\right) = 0.2 \times \tan(0°) = 0$$

$$k_6 = 0.25 - 1.5 \times \tan\left(\alpha_{apex}\right) + 2.6 \times \left(\tan\left(\alpha_{apex}\right)\right)^2 = 0.25 - 1.5 \times \tan(0°) + 2.6 \times \tan(0°)^2 = 0.25$$

$$k_7 = 2.1 \times \tan\left(\alpha_{apex}\right) - 4 \times \left(\tan\left(\alpha_{apex}\right)\right)^2 = 2.1 \times \tan(0°) - 4 \times \tan(0°)^2 = 0$$

$$k_p = k_5 + k_6 \times \left(\frac{h_{apex}}{R}\right) + k_7 \times \left(\frac{h_{apex}}{R}\right)^2 = 0 + 0.25 \times \frac{1575}{18787.5} + 0 \times \left(\frac{1575}{18787.5}\right)^2 = 0.021$$

Tensile stress (*EN 1995-1-1, equation 6.55*):

$$\sigma_{t,90,d} = k_p \times \frac{6 \times M_{max}}{b \times {h_{apex}}^2} - 0.6 \times \frac{q_{dII}}{b} = 0.02 \times \frac{6 \times 992 \times 10^6}{165 \times 1575^2} - 0.6 \times \frac{19.8}{165} = 0.23 \text{ MPa}$$

Tensile strength perpendicular to the grain shall be modified by factors k_{vol} and k_{dis} (*EN 1995-1-1, equation 6.51* and *6.52*):

$$Vol = b \times h_{apex} \times \left(2 \times R_{int} + h_{apex}\right) \times \alpha \times \frac{\pi}{180°} = 0.17 \times 1.58 \times \left(2 \times 18 + 1.58\right) \times 9° \times \frac{3.14}{180°} = 1.53 \text{ m}^3$$

$$k_{dis} = 1.4$$

$$k_{vol} = \left(\frac{V_0}{Vol}\right)^{0.2} = \left(\frac{0.01}{1.533}\right)^{0.2} = 0.365$$

Tensile stress perpendicular to the grain verification (*EN 1995-1-1, equation 6.50*):

$$\frac{\sigma_{t,90,d}}{k_{dis} \times k_{vol} \times f_{t,90,d}} = \frac{0.23}{1.4 \times 0.37 \times 0.32} = 1.43 \quad > 1 \quad \textbf{NOT OK}$$

The beam needs to be reinforced for tensile stress perpendicular to the grain at the apex zone.

e) Stability check for lateral torsional buckling

The beam is stiffened by means of a bracing system; braced points are 1.80 m apart.

Length between two supports:

$$l_{0,z} = 1.80 \text{ m}$$

Critical bending stress:

$$\sigma_{cr,m} = \frac{\pi}{l_{0,z} \times W_y} \times \sqrt{E_{0.05} \times I_z \times G_{05} \times I_{tor}} = \frac{\pi}{1.80 \times 10^3 \times \frac{1575^2 \times 165}{6}} \times \sqrt{10800 \times \frac{1575 \times 165^3}{12} \times 542 \times \frac{165^3 \times 1575}{3}} = 73 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{cr,m}}} = \sqrt{\frac{30}{73}} = 0.6$$

Critical factor for lateral torsional buckling:

$$k_{crit} = 1$$

The lateral torsional buckling critical factor is equal to 1. Hence, no need for lateral torsional buckling verification.

4.7 SLS verifications

Two different load combinations are considered:

Combination SLS 1 (permanent loads):

$$q_{sls,1} = g_{k,1} + g_{k,2} = 5 \text{ kN/m}$$

Combination SLS 2 (snow load):

$$q_{sls,2} = s_k = 9.3 \text{ kN/m}$$

The deflection is calculated for a uniformly distributed unit load q_1, *see The Glulam Handbook Volume 2, section 6.2.6, page 90*:

$$w_1 = \left(\frac{5}{384} \times \frac{1 \times l_{tot}{}^4}{E_{0,mean} \times \frac{b \times h_0{}^3}{12}} + 1.2 \times \frac{l_{tot}{}^2}{8 \times G_{mean} \times b \times h_0} \right) \times \frac{1}{\cos\left(\frac{2\alpha}{2}\right)} = \left[\frac{5}{384} \times \frac{\left(20 \times 10^3\right)^4}{13000 \times \frac{165 \times 1559^3}{12}} + 1.2 \times \frac{\left(20 \times 10^3\right)^2}{8 \times 650 \times 165 \times 1559} \right] \times \frac{1}{\cos\left(\frac{2.9°}{2}\right)} = 3.5 \text{ mm}$$

where:
The deflection due to the shear force is:

$$w_{shear} = 1.2 \times \frac{l_{tot}{}^2}{8 \times G_{mean} \times b \times h_{apex}} = 0.36 \qquad \frac{w_{shear}}{w_1} = 10\ \%$$

The deflection due to the bending moment is:

$$w_{bending} = \frac{5}{384} \times \frac{1 \times l_{tot}{}^4}{E_{0,mean} \times \frac{b \times h_0{}^3}{12}} = 3.08 \qquad \frac{w_{bending}}{w_1} = 90\ \%$$

Instantaneous deflection due to permanent load:

$$w_{inst,permanent} = w_1 \times q_{sls,1} = 3.5 \times 5 = 17.2 \text{ mm}$$

Instantaneous deflection due to snow load:

$$w_{inst,snow} = w_1 \times q_{sls,2} = 3.5 \times 9.3 = 31.8 \text{ mm}$$

Instantaneous deflection verification, *see table 11.4, page 51*:

$$w_{inst,permanent} + w_{inst,snow} = 49.0 \text{ mm} \quad < \quad \frac{l_{tot}}{300/1.5} = 100 \text{ mm} \quad \textbf{OK}$$

Final deflection due to permanent load:

$$w_{final,perm} = w_{inst,permanent} \times \left(1 + k_{def}\right) = 17.2 \times \left(1 + 0.6\right) = 27.6 \text{ mm}$$

Final deflection due to snow load:

$$w_{final,snow} = w_{inst,snow} \times \left(1 + \psi_{2,snow} \times k_{def}\right) = 31.8 \times \left(1 + 0.1 \times 0.6\right) = 33.8 \text{ mm}$$

Total final deflection:

$$w_{final,tot} = w_{final,snow} + w_{final,perm} = 33.8 + 27.6 = 61.4 \text{ mm}$$

Total deflection verification, *see table 11.4, page 51*:

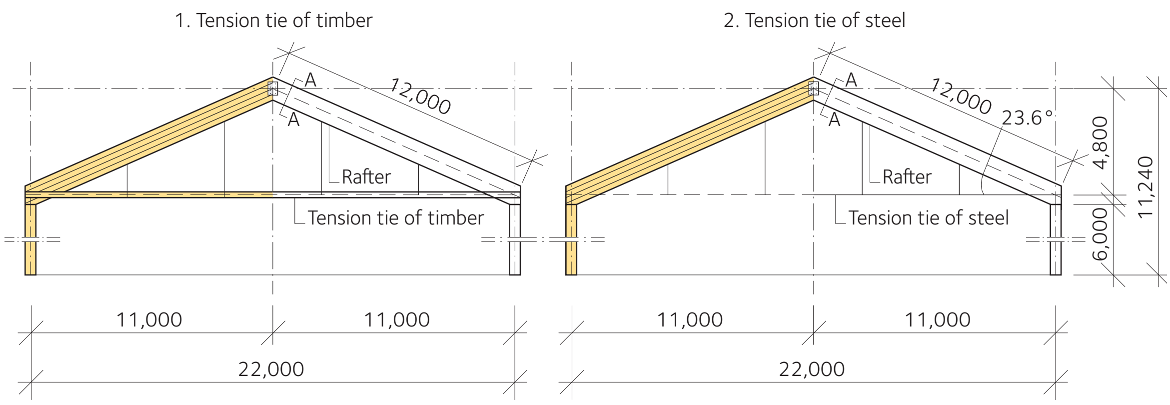
$$w_{final,tot} = 61.4 \text{ mm} \quad < \quad \frac{l_{tot}}{250/1.5} = 120 \text{ mm} \quad \textbf{OK}$$

Example 5: Design of a three-hinged truss

5.1 System, dimensions and design parameters

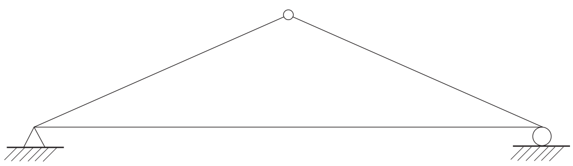
Design and verify the truss below.

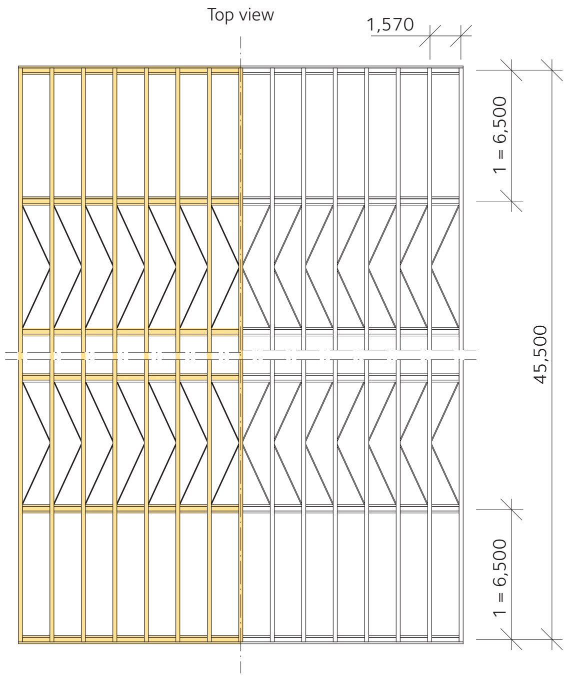
Two structural systems are considered: 1. Tension tie of timber. 2. Tension tie of steel.



The cross-section dimensions are determined using the preliminary design shown in *section 5.4, page 108*.

Static system





Glulam roof truss, strength class	GL30c
Tension tie of timber (1), strength class	GL30c
Tension tie of steel (2), strength class	5.6
Glulam columns, strength class	GL30c
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for snow load	$\gamma_s = 1.5$
Material partial factors for steel	$\gamma_{M0} = 1.0$
	$\gamma_{M2} = 1.2$
Material partial factor for GL	$\gamma_M = 1.25$

5.2 Loads

The loads considered in the design of the truss are:

Structural

$G_{k,1} = 0.1 \text{ kN/m}^2$ $\quad g_{k,1} = G_{k,1} \times i = 0.1 \times 6.5 = 0.65 \text{ kN/m}$

Non-structural

$G_{k,2} = 0.4 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.4 \times 6.5 \times 1.1 = 2.9 \text{ kN/m}$

Snow load

$S_k = 2.0 \text{ kN/m}^2$

Leeward $\quad s_{k,l} = S_k \times i \times \mu \times 1.1 = 2 \times 6.5 \times 1.03 \times 1.1 = 14.75 \text{ kN/m}$

Windward $\quad s_{k,r} = S_k \times i \times \mu \times 1.1 = 2 \times 6.5 \times 0.8 \times 1.1 = 11.4 \text{ kN/m}$

The self-weight considered in the equations above is the load projection onto the horizontal plane.
Factor 1.1 used in the equations above accounts for the continuity of purlins over trusses.
Assumed unsymmetric snow load as per EKS 10. Snow guards may be added.

5.3 Load combinations

Three different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load), $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times \left(0.65 + 2.86\right) = 4.21 \text{ kN/m}$$

Combination 2a (self-load leading + snow load, unsymmetric medium term load, $k_{mod} = 0.8$):

Leeward
$$q_{dIIA,l} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,l}\right] = 1 \times \left[1.2 \times \left(0.65 + 2.86\right) + 1.5 \times 14.75\right] = 26.41 \text{ kN/m}$$

Windward
$$q_{dIIA,r} = \gamma_d \times \left[\gamma_s \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,r}\right] = 1 \times \left[1.2 \times \left(0.65 + 2.86\right) + 1.5 \times 11.44\right] = 21.37 \text{ kN/m}$$

Combination 2b (self-load leading + snow load, unsymmetric medium term load, $k_{mod} = 0.8$):

Leeward
$$q_{dIIB,l} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,l}\right] = 1 \times \left[1.2 \times \left(0.65 + 2.86\right) + 1.5 \times 14.75\right] = 26.41 \text{ kN/m}$$

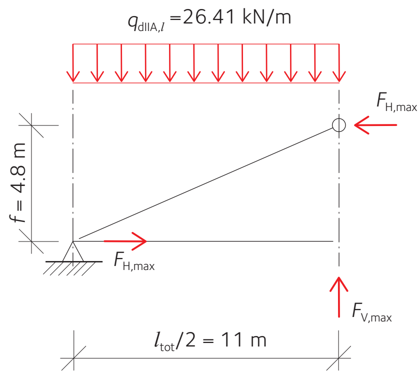
Windward
$$q_{dIIB,r} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times 0.5 \times s_{k,r}\right] = 1 \times \left[1.2 \times \left(0.65 + 2.86\right) + 1.5 \times 0.5 \times 11.44\right] = 12.79 \text{ kN/m}$$

5.4 Preliminary design

The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, section 9.1, page 132 and 9.2, page 135*:

$$F_{V,max} = \frac{\left(3 \times q_{dIIA,l} + q_{dIIA,r}\right) \times l_{tot}}{8} = \frac{\left(3 \times 26.41 + 21.37\right) \times 22}{8} = 277 \text{ kN}$$

$$F_{H,max} = \frac{\left(q_{dIIA,l} + q_{dIIA,r}\right) \times l_{tot}^2}{16 \times f} = \frac{\left(26.41 + 21.37\right) \times 22^2}{16 \times 4.8} = 301 \text{ kN}$$



Rafter:

$$b = \frac{l_{tot}}{170} = \frac{22}{170} = 0.13 \text{ m} \qquad \rightarrow \qquad b = 140 \text{ mm}$$

$$h = \frac{k_1}{2} + 0.5 \times \sqrt{k_1^{\,2} + 4 \times k_2} = \frac{190.7}{2} + 0.5 \times \sqrt{190.7^2 + 4 \times 891632.5} = 1044 \text{ mm} \qquad \rightarrow \qquad h_{apex} = 1035 \text{ mm}$$

where k_1 and k_2 are defined as:

$$k_1 = \frac{q_{dIIA,l} \times l_{tot}}{54.4 \times b \times \sin(\alpha)} = \frac{26.41 \times 22 \times 10^3}{54.4 \times 140 \times \sin\left(23.6°\right)} = 190.7 \text{ mm}$$

$$k_2 = \frac{q_{dIIA,l} \times l_{tot}^{\,2}}{102.4 \times b} = \frac{26.41 \times \left(22 \times 10^3\right)^2}{102.4 \times 140} = 891632.5 \text{ mm}^2$$

Tension tie of timber (it consists of two separate members):

$$b = 90 \text{ mm}$$

$$A_{min} = \frac{F_{H,max}}{0.7 \times f_{t,0,d}} = \frac{301.12 \times 10^3}{0.7 \times 12.48} = 34468.86 \text{ mm}^2$$

$$h_{min} = \frac{A_{min}}{2 \times b} = \frac{34468.86}{2 \times 90} = 191.49 \text{ mm} \qquad \rightarrow \qquad h = 315 \text{ mm}$$

The minimum thickness recommended for the glulam tie rod is 90 mm.
The depth of the two tie rod members should be chosen with regard to:

- compensation for the reduction of net area due to the fastener holes.
- allow for placing of the connectors.

For this example a minimum depth of the timber tension tie h = 315 mm is recommended.
A factor 0.7 is considered to take into account the reduction of the cross-section due to the presence of fasteners.

Tension tie of steel (it consists of two separate members):

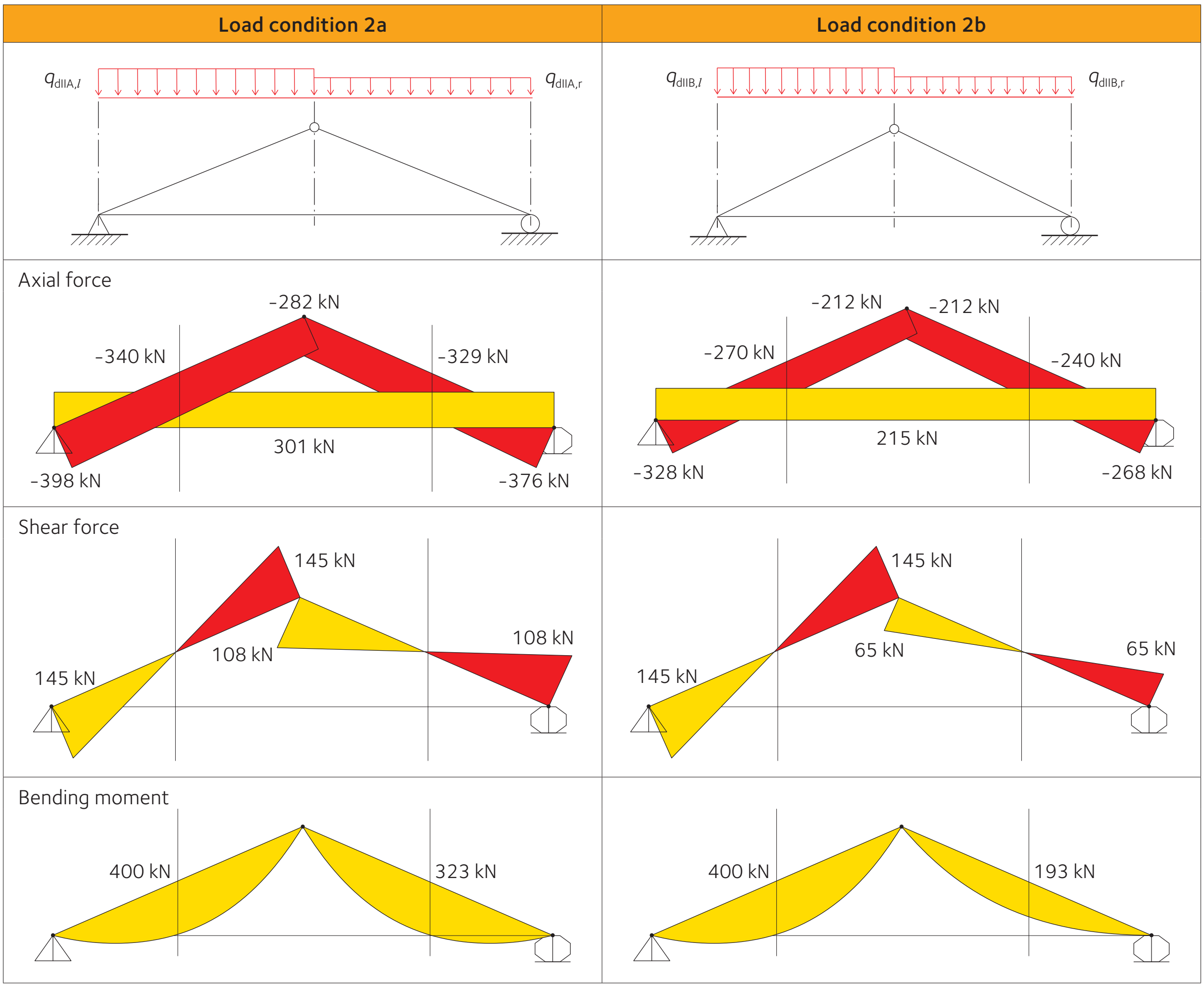
$$A_{s,min} = \frac{1}{2} \times \left(F_{H,max} \times \frac{1.4}{f_{uk}}\right) = \frac{1}{2} \times 301.12 \times 10^3 \times \frac{1.4}{500} = 421.6 \text{ mm}^2 \qquad \rightarrow \qquad \text{choose M27} \quad A_s = 459 \text{ mm}^2$$

Timber column:

$$b = 140 \text{ mm} \qquad h_{min} = \frac{f_{V,max}}{f_{c,\beta,d} \times b} = \frac{276.87 \times 10^3}{4.95 \times 140} = 400 \text{ mm} \qquad \rightarrow \qquad h = 405 \text{ mm}$$

For the value of $f_{c,\alpha,d}$ *see section 5.6 c), page 112*.

5.5 Internal forces and moments



Load condition 2a is the governing condition.

5.6 Verification of rafter

a) Shear

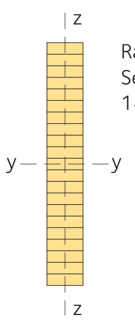
$$\tau_{\mathrm{d}} = \frac{3 \times V_{\mathrm{Ed}}}{2 \times b \times h} = \frac{3 \times 145 \times 10^3}{2 \times 140 \times 1035} = 1.50 \text{ MPa}$$

Shear verification (*EN 1995-1-1, equation 6.13*)

$$\frac{\tau_{\mathrm{d}}}{k_{\mathrm{cr}} \times f_{\mathrm{v,d}}} = \frac{1.50}{0.86 \times 2.24} = 0.78 \quad < 1 \quad \textbf{OK}$$

b) Stability check for combined bending and compression

The rafter is laterally stiffened by means of a bracing system; braced points are 1.57 m apart.



Rafter
Section A-A
140 × 1,035 mm

$$\sigma_{\mathrm{m,d}} = \frac{6 \times M_{\mathrm{Ed}}}{b \times h^2} = \frac{6 \times 400 \times 10^6}{140 \times 1035^2} = 16.0 \text{ MPa}$$

$$\sigma_{\mathrm{c,0,d}} = \frac{N_{\mathrm{Ed}}}{b \times h} = \frac{340 \times 10^3}{140 \times 1035} = 2.3 \text{ MPa}$$

Stability about the z-axis (deflection in the y-direction)

Buckling length:

$$l_{0,z} = \frac{1.57}{\cos(\alpha)} = \frac{1.57}{\cos\left(23.6°\right)} = 1.71 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,z} = \frac{\pi^2 \times E_{0.05} \times I_z}{A \times \left(l_{0,z}\right)^2} = \frac{\pi^2 \times 10800 \times \frac{140^3 \times 1035}{12}}{140 \times 1035 \times \left(1.71 \times 10^3\right)^2} = 59.33 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,z}}} = \sqrt{\frac{24.5}{59.33}} = 0.64$$

k factor:

$$k_z = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,z} - 0.3\right) + \lambda_{rel,z}{}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times \left(0.64 - 0.3\right) + 0.64^2\right] = 0.72$$

Reduction factor for buckling:

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z{}^2 - \lambda_{rel,z}{}^2}} = \frac{1}{0.72 + \sqrt{0.72^2 - 0.64^2}} = 0.95$$

Stability about the y-axis (deflection in the z-direction)

Buckling length:

$$l_{0,y} = \frac{\frac{l_{tot}}{2}}{\cos(\alpha)} = \frac{\frac{22}{2}}{\cos\left(23.6°\right)} = 12 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,y} = \frac{\pi^2 \times E_{0.05} \times I_y}{A \times \left(l_{0,y}\right)^2} = \frac{\pi^2 \times 10800 \times \frac{140 \times 1035^2}{12}}{140 \times 1035 \times \left(12 \times 10^3\right)^2} = 66.06 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,y}}} = \sqrt{\frac{24.5}{66.06}} = 0.61$$

k factor:

$$k_y = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}{}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times \left(0.61 - 0.3\right) + 0.61^2\right] = 0.70$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y{}^2 - \lambda_{rel,y}{}^2}} = \frac{1}{0.70 + \sqrt{0.70^2 - 0.61^2}} = 0.95$$

Lateral torsional buckling

Effective buckling length:

$$l_{0,z} = \frac{1.57}{\cos(\alpha)} = \frac{1.57}{\cos\left(23.6°\right)} = 1.71 \text{ m}$$

Critical bending stress:

$$\sigma_{cr,m} = \frac{\pi}{W_y \times l_{0,z}} \times \sqrt{E_{0.05} \times I_z \times G_{05} \times I_{tor}} = \frac{\pi}{\frac{140 \times 1035^2}{6} \times 1713.3} \times \sqrt{10800 \times \frac{1035 \times 140^3}{12} \times 540 \times \frac{140^3 \times 1035}{3}} = 83.87 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{cr,m}}} = 0.598$$

Critical factor for lateral torsional buckling:

$$\text{for} \quad \lambda > 0.75 \quad \rightarrow \quad k_{crit} = 1$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

$$\frac{\sigma_{c,0,d}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} = \frac{2.35}{0.95 \times 15.68} + \frac{16.0}{19.2} = 0.99 \quad < 1 \quad \textbf{OK}$$

Check for axial buckling about z-axis and bending about y-axis (*EN 1995-1-1, equation 6.24*):

$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} + 0.7 \times \frac{\sigma_{m,d}}{f_{m,d}} = \frac{2.35}{0.95 \times 15.68} + 0.7 \times \frac{16.0}{19.2} = 0.74 \quad < 1 \quad \textbf{OK}$$

Check for lateral torsional buckling and axial buckling about z-axis and bending about y-axis (*EN 1995-1-1, equation 6.35*):

$$\left(\frac{\sigma_{m,d}}{k_{crit} \times f_{m,d}}\right)^2 + \frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \left(\frac{16.0}{19.2}\right)^2 + \frac{2.35}{0.95 \times 15.68} = 0.85 \quad < 1 \quad \textbf{OK}$$

c) Check for compression at an angle β to the grain

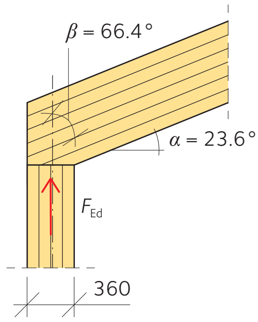
$$\sigma_{c,\beta,d} = \frac{F_{Ed}}{\left(h_{col} + 30 \times \cos(\alpha)\right) \times b_{col}} = \frac{277 \times 10^3}{\left(405 + 30 \times \cos\left(23.6°\right)\right) \times 140} = 4.57 \text{ MPa}$$

$f_{c,90,d}$ can be replaced by $f_{c,90,k}$ due to the fact that $g_k/s_k < 0.4$, *see table 8.11, page 25, 8.12, page 25* and *8.13, page 26*.

$$f_{c,\beta,d} = \frac{f_{c,0,d}}{\frac{f_{c,0,d}}{\left(1.75 \times f_{c,90,k}\right)} \times \sin(\beta)^2 + \cos(\beta)^2} = \frac{15.68}{\frac{15.68}{1.75 \times 2.5} \times \sin\left(66.4°\right)^2 + \cos\left(66.4°\right)^2} = 4.95 \text{ MPa}$$

Compression at an angle β to the grain (*EN 1995-1-1, equation 6.16*):

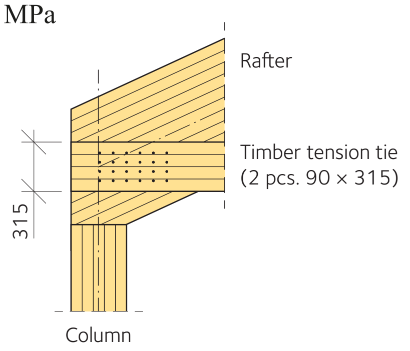
$$\frac{\sigma_{c,\beta,d}}{f_{c,\beta,d}} = \frac{4.57}{4.95} = 0.92 \quad < 1 \quad \textbf{OK}$$



5.7 Verification of the timber tension tie

The verification is performed on the net cross section. In this case 4 rows of connectors, each with a diameter of d = 11 mm, are used.

$$\sigma_{t,d} = \frac{T_{Ed}}{2 \times \left[b \times (h - 4 \times 11)\right]} = \frac{301 \times 10^3}{2 \times 90 \times (315 - 4 \times 11)} = 6.17 \text{ MPa}$$



The design of the connection is shown in *example 17, page 193*.

Tension parallel to the grain (*EN 1995-1-1, equation 6.1*):

$$\frac{\sigma_{t,d}}{f_{t,0,d}} = \frac{6.17}{12.48} = 0.49 \quad < 1 \quad \textbf{OK}$$

5.8 Verification of steel tension tie

a) Tension force

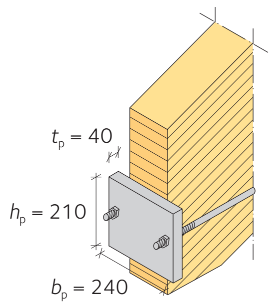
$$T_{Ed} = 275 \times 10^3 \text{ N}$$

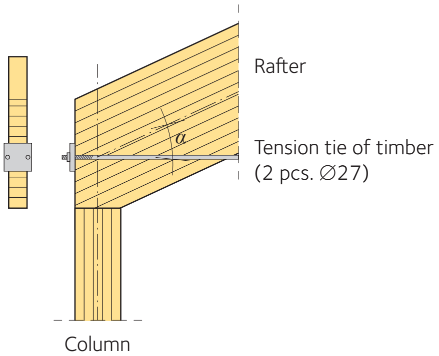
The capacity is determined according to (*EN 1993-1-8, table 3.4*):

$$T_{Rd} = 2 \times \frac{A_s \times f_{uk} \times 0.9}{\gamma_{M2}} \times 10^{-3} = \frac{459 \times 500 \times 0.9}{1.2} \times 10^{-3} = 344.25 \text{ kN}$$

Verify the figure for tension (*EN 1993-1-1, equation 6.5*):

$$\frac{T_{Ed}}{T_{Rd}} = \frac{301}{344.3} = 0.87 \quad < 1 \quad \textbf{OK}$$





b) Compression at an angle α to the grain

The steel plate dimensions shall be chosen in order to avoid local compression failure of the rafter. A steel plate of S355 with dimensions h_p = 210 mm, t_p = 40 mm is adopted:

$$T_{Ed} \leq f_{c,\alpha,d} \times A_{ef,plate}$$

$$A_{ef,plate} = \beta \times b_{rafter} \times h_{plate} = 1 \times 140 \times 210 = 29400 \text{ mm}^2$$

where β is the area reduction factor:

$$\beta = \min\left(1, \frac{2 \times t_{plate}}{b_{rafter}} \times \sqrt{\frac{f_{y,d}}{2 \times f_{c,\alpha,d}}}\right) = \min\left(1, \frac{2 \times 40}{140} \times \sqrt{\frac{300}{2 \times 11.09}}\right) = 1$$

$f_{c,90,d}$ can be replaced by $f_{c,90,k}$ due to the fact that g_k/s_k < 0.4, *see tables 8.11, page 25, 8.12, page 25* and *8.13, page 26*:

$$f_{c,\alpha,d} = \frac{f_{c,0,d}}{\frac{f_{c,0,d}}{1.75 \times f_{c,90,k}} \times \sin(\alpha)^2 + \cos(\alpha)^2} = \frac{15.68}{\frac{15.68}{1.75 \times 2.5} \times \sin(23.6°)^2 + \cos(23.6°)^2} = 11.09 \text{ MPa}$$

Verify the figure for compression at an angle α to the grain (*EN 1995-1-1, equation 6.16*):

$$\frac{T_{Ed}}{A_{ef,plate} \times f_{c,\alpha,d}} = \frac{301 \times 10^3}{29400 \times 11.09} = 0.92 \quad < 1 \quad \textbf{OK}$$

c) Actions on the column

The columns are assumed to be clamped at their base. The deformation of the steel tie rod causes an additional bending stress in the columns:

$$\delta_{tie} = \frac{T_{Ed}}{E_{steel} \times A} \times l_{tot} = \frac{301 \times 10^3}{210000 \times 2 \times \frac{\pi \times 27^2}{4}} \times 22 \times 10^3 = 27.5 \text{ mm}$$

Shear force in the column:

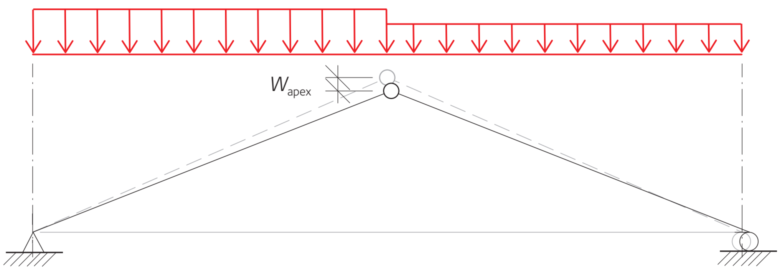
$$F_{column} = \frac{\delta_{tie}}{2} \times \frac{3 \times E_{0.05} \times \frac{b_{col} \times h_{col}}{12}}{(l_{col})^3} \times 10^{-3} = \frac{27.5}{2} \times \frac{3 \times 10800 \times \frac{140 \times 405^3}{12}}{6000^3} \times 10^{-3} = 1.60 \text{ kN}$$

Bending moment at the base of the column:

$$M_{column} = F_{column} \times l_{col} = 1.60 \times 6 = 9.61 \text{ kNm}$$

5.9 SLS Verifications

Deflection at the apex:



Two load combinations are considered:

Combination SLS 1 (permanent loads):

$$q_{\mathrm{sls},1} = g_{\mathrm{k},1} + g_{\mathrm{k},2} = 0.7 + 2.9 = 3.58 \text{ kN/m}$$

Combination SLS 2 (unsymmetric snow load):

$$q_{\mathrm{sls},2,l} = s_{\mathrm{k},l} = 14.75 = 14.75 \text{ kN/m}$$

$$q_{\mathrm{sls},2,\mathrm{r}} = s_{\mathrm{k},\mathrm{r}} = 11.44 = 11.44 \text{ kN/m}$$

a) Structural system 1 (tension tie of timber)

Calculate the instantaneous deflection at the apex under a uniformly distributed load q_1, *see The Glulam Handbook Volume 2, section 6.2, page 84*:

$$w_{\mathrm{inst},1} = \frac{q_1 l_{\mathrm{tot}}^2}{16 \times E_{0,\mathrm{m}} \times b_{\mathrm{raf}} \times h_{\mathrm{raf}} \times \tan(\alpha)^2} \left(\frac{1}{\cos(\alpha)^3} + \frac{E_{0,\mathrm{m}} \times b_{\mathrm{raf}} \times h_{\mathrm{raf}}}{E_{0,\mathrm{m}} \times b_{\mathrm{tie}} \times h_{\mathrm{tie}} \times 2} \right) = \frac{1 \times (22 \times 10^3)^2}{16 \times 13000 \times 140 \times 1035 \times \tan(\alpha)^2} \times \left(\frac{1}{\cos(\alpha)^3} + \frac{13000 \times 140 \times 1035}{13000 \times 90 \times 315 \times 2} \right) = 0.33 \text{ mm}$$

Instantaneous deflection due to permanent loads:

$$w_{\mathrm{inst,permanent}} = w_{\mathrm{inst},1} \times q_{\mathrm{sls},1} = 0.33 \times 2 \times 3.58 = 2.32 \text{ mm}$$

Instantaneous deflection due to snow load:

$$w_{\mathrm{inst,snow}} = w_{\mathrm{inst},1} \times q_{\mathrm{sls},2} = 0.33 \times (14.75 + 11.44) = 8.51 \text{ mm}$$

Final deflection due to permanent loads:

$$w_{\mathrm{final,perm}} = w_{\mathrm{inst,permanent}} \times (1 + k_{\mathrm{def}}) = 2.32 \times (1 + 0.6) = 3.71 \text{ mm}$$

Final deflection due to snow load:

$$w_{\mathrm{final,snow}} = w_{\mathrm{inst,snow}} \times (1 + \psi_{2,\mathrm{snow}} \times k_{\mathrm{def}}) = 8.51 \times (1 + 0.2 \times 0.6) = 9.53 \text{ mm}$$

Total final deflection:

$$w_{\mathrm{fin,tot}} = w_{\mathrm{final,snow}} + w_{\mathrm{final,perm}} = 9.53 + 3.71 = 13.24 \text{ mm}$$

b) Structural system 2 (tension tie of steel)

Instantaneous deflection due to permanent loads:

$$w_{\text{inst,per}} = \frac{2 \times q_{\text{sls,1}} \times l_{\text{tot}}^{2}}{16 \times E_{0,\text{m}} \times b_{\text{raf}} \times h_{\text{raf}} \times \tan(\alpha)^2}\left(\frac{1}{\cos(\alpha)^3} + \frac{E_{0,\text{m}} \times b_{\text{raf}} \times h_{\text{raf}}}{E_{\text{steel}} \times A_{\text{tie}}}\right) = \frac{2 \times 3.58 \times \left(22 \times 10^3\right)^2}{16 \times 13000 \times 140 \times 1035 \times \tan(\alpha)^2} \times \left(\frac{1}{\cos(\alpha)^3} + \frac{13000 \times 140 \times 1035}{210000 \times 1145}\right) = 5.5 \text{ mm}$$

Final deflection due to due to permanent loads:

$$w_{\text{fin,per}} = \frac{2 \times q_{\text{sls,1}} \times l_{\text{tot}}^{2}}{16 \times E_{0,\text{m,F}} \times b_{\text{raf}} \times h_{\text{raf}} \times \tan(\alpha)^2}\left(\frac{1}{\cos(\alpha)^3} + \frac{E_{0,\text{m,F}} \times b_{\text{raf}} \times h_{\text{raf}}}{E_{\text{steel}} \times A_{\text{tie}}}\right) = \frac{2 \times 3.58 \times \left(22 \times 10^3\right)^2}{16 \times 8125 \times 140 \times 1035 \times \tan(\alpha)^2} \times \left(\frac{1}{\cos(\alpha)^3} + \frac{8125 \times 140 \times 1035}{210000 \times 1145}\right) = 6.0 \text{ mm}$$

where the final mean value of the modulus of elasticity for permanent loads is:

$$E_{0,\text{m,F}} = \frac{E_{0,\text{m}}}{1 + k_{\text{def}}} = \frac{13000}{1 + 0.6} = 8125 \text{ MPa}$$

Instantaneous deflection due to snow load:

$$w_{\text{inst,s}} = \frac{\left(q_{\text{sls,2},l} + q_{\text{sls,2,r}}\right) \times l_{\text{tot}}^{2}}{16 \times E_{0,\text{m}} \times b_{\text{raf}} \times h_{\text{raf}} \times \tan(\alpha)^2}\left(\frac{1}{\cos(\alpha)^3} + \frac{E_{0,\text{m}} \times b_{\text{raf}} \times h_{\text{raf}}}{E_{\text{steel}} \times A_{\text{tie}}}\right) = \frac{\left(14.75 + 11.44\right) \times \left(22 \times 10^3\right)^2}{16 \times 13000 \times 140 \times 1035 \times \tan(\alpha)^2} \times \left(\frac{1}{\cos(\alpha)^3} + \frac{13000 \times 140 \times 1035}{210000 \times 1145}\right) = 20.2 \text{ mm}$$

Final deflection due to due to snow load:

$$w_{\text{fin,s}} = \frac{\left(q_{\text{sls,2},l} + q_{\text{sls,2,r}}\right) \times l_{\text{tot}}^{2}}{16 \times E_{0,\text{m,F}} \times b_{\text{raf}} \times h_{\text{raf}} \times \tan(\alpha)^2}\left(\frac{1}{\cos(\alpha)^3} + \frac{E_{0,\text{m,F}} \times b_{\text{raf}} \times h_{\text{raf}}}{E_{\text{steel}} \times A_{\text{tie}}}\right) = \frac{\left(14.75 + 11.44\right) \times \left(22 \times 10^3\right)^2}{16 \times 11607.1 \times 140 \times 1035 \times \tan(\alpha)^2} \times \left(\frac{1}{\cos(\alpha)^3} + \frac{11607.1 \times 140 \times 1035}{210000 \times 1145}\right) = 23.7 \text{ mm}$$

where the final mean value of elasticity for snow load is:

$$E_{0,\text{m,F}} = \frac{E_{0,\text{m}}}{1 + k_{\text{def}} \times \psi_{2,\text{snow}}} = \frac{13000}{1 + 0.6 \times 0.2} = 11607.1 \text{ MPa}$$

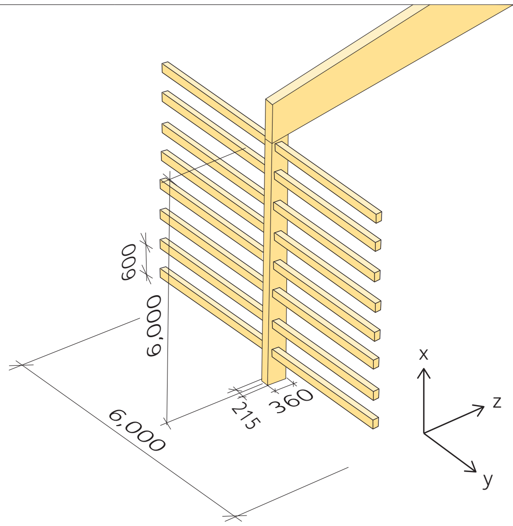
Total final deflection:

$$w_{\text{fin,tot}} = w_{\text{fin,per}} + w_{\text{fin,s}} = 5.98 + 23.71 = 29.7 \text{ mm}$$

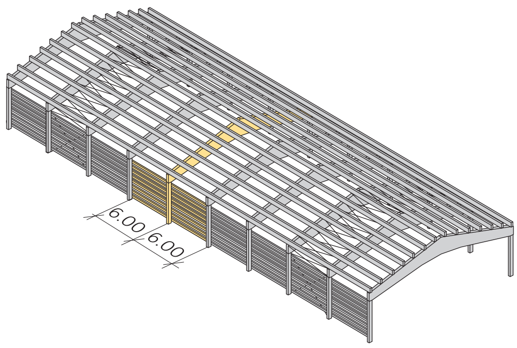
Example 6: Design of a column

6.1 System, dimensions and design parameters

Design and verify the column below. The column is clamped at the base for rotations about y-axis and it is free at its top. The column refers to the structure with pitched-cambered beams described in *example 3, page 94*.



The column is made of glulam, strength class	GL30c
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$



6.2 Loads

The loads considered in the design are:

Structural

$$g_{k,beam} = 1.2 \text{ kN/m}$$

Non-structural

$$g_{k,column} = 0.5 \text{ kN/m}$$

Other permanent load

$G_{k,2} = 0.5 \text{ kN/m}^2$

$$g_{k,2} = G_{k,2} \times i \times 1.1 = 0.6 \times 6 \times 1.1 = 3.96 \text{ kN/m}$$

Snow load

$S_k = 1.5 \text{ kN/m}^2$

$$s_k = S_k \times i \times \mu \times 1.1 = 1.5 \times 6 \times 0.98 \times 1.1 = 9.7 \text{ kN/m}$$

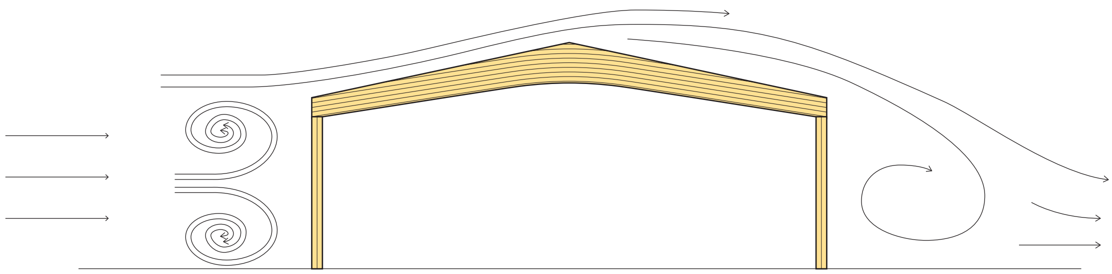
Wind load

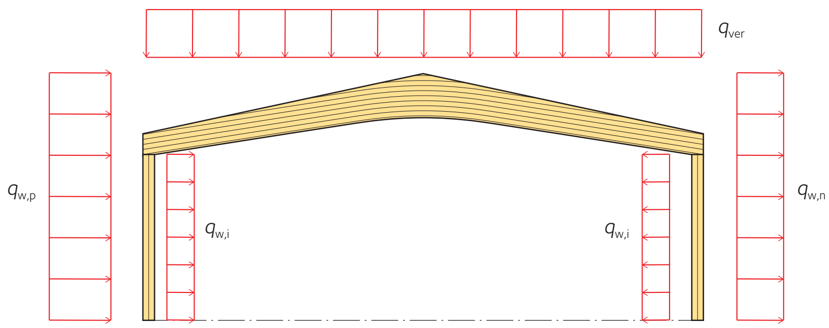
$Q_{w,k} = 0.6 \text{ kN/m}^2$

$$q_{w,k,pos} = Q_{w,k} \times i \times C_{e,pos} = 0.6 \times 6 \times 0.75 = 2.7 \text{ kN/m}$$
$$q_{w,k,neg} = Q_{w,k} \times i \times C_{e,neg} = 0.6 \times 6 \times 0.4 = 1.44 \text{ kN/m}$$
$$q_{w,k,int} = Q_{w,k} \times i \times C_{int} = 0.6 \times 6 \times 0.35 = 1.26 \text{ kN/m}$$

Factor 1.1 used in the equations above accounts for the continuity of purlins over trusses.

The effect of the wind load can be simplified by means of the following uniformly distributed loads:





6.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (snow load leading, $k_{mod} = 0.8$):

$$q_{ver,1} = \gamma_d \times \left[\gamma_g \times (g_{k,beam} + g_{k,2}) + \gamma_q \times s_k\right] = 1 \times [1.2 \times (1.2 + 4) + 1.5 \times 9.7] = 20.8 \text{ kN/m}$$

$$q_{w,p,1} = \gamma_d \times \gamma_q \times q_{w,k,pos} \times \psi_{0,w} = 1 \times 1.5 \times 2.7 \times 0.3 = 1.21 \text{ kN/m}$$

$$q_{w,n,1} = \gamma_d \times \gamma_q \times q_{w,k,neg} \times \psi_{0,w} = 1 \times 1.5 \times 1.44 \times 0.3 = 0.65 \text{ kN/m}$$

$$q_{w,i,1} = \gamma_d \times \gamma_q \times q_{w,k,int} \times \psi_{0,w} = 1 \times 1.5 \times 1.26 \times 0.3 = 0.57 \text{ kN/m}$$

Combination 2 (wind load leading, $k_{mod} = 0.9$):

$$q_{ver,2} = \gamma_d \times \left[\gamma_g \times (g_{k,beam} + g_{k,2}) + \gamma_q \times \psi_{0,s} \times s_k\right] = 1 \times [1.2 \times (1.2 + 4) + 1.5 \times 0.6 \times 9.7] = 15.0 \text{ kN/m}$$

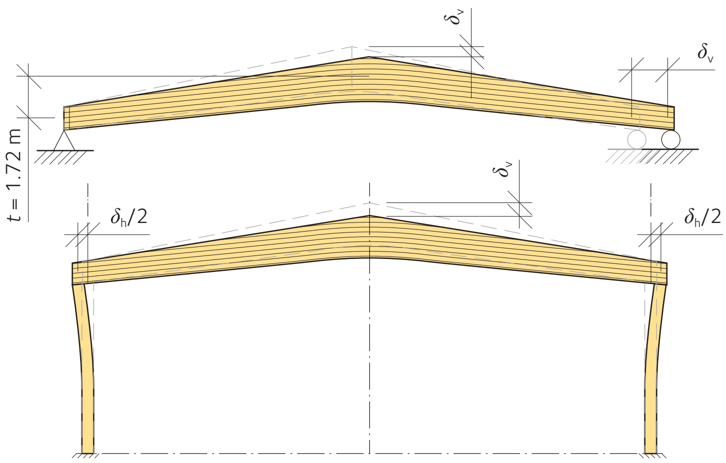
$$q_{w,p,2} = \gamma_d \times \gamma_q \times q_{w,k,pos} = 1 \times 1.5 \times 2.7 = 4.1 \text{ kN/m}$$

$$q_{w,n,2} = \gamma_d \times \gamma_q \times q_{w,k,neg} = 1 \times 1.5 \times 1.44 = 2.2 \text{ kN/m}$$

$$q_{w,i,2} = \gamma_d \times \gamma_q \times q_{w,k,int} = 1 \times 1.5 \times 1.26 = 1.9 \text{ kN/m}$$

6.4 Horizontal displacement of the supports due to vertical deflection of the beam

The deflection of the beam causes a horizontal displacement at the support. In a real structure this displacement occurs at both supports and its magnitude is $\delta_h/2$. t is the height difference between the beam's system lines at the support and the apex.



a) Vertical deflection

The instantaneous deflection due to permanent load and snow load is calculated in *example 3, page 94*:

$$\delta_{\text{inst,snow}} = w_{\text{unitary}} \times \left(s_{\text{k}}\right) = 3.9 \times 9.7 = 37.8 \text{ mm}$$

$$\delta_{\text{inst,perm}} = w_{\text{unitary}} \times \left(g_{\text{k,beam}} + g_{\text{k,2}}\right) = 3.9 \times (1.2 + 4) = 19.9 \text{ mm}$$

The final deflections are:
Combination 1:

$$\delta_{\text{v,1}} = \gamma_{\text{g}}\delta_{\text{inst,perm}} \times \left(1 + k_{\text{def}}\right) + \gamma_{\text{q}}\delta_{\text{inst,snow}} \times \left(1 + \psi_{2,\text{s}} \times k_{\text{def}}\right) = 1.2 \times 19.9 \times \left(1 + 0.6\right) + 1.5 \times 37.8 \times \left(1 + 0.1 \times 0.6\right) = 98.4 \text{ mm}$$

Combination 2:

$$\delta_{\text{v,2}} = \gamma_{\text{g}}\delta_{\text{inst,perm}} + \gamma_{\text{q}} \times \psi_{0,\text{s}}\delta_{\text{inst,snow}} = 1.2 \times 19.9 + 1.5 \times 0.6 \times 37.8 = 57.9 \text{ mm}$$

b) Horizontal displacement

The horizontal displacement depends on the mid-span deflection δ_{v} and it can be evaluated by the following equation, *see The Glulam Handbook Volume 2, section 6.2, page 93*:

$$\delta_{\text{h}} = \left(4 \times \frac{t}{l_{\text{tot}}} + 3.2 \times \frac{h_0}{l_{\text{tot}}}\right) \times \delta_{\text{v}}$$

Combination 1:

$$\delta_{\text{h,1}} = \left(4 \times \frac{t}{l_{\text{tot}}} + 3.2 \times \frac{h_0}{l_{\text{tot}}}\right) \times \delta_{\text{v,1}} = \left(4 \times \frac{1724}{20 \times 10^3} + 3.2 \times \frac{800}{20 \times 10^3}\right) \times 98.4 = 46.5 \text{ mm}$$

Combination 2:

$$\delta_{\text{h,2}} = \left(4 \times \frac{t}{l_{\text{tot}}} + 3.2 \times \frac{h_0}{l_{\text{tot}}}\right) \times \delta_{\text{v,2}} = \left(4 \times \frac{1724}{20 \times 10^3} + 3.2 \times \frac{800}{20 \times 10^3}\right) \times 57.9 = 27.4 \text{ mm}$$

c) Effects of the horizontal displacement

The horizontal displacement of the supports generates the following additional actions on the columns:

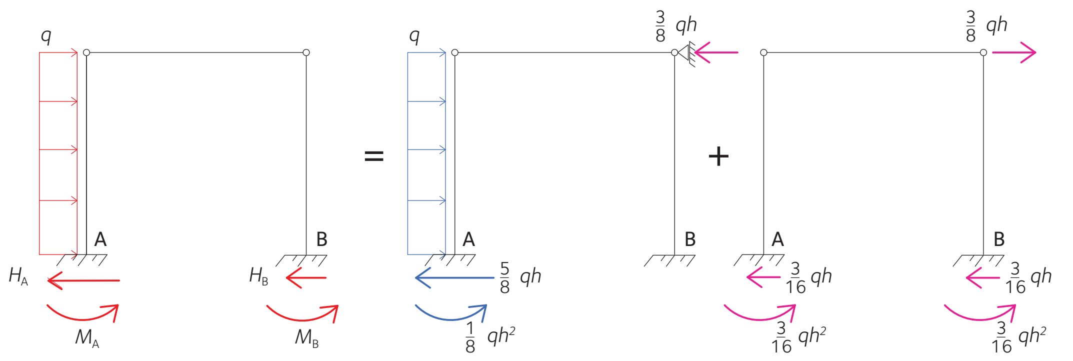
$$V_{\delta,\text{h,1}} = \frac{\delta_{\text{h,1}}}{2} \times \frac{3 \times E_{0.05} \times I_{\text{col}}}{{l_{\text{col}}}^3} \times 10^{-3} = \frac{46.5}{2} \times \frac{3 \times 10800 \times \frac{215 \times 360^3}{12}}{6000^3} \times 10^{-3} = 2.9 \text{ kN}$$

$$V_{\delta,\text{h,2}} = \frac{\delta_{\text{h,2}}}{2} \times \frac{3 \times E_{0.05} \times I_{\text{col}}}{{l_{\text{col}}}^3} \times 10^{-3} = \frac{27.4}{2} \times \frac{3 \times 10800 \times \frac{215 \times 360^3}{12}}{6000^3} \times 10^{-3} = 1.7 \text{ kN}$$

$$M_{\delta,\text{h,2}} = V_{\delta,\text{h,2}} \times l_{\text{col}} = 1.7 \times 6 = 10.3 \text{ kNm} \qquad M_{\delta,\text{h,1}} = V_{\delta,\text{h,1}} \times l_{\text{col}} = 2.9 \times 6 = 17.4 \text{ kNm}$$

6.5 ULS verifications

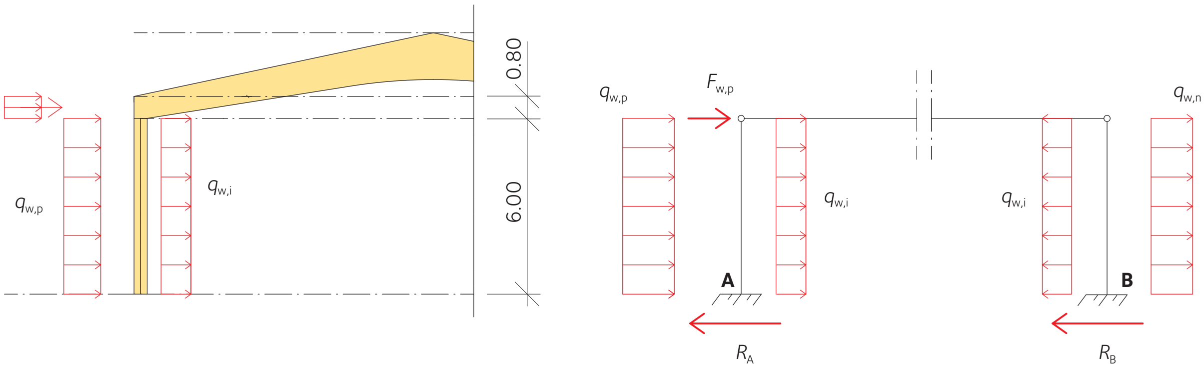
Load distribution between connected columns:



$$H_A = \frac{5}{8} \times q \times h + \frac{3}{16} \times q \times h \rightarrow \frac{13 \times h \times q}{16} \qquad M_A = \frac{1}{8} \times q \times h^2 + \frac{3}{16} \times q \times h^2 \rightarrow \frac{13 \times h^2 \times q}{16}$$

$$H_B = \frac{3}{16} \times q \times h \qquad M_B = \frac{3}{16} \times q \times h^2 \rightarrow \frac{3 \times h^2 \times q}{16}$$

Both external and internal wind loads generates shear and moment in the columns:



$$F_{w,p} = q_{w,p,2} \times 0.8 = 4.05 \times 0.8 = 3.24 \text{ kN}$$

$$R_A = \left(\frac{13}{16} \times q_{w,p,2} + \frac{3}{16} \times q_{w,n,2}\right) \times l_{col} + \frac{1}{2} \times F_{w,p} + \frac{5}{8} \times q_{w,i,2} \times l_{col} = \left(\frac{13}{16} \times 4.1 + \frac{3}{16} \times 2.2\right) \times 6 + \frac{1}{2} \times 3.2 + \frac{5}{8} \times 1.9 \times 6 = 30.9 \text{ kN}$$

$$R_B = \left(\frac{13}{16} \times q_{w,n,2} + \frac{3}{16} \times q_{w,p,2}\right) \times l_{col} + \frac{1}{2} \times F_{w,p} + \frac{5}{8} \times q_{w,i,2} \times l_{col} = \left(\frac{13}{16} \times 2.2 + \frac{3}{16} \times 4.1\right) \times 6 + \frac{1}{2} \times 3.2 + \frac{5}{8} \times 1.9 \times 6 = 9.6 \text{ kN}$$

a) Shear

Load combination 2 governs the design for shear at the base of the column:

$$V_{windwar,2} = R_A = 30.9 \text{ kN}$$

$$\tau_{d,2} = \frac{3 \times V_{windwar}}{2 \times (b_{col} \times h_{col}) \times k_r} = \frac{2 \times 30881.25}{2 \times 215 \times 360 \times 0.8} = 0.75 \text{ MPa}$$

k_r is a reduction factor which accounts for the presence of fasteners.

Shear verification (*EN 1995-1-1 equation 6.13*):

$$f_{v,d,2} = \frac{f_{v,k} \times k_{mod,2}}{\gamma_M} = \frac{3.5 \times 0.9}{1.25} = 2.52 \text{ MPa} \qquad \frac{\tau_{d,2}}{k_{cr} \times f_{v,d,2}} = \frac{0.75}{0.86 \times 2.52} = 0.35 \quad < 1 \quad \textbf{OK}$$

b) Combined bending and compression at the base

Load combination 2 governs the design for bending at the base of the column:

$$M_{d,2} = \left(\frac{5 \times q_{w,p,2}}{16} + \frac{3 \times q_{w,n,2}}{16}\right) \times l_{col}^{2} + \frac{1}{2} \times F_{w,p} \times l_{col} + \frac{q_{w,i,2}}{8} \times l_{col}^{2} = \left(\frac{5 \times 4.1}{16} + \frac{3 \times 2.2}{16}\right) \times 6^2 + \frac{1}{2} \times 3.2 \times 6 + \frac{1.9}{8} \times 6^2 = 78.4 \text{ kNm}$$

$$F_{v,2} = q_{ver,2} \times \frac{l_{tot}}{2} + \gamma_g \times g_{k,column} \times l_{col} = 15.0 \times \frac{20}{2} + 1.2 \times 0.5 \times 6 = 153.3 \text{ kN}$$

$$\sigma_{c,0,d,2} = \frac{F_{v,2}}{k_r \times b_{col} \times h_{col}} = \frac{153.3 \times 10^3}{0.8 \times 215 \times 360} = 2.47 \text{ MPa}$$

$$\sigma_{m,d,2} = \frac{6 \times M_{d,2}}{b_{col} \times h_{col}^{2}} = \frac{6 \times 78.37 \times 10^6}{215 \times 360^2} = 16.88 \text{ MPa}$$

The bending moment $M_{2,\delta,h}$ generated by the horizontal displacement of the pitched-cambered beam is not taken into consideration in the design of the columns because it acts in the opposite direction compared to the wind effects.

Combined bending and compression verification (*EN 1995-1-1, equation 6.19*):

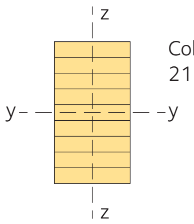
$$f_{m,d,2} = \frac{k_h \times f_{m,k} \times k_{mod,2}}{\gamma_M} = \frac{1.05 \times 30 \times 0.9}{1.25} = 22.73 \text{ MPa}$$

$$f_{c,0,d,2} = \frac{f_{c,0,k} \times k_{mod,2}}{\gamma_M} = \frac{24.5 \times 0.9}{1.25} = 17.64 \text{ MPa}$$

$$\frac{\sigma_{m,d,2}}{f_{m,d,2}} + \left(\frac{\sigma_{c,0,d,2}}{f_{c,0,d,2}}\right)^2 = \frac{16.88}{22.73} + \left(\frac{2.47}{17.64}\right)^2 = 0.76 \quad < 1 \quad \textbf{OK}$$

c) Stability check for combined bending and compression

$$\sigma_{c,0,d,2} = \frac{F_{v,2}}{b_{col} \times h_{col} \times k_r} = \frac{153.3 \times 10^3}{215 \times 360 \times 0.8} = 2.47 \text{ MPa}$$



$$\sigma_{m,d,2} = \frac{6 \times M_{d,2}}{b_{col} \times h_{col}^{2}} = \frac{6 \times 78.37 \times 10^6}{215 \times 360^2} = 16.88 \text{ MPa}$$

Stability about the y-axis (deflection in the z-direction)

Buckling length:

$$l_{0,y} = 2.25 \times 6 = 13.5 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,y} = \frac{\pi^2 \times E_{0.05} \times I_y}{b_{col} \times h_{col} \times l_{0,y}^{2}} = \frac{\pi^2 \times 10800 \times \frac{215 \times 360^3}{12}}{215 \times 360 \times \left(13.5 \times 10^3\right)^2} = 6.32 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,y}}} = \sqrt{\frac{24.5}{6.32}} = 1.97$$

Factor k:

$$k_y = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}^{\ 2}\right] = \frac{1}{2} \times \left[1 + 0.1 \times (1.97 - 0.3) + 1.97^2\right] = 2.52$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^{\ 2} - \lambda_{rel,y}^{\ 2}}} = \frac{1}{2.52 + \sqrt{2.52^2 - 1.97^2}} = 0.24$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

$$\frac{\sigma_{c,0,d,2}}{k_{c,y} \times f_{c,0,d,2}} + \frac{\sigma_{m,d,2}}{f_{m,d,2}} = \frac{2.47}{0.24 \times 17.64} + \frac{16.88}{22.73} = 1.33 \quad > 1 \quad \textbf{NOT OK}$$

The verification is not satisfied. The depth of the column is increased from 360 mm to 405 mm:

$$h_{col} = 405 \text{ mm} \qquad b_{col} = 215 \text{ mm}$$

$$\sigma_{c,0,d,2} = \frac{F_{v,2}}{b_{col} \times h_{col} \times k_r} = \frac{153.3 \times 10^3}{215 \times 405 \times 0.8} = 2.21 \text{ MPa} \qquad \sigma_{m,d,2} = \frac{6 \times M_{d,2}}{b_{col} \times h_{col}^{\ 2}} = \frac{6 \times 78.37 \times 10^6}{215 \times 405^2} = 13.33 \text{ MPa}$$

Euler critical stress:

$$\sigma_{cr,y} = \frac{\pi^2 \times E_{0.05} \times I_y}{b_{col} \times h_{col} \times l_{0,y}^{\ 2}} = \frac{\pi^2 \times 10800 \times \frac{215 \times 405^3}{12}}{215 \times 405 \times \left(13.5 \times 10^3\right)^2} = 7.99 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,y}}} = \sqrt{\frac{24.5}{7.99}} = 1.75$$

k factor:

$$k_y = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}^{\ 2}\right] = \frac{1}{2} \times \left[1 + 0.1 \times \left(1.75 - 0.3\right) + 1.75^2\right] = 2.1$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^{\ 2} - \lambda_{rel,y}^{\ 2}}} = \frac{1}{2.1 + \sqrt{2.1^2 - 1.75^2}} = 0.31$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

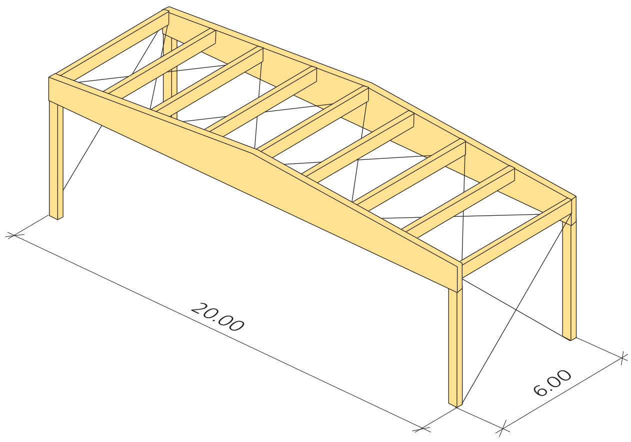
$$\frac{\sigma_{c,0,d,2}}{k_{c,y} \times f_{c,0,d,2}} + \frac{\sigma_{m,d,2}}{f_{m,d,2}} = \frac{2.21}{0.31 \times 17.64} + \frac{13.33}{20.77} = 1.05 \quad > 1 \quad \textbf{NOT OK}$$

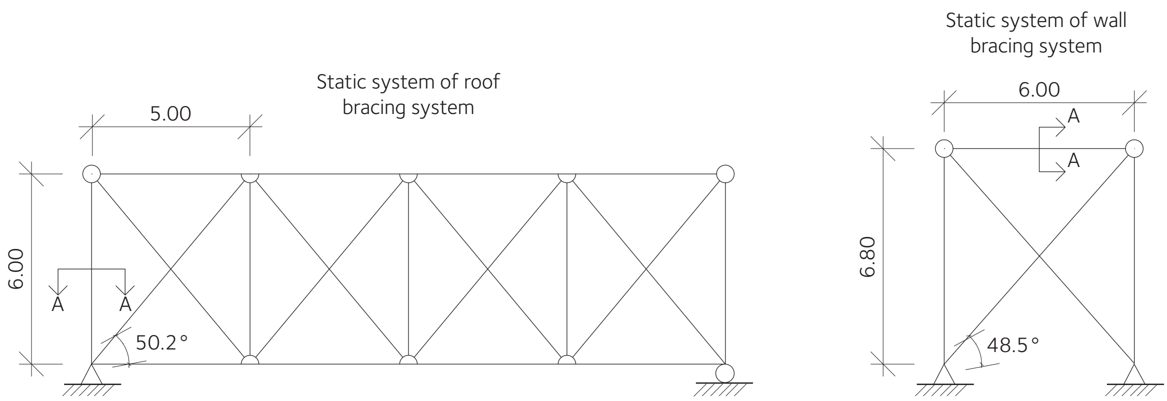
The condition is not met, increase the column dimensions.

Example 7: Design of a bracing system

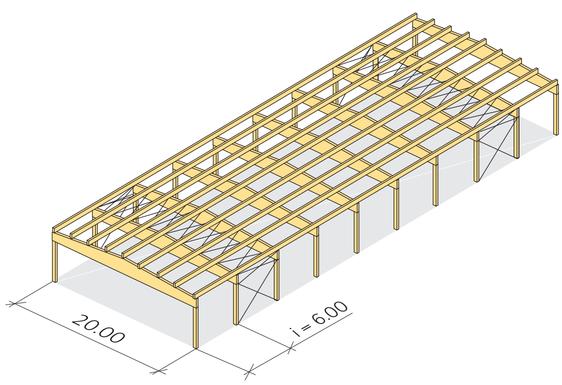
7.1 System, dimensions and design parameters

Design and verify the bracing system below. The bracing system refers to the structure with tapered beams described in *example 2, page 87*.





Glulam pitched beams, strength class	GL30c
Glulam compression struts, strength class	GL30c
Steel tie rods, strength class	5.6
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable loads	$\gamma_q = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$
Material partial factor for steel	$\gamma_{M2} = 1.2$



7.2 Loads

The loads considered in the design are:

Structural

$$g_{k,beam} = 1.1 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.6 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.6 \times 6 \times 1.1 = 3.96 \text{ kN/m}$

Snow load

$S_k = 1.5 \text{ kN/m}^2$ $\quad s_k = S_k \times i \times \mu \times 1.1 = 1.5 \times 6 \times 0.854 \times 1.1 = 8.46 \text{ kN/m}$

Wind load

$Q_{w,k} = 0.7 \text{ kN/m}^2$ $\quad q_{w,k,pos} = Q_{w,k} \times C_{e,pos} = 0.7 \times 0.75 = 0.53 \text{ kN/m}^2$

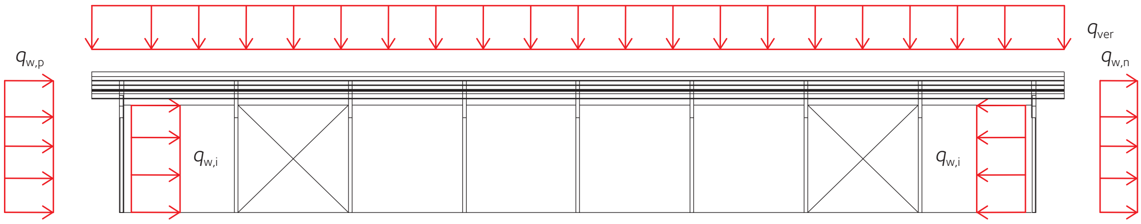
$$q_{w,k,neg} = Q_{w,k} \times C_{e,neg} = 0.7 \times 0.4 = 0.28 \text{ kN/m}^2$$

$$q_{w,k,int} = Q_{w,k} \times C_{int} = 0.7 \times 0.35 = 0.25 \text{ kN/m}^2$$

Factor 1.1 used in the equations above accounts for the continuity of purlins over trusses.

7.3 Load combinations

The effect of the wind load can be simplified by means of the following uniformly distributed loads:



Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (snow load leading, $k_{mod} = 0.8$):

$$q_{ver,1} = \gamma_d \times \left[\gamma_g \times \left(g_{k,beam} + g_{k,2}\right) + \gamma_q \times s_k\right] = 1 \times \left[1.2 \times \left(1.1 + 3.96\right) + 1.5 \times 8.46\right] = 18.75 \text{ kN/m}$$

$$q_{w,p,1} = \gamma_d \times \gamma_q \times q_{w,k,pos} \times \psi_{0,w} = 1 \times 1.5 \times 0.53 \times 0.3 = 0.24 \text{ kN/m}^2$$

$$q_{w,n,1} = \gamma_d \times \gamma_q \times q_{w,k,neg} \times \psi_{0,w} = 1 \times 1.5 \times 0.28 \times 0.3 = 0.13 \text{ kN/m}^2$$

$$q_{w,i,1} = \gamma_d \times \gamma_q \times q_{w,k,int} \times \psi_{0,w} = 1 \times 1.5 \times 0.25 \times 0.3 = 0.11 \text{ kN/m}^2$$

Combination 2 (wind load leading, $k_{mod} = 0.9$):

$$q_{ver,2} = \gamma_d \times \left[\gamma_g \times \left(g_{k,beam} + g_{k,2}\right) + \gamma_q \times \psi_{0,s} \times s_k\right] = 1 \times \left[1.2 \times \left(1.1 + 3.96\right) + 1.5 \times 0.6 \times 8.46\right] = 13.68 \text{ kN/m}$$

$$q_{w,p,2} = \gamma_d \times \gamma_q \times q_{w,k,pos} = 1 \times 1.5 \times 0.53 = 0.79 \text{ kN/m}^2$$

$$q_{w,n,2} = \gamma_d \times \gamma_q \times q_{w,k,neg} = 1 \times 1.5 \times 0.28 = 0.42 \text{ kN/m}^2$$

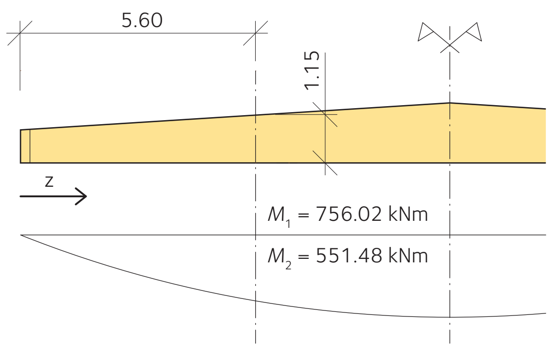
$$q_{w,i,2} = \gamma_d \times \gamma_q \times q_{w,k,int} = 1 \times 1.5 \times 0.25 = 0.37 \text{ kN/m}^2$$

7.4 Stability load

The bracing system is loaded by wind and a stability load (*EN 1995-1-1, clause 9.2.5.3*). The in-plane bending moment at the most stressed section of the tapered beam is calculated in *example 2, page 87*.

$$\text{at } x = x_{max}: \quad M_1 = \frac{q_{ver,1} \times x_{max}}{2} \times (l_{tot} - x_{max}) = \frac{18.75 \times 5.6}{2} \times (20 - 5.6) = 756.02 \text{ kNm}$$

$$M_2 = \frac{q_{ver,2} \times x_{max}}{2} \times (l_{tot} - x_{max}) = \frac{13.68 \times 5.6}{2} \times (20 - 5.6) = 551.48 \text{ kNm}$$



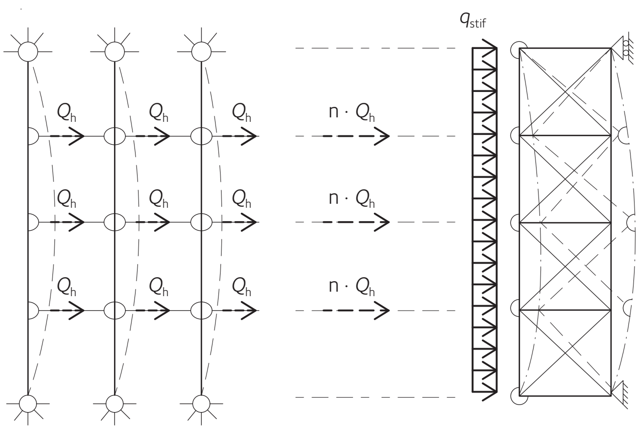
The structure is stiffened by two bracing systems. Each system brace one half of the total number of tapered beams, i.e. $n = 4$.

Note that the number of tapered beams is 9. However the beams at the gables carry only one half of the vertical load.

Stability loads, *see The Glulam Handbook Volume 2, Chapter 13, page 170*:

$$q_{st,1} = n \times \frac{1}{20} \times \frac{M_1}{l_{tot} \times h_{x,max}} = 4 \times \frac{1}{20} \times \frac{756.02}{20 \times 1.15} = 6.56 \text{ kN/m}$$

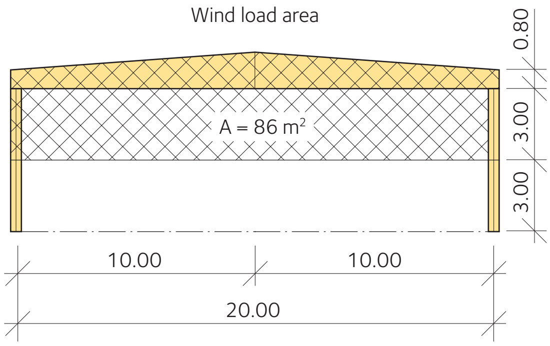
$$q_{st,2} = n \times \frac{1}{20} \times \frac{M_2}{l_{tot} \times h_{x,max}} = 4 \times \frac{1}{20} \times \frac{551.48}{20 \times 1.15} = 4.77 \text{ kN/m}$$



7.5 Roof bracing system

The wind load area is shown below. The factor 1.1 accounts for the presence of purlins and curtain walls.

$$A_{wind} = 86 \times 1.1 = 94.6 \text{ m}^2$$



The wind can be considered as a uniformly distributed load acting on the roof plane. It is assumed that the bracing system closest to the windward side takes the entire load. This assumption is based on the fact that the roof sheeting is normally not stiff enough to allow for an even distribution of the wind load between the bracing systems.

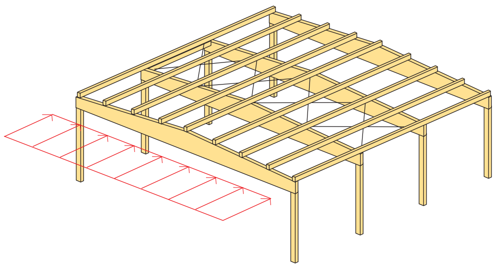
$$q_{w,1} = \left(q_{w,p,1} + q_{w,i,1}\right) \times \frac{A_{wind}}{l_{tot}} = (0.24 + 0.11) \times \frac{94.6}{20} = 1.64 \text{ kN/m}$$

$$q_{w,2} = \left(q_{w,p,2} + q_{w,i,2}\right) \times \frac{A_{wind}}{l_{tot}} = (0.79 + 0.37) \times \frac{94.6}{20} = 5.46 \text{ kN/m}$$

The total load acting on the bracing system is:

$$q_{tot,1} = q_{w,1} + q_{st,1} = 1.64 + 6.56 = 8.20 \text{ kN/m}$$

$$q_{tot,2} = q_{w,2} + q_{st,2} = 5.46 + 4.77 = 10.23 \text{ kN/m}$$

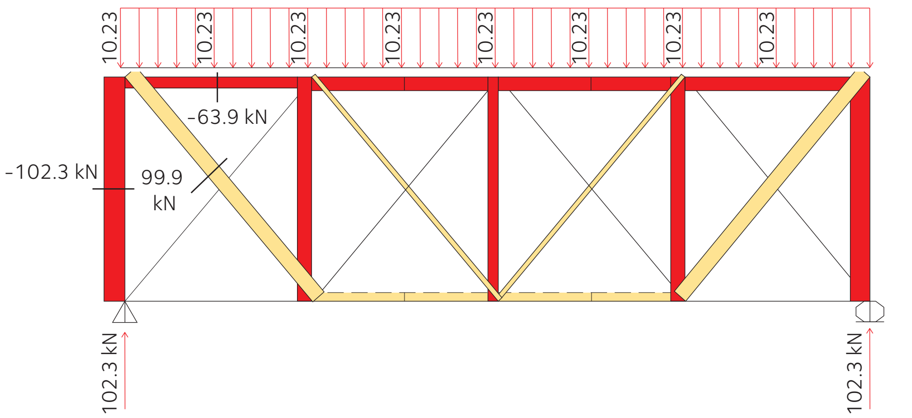


Leading design combination:

$$\frac{q_{tot,1}}{k_{mod,1}} = \frac{8.20}{0.8} = 10.3 \quad < \quad \frac{q_{tot,2}}{k_{mod,2}} = \frac{10.23}{0.9} = 11.4$$

Thus combination 2 is leading at ULS.

a) Internal forces and reactions



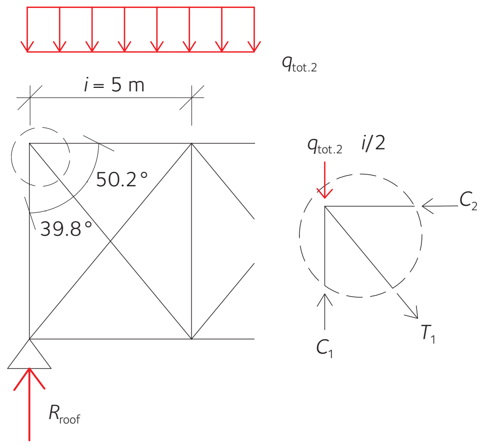
$$R_{\text{roof}} = q_{\text{tot},2} \times \frac{l_{\text{tot}}}{2} = 10.23 \times \frac{20}{2} = 102.3 \text{ kN}$$

$$C_1 = R_{\text{roof}} = 102.3 \text{ kN}$$

$$q_{\text{tot},2} \times \frac{i}{2} = 10.23 \times \frac{5}{2} = 25.6 \text{ kN}$$

$$T_1 = \frac{\left(C_1 - q_{\text{tot},2} \times \frac{i}{2}\right)}{\cos(39.8°)} = \frac{102.3 - 10.23 \times \frac{5}{2}}{\cos(39.8°)} = 99.87 \text{ kN}$$

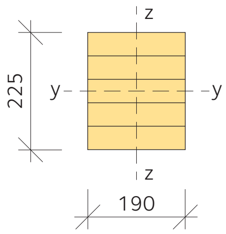
$$C_2 = T_1 \times \cos(50.2°) = 99.87 \times \cos(50.2°) = 64 \text{ kN}$$



b) Verification of the strut

$$\sigma_{\text{c},0,\text{d}} = \frac{N_{\text{Ed}}}{b \times h} = \frac{102.3 \times 10^3}{190 \times 225} = 2.4 \text{ MPa}$$

Strut
Section A-A
(*see 7.1, page 124*)



Stability about the z-axis (deflection in the y-direction)

Buckling length:

$$l_{0,\text{z}} = 6 \text{ m}$$

Euler critical stress:

$$\sigma_{\text{cr,z}} = \frac{\pi^2 \times E_{0.05} \times I_{\text{z}}}{(b \times h) \times l_{0,\text{z}}^2} = \frac{\pi^2 \times 10800 \times \frac{190^3 \times 225}{12}}{190 \times 225 \times (6 \times 10^3)^2} = 8.9 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{\text{rel,z}} = \sqrt{\frac{f_{\text{c},0,\text{k}}}{\sigma_{\text{cr,z}}}} = \sqrt{\frac{24.5}{8.91}} = 1.66$$

k factor:

$$k_{\text{z}} = \frac{1}{2} \times \left[1 + \beta_{\text{c}} \times (\lambda_{\text{rel,z}} - 0.3) + \lambda_{\text{rel,z}}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times (1.66 - 0.3) + 1.66^2\right] = 1.94$$

Reduction factor for buckling:

$$k_{\text{c,z}} = \frac{1}{k_{\text{z}} + \sqrt{k_{\text{z}}^2 - \lambda_{\text{rel,z}}^2}} = \frac{1}{1.94 + \sqrt{1.94^2 - 1.66^2}} = 0.34$$

Check for axial buckling about z-axis (*EN 1995-1-1, equation 6.23*):

$$\frac{\sigma_{\text{c},0,\text{d}}}{k_{\text{c,z}} \times f_{\text{c},0,\text{d}}} = \frac{2.39}{0.34 \times 17.64} = 0.4 \quad < 1 \quad \textbf{OK}$$

c) Verification of the tie rod

Tie-rods with diameter d = 24 mm (A_{net} = 353 mm²) are utilized for the roof bracing system. The verification is carried out following the rules for the design of bolts since the ends of the tie-rods are threaded:

$$T_{Ed} = 99.9 \text{ kN}$$

Tension capacity (*EN 1993-1-8, table 3.4*):

$$T_{Rd} = \frac{A_{net} \times f_{uk} \times 0.9}{\gamma_{M2}} = \frac{353 \times 500 \times 0.9}{1.2} = 132375 \text{ N}$$

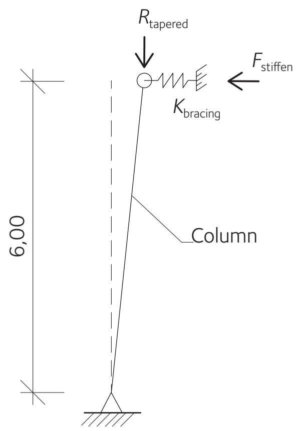
Verification (*EN 1993-1-1, equation 6.5*):

$$\frac{T_{Ed}}{T_{Rd}} = \frac{99.9}{132.38} = 0.75 \quad < 1 \quad \textbf{OK}$$

7.6 Wall bracing system

The wall bracing system is subjected to a concentrate force which is the sum of the support-reaction from the roof bracing system and the stability force acting on the columns.

The stability force on the columns is derived by means of the following static model, *see The Glulam Handbook Volume 2, Chapter 13, page 170*:



$$R_{tapered} = q_{ver,2} \times \frac{l_{tot}}{2} = 13.68 \times \frac{20}{2} = 137 \text{ kN}$$

$$F_{stiffen} = \frac{n \times R_{tapered}}{100} = \frac{4 \times 137}{100} = 5.5 \text{ kN}$$

$$K_{min} = 2 \times n \times \frac{R_{tapered}}{h} = 2 \times 4 \times \frac{137}{6.8} = 160.9 \text{ N/mm}$$

$$K_{bracing} = \frac{E_{steel} \times A_{tierod} \times \left(\cos\left(48.5°\right)\right)^3}{h} = \frac{210000 \times \frac{\pi \times 27^2}{4} \times \cos\left(48.5°\right)^3}{6.8 \times 10^3} = 5144.2 \text{ N/mm}$$

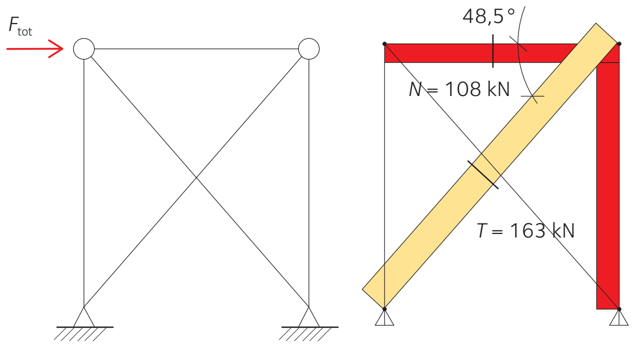
Minimum brace stiffness required:

$$K_{bracing} = 5144.2 \text{ N/mm} \quad > \quad K_{min} = 160.9 \text{ N/mm} \quad \textbf{OK}$$

a) Verification of the strut

Axial force:

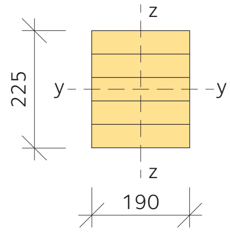
$$F_{tot} = R_{roof} + F_{stiffen} = 102.3 + 5.5 = 107.8 \text{ kN}$$



A new check is performed on the 190 × 225 mm strut, this time tacking also into consideration the action of the stability force:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{107.8 \times 10^3}{190 \times 225} = 2.52 \text{ MPa}$$

Strut
Section A–A
(*see 7.1, page124*)



Check for axial buckling about z-axis (*EN 1995-1-1, equation 6.23*):

$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \frac{2.52}{0.34 \times 17.64} = 0.42 \quad < 1 \quad \textbf{OK}$$

b) Verification of the steel tie rod

Tie-rods with diameter d = 27 mm (A_{net} = 459 mm²) are utilized for the wall bracing system:

$$T_{Ed} = 162.7 \text{ kN}$$

Tension capacity (*EN 1993-1-8, table 3.4*):

$$T_{Rd} = \frac{A_{net} \times f_{uk} \times 0.9}{\gamma_{M2}} = \frac{459 \times 500 \times 0.9}{1.2} = 172125 \text{ N}$$

Verification (*EN 1993-1-1, equation 6.5*):

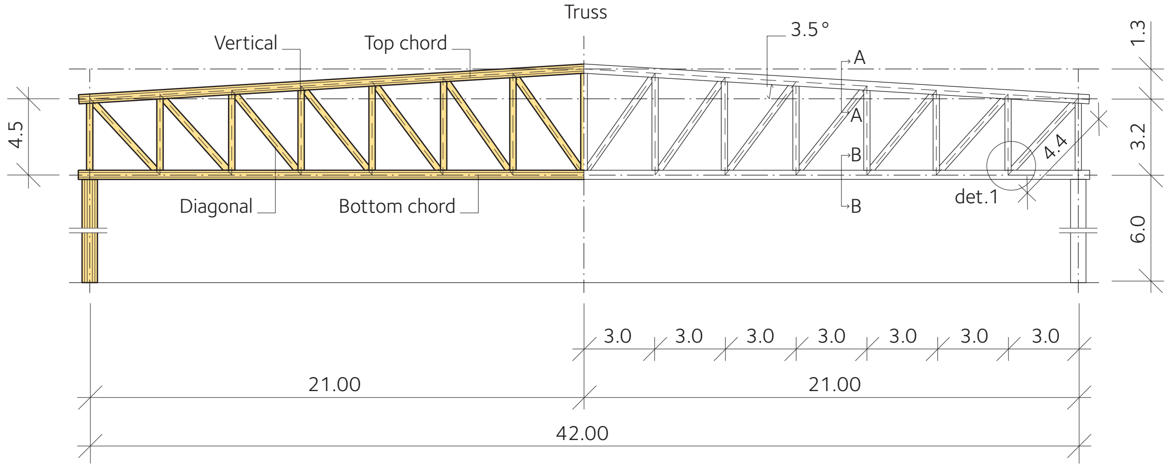
$$\frac{T_{Ed}}{T_{Rd}} = \frac{162.7}{172.1} = 0.95 \quad < 1 \quad \textbf{OK}$$

Example 8: Design of a truss

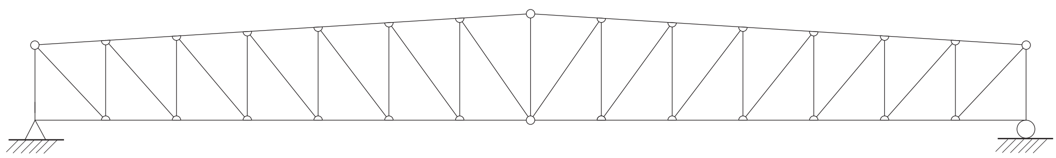
8.1 System, dimensions and design parameters

Design and verify the truss below.

Truss

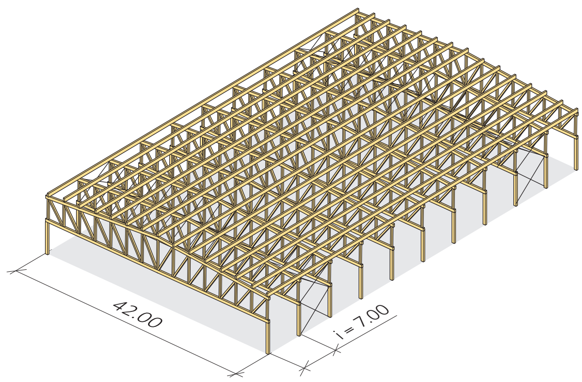


Static system



The cross-section dimensions are determined using the preliminary design method shown in *section 8.4, page 132*.

The truss is made of glulam, strength class	GL30c
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$



8.2 Loads

The loads considered in the design of the truss are:

Truss

$$g_{k,1} = 2.4 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.6 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.6 \times 7 \times 1.1 = 4.62 \text{ kN/m}$

Snow load

$S_k = 1.5 \text{ kN/m}^2$ $\quad s_k = S_k \times i \times \mu \times 1.1 = 1.5 \times 7 \times 0.853 \times 1.1 = 9.85 \text{ kN/m}$

Factor 1.1 used in the equations above accounts for the continuity of purlins over trusses.

8.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times \left(2.4 + 4.62\right) = 8.42 \text{ kN/m}$$

Combination 2 (self-load leading + snow load, medium term symmetric load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s + s_k\right] = 1 \times \left[1.2 \times \left(2.4 + 4.62\right) + 1.5 \times 9.85\right] = 23.20 \text{ kN/m}$$

8.4 Preliminary design

The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, section 8.2, page 123*:

Truss depth at the apex:

$$h_{apex} = \frac{l_{tot}}{10} = \frac{42}{10} = 4.2 \text{ m} \quad \rightarrow \quad h_{apex} = 4.5 \text{ m}$$

Truss depth at the edge:

$$h_{edge} = h_{apex} - \frac{l_{tot}}{2} \times \sin(\alpha) = 4.5 - \frac{42}{2} \times \sin\left(3.5°\right) = 3.22 \text{ m} \quad \rightarrow \quad h_{edge} = 3.2 \text{ m}$$

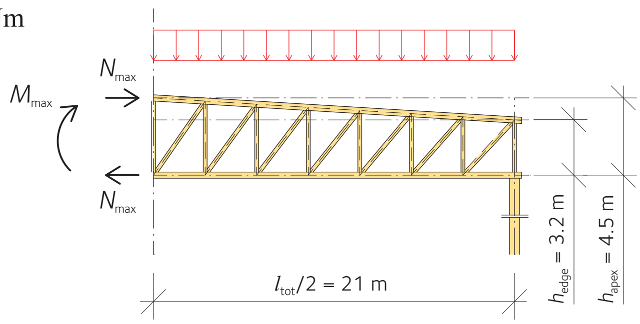
The most stressed truss chord elements are located at mid-span.

Maximum bending moment:

$$M_{max} = \frac{q_{dII} \times l_{tot}^{\ 2}}{8} = \frac{23.20 \times 42^2}{8} = 5116.6 \text{ kNm}$$

Maximum compression / tension force:

$$N_{max} = \frac{M_{max}}{h_{apex}} = \frac{5116.6}{4.5} = 1137 \text{ kN}$$



Chords:

$$A = \frac{N_{\max}}{0.7 \times f_{t,0,d}} = \frac{1137 \times 10^3}{0.7 \times 12.5} = 129946 \text{ mm}^2$$

$$d = \sqrt{A} = 360 \text{ mm} \quad \rightarrow \quad h = 360 \text{ mm} \quad b = 355 \text{ mm}$$

The cross-section should be relatively wide in order to allow for several slotted-in plates.
A reduction factor of 0.7 is assumed to account for the cross-section reduction due to slots and holes.

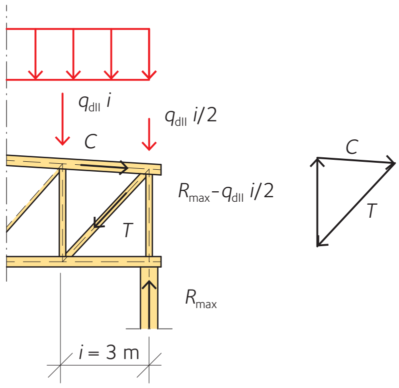
The reaction force is:

$$R_{\max} = \frac{q_{dII} \times l_{tot}}{2} = 482 \text{ kN}$$

$$R_{\max} - \frac{q_{dII} \times i}{2} = 448 \text{ kN}$$

The reaction force generates the follow internal force:

$$C = 398 \text{ kN}$$
$$T = 581 \text{ kN}$$

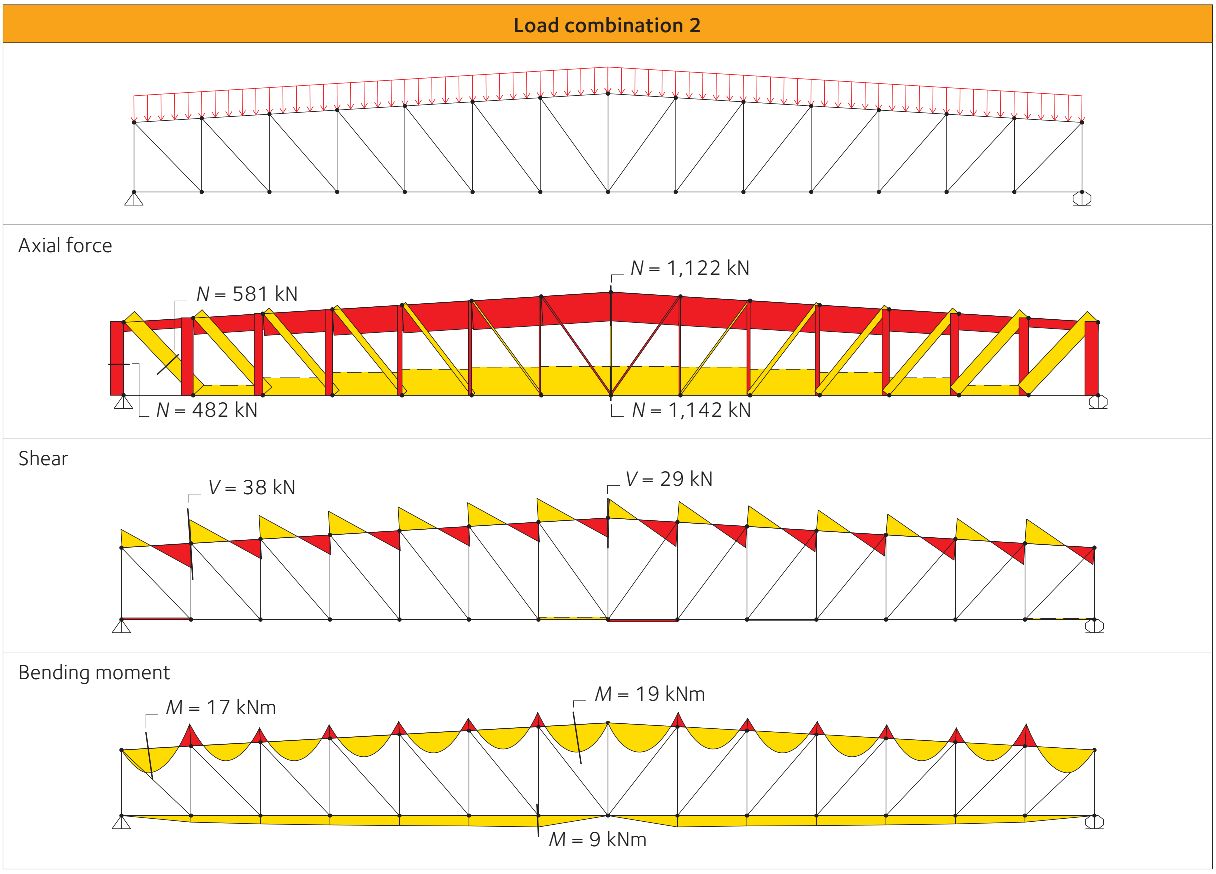


Verticals and diagonals:

$$b = 355 \text{ mm}$$

$$h_{\min} = \frac{T}{0.7 \times f_{t,0,d} \times b} = 186 \text{ mm} \quad \rightarrow \quad h = 225 \text{ mm}$$

8.5 Internal forces and moments



8.6. Verification of the top chord

a) Compression parallel to the grain

$$\sigma_{c,0,d} = \frac{N_{Ed}}{\left(b - 4 \times d_{plate}\right) \times \left(h - 3 \times d_{dowel}\right)} = \frac{1122 \times 10^3}{(355 - 4 \times 8) \times (360 - 3 \times 12)} = 10.72 \text{ MPa}$$

The compression stress is computed in the net cross-section. For example a connection with 4 slotted-in steel plates with a 3 × 3 dowel net d = 12 mm is assumed.

Compression parallel to the grain verification (*EN 1995-1-1, equation 6.2*):

$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} = \frac{10.72}{15.68} = 0.68 \quad < 1 \quad \textbf{OK}$$

b) Stability check for combined bending and compression

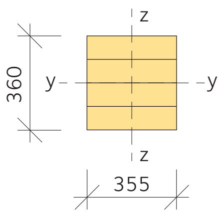
The truss is laterally stiffened by means of a bracing system; braced points are 3 m apart.

Since the cross-section is nearly square, the check is performed only for buckling about y-axis (bending moments caused by gravity loads also tend to reduce the buckling strength about y-axis):

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{1122 \times 10^3}{355 \times 360} = 8.78 \text{ kN}$$

$$\sigma_{m,y,d} = \frac{M_{Ed}}{b \times \frac{h^2}{6}} = \frac{20 \times 10^6}{355 \times \frac{360^2}{6}} = 2.65 \text{ MPa}$$

Top chord,
Section A-A
(*see 8.1, page 131*)



Stability about the y-axis (deflection in the z-direction)

Buckling length:

$$l_{0,y} = 3 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,y} = \frac{\pi^2 \times E_{0.05} \times I_y}{\left(b \times h\right) \times l_{0,y}^{\ 2}} = \frac{\pi^2 \times 10800 \times \frac{355 \times 360^3}{12}}{355 \times 360 \times \left(3 \times 10^3\right)^2} = 127.91 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,y}}} = \sqrt{\frac{24.5}{127.91}} = 0.44$$

k factor:

$$k_y = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}^{\ 2}\right] = \frac{1}{2} \times \left[1 + 0.1 \times (0.44 - 0.3) + 0.44^2\right] = 0.6$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^{\ 2} - \lambda_{rel,y}^{\ 2}}} = \frac{1}{0.6 + \sqrt{0.6^2 - 0.44^2}} = 0.98$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

$$\frac{\sigma_{c,0,d}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,d}} = \frac{8.78}{0.98 \times 15.68} + \frac{2.65}{19.2} = 0.71 \quad < 1 \quad \textbf{OK}$$

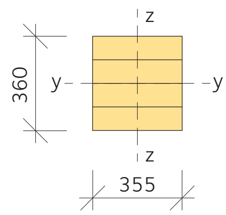
8.7 Verification of the bottom chord

Tension parallel to the grain shall be evaluated with regards to the net cross-section. For example a connection with 4 slotted-in steel plates with a 3 × 3 dowel net d = 12 mm is assumed:

$$\sigma_{m,y,d} = \frac{M_{Ed}}{\left(b - 4 \times d_{plate}\right) \times \frac{h^2}{6}} = \frac{9 \times 10^6}{\left(355 - 4 \times 8\right) \times \frac{360^2}{6}} = 1.29 \text{ MPa}$$

$$\sigma_{t,0,d} = \frac{N_{Ed}}{\left(b - 4 \times d_{plate}\right) \times h} = \frac{1142 \times 10^3}{\left(355 - 4 \times 8\right) \times 360} = 9.82 \text{ MPa}$$

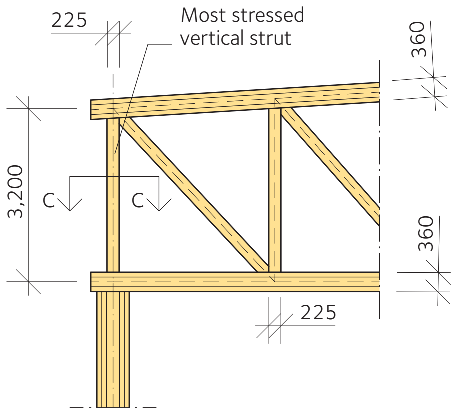
Bottom chord,
Section B-B
(*see 8.1, page 131*)



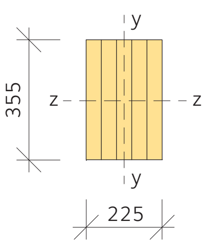
Combined bending and tension verification (*EN 1995-1-1, equation 6.17*):

$$\frac{\sigma_{t,0,d}}{f_{t,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,d}} = \frac{9.82}{12.48} + \frac{1.29}{19.2} = 0.85 \quad < 1 \quad \textbf{OK}$$

8.8 Verification of the verticals



Vertical
Section C-C
(*see 8.1, page 131*)



$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{482 \times 10^3}{355 \times 225} = 6.04 \text{ MPa}$$

Stability about the y-axis (deflection in the z-direction)

Buckling length:

$$l_{0,y} = 3.2 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,y} = \frac{\pi^2 \times E_{0.05} \times I_y}{\left(b \times h\right) \times l_{0,y}^{\ 2}} = \frac{\pi^2 \times 10800 \times \frac{355 \times 225^3}{12}}{355 \times 225 \times \left(3.2 \times 10^3\right)^2} = 43.91 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{\text{rel,y}} = \sqrt{\frac{f_{\text{c,0,k}}}{\sigma_{\text{cr,y}}}} = \sqrt{\frac{24.5}{43.91}} = 0.75$$

k factor:

$$k_{\text{y}} = \frac{1}{2} \times \left[1 + \beta_{\text{c}} \times \left(\lambda_{\text{rel,y}} - 0.3\right) + \lambda_{\text{rel,y}}{}^{2}\right] = \frac{1}{2} \times \left[1 + 0.1 \times \left(0.75 - 0.3\right) + 0.75^{2}\right] = 0.8$$

Reduction factor for buckling:

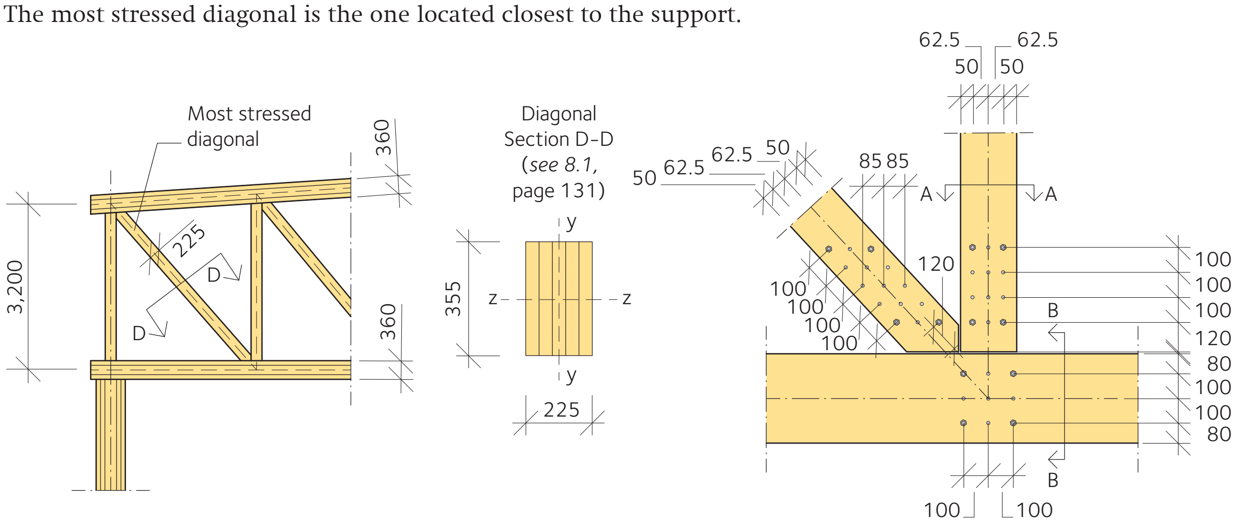
$$k_{\text{c,y}} = \frac{1}{k_{\text{y}} + \sqrt{k_{\text{y}}{}^{2} - \lambda_{\text{rel,y}}{}^{2}}} = \frac{1}{0.8 + \sqrt{0.8^{2} - 0.75^{2}}} = 0.92$$

Check for axial buckling about y-axis (*EN 1995-1-1, equation 6.23*):

$$\frac{\sigma_{\text{c,0,d}}}{k_{\text{c,y}} \times f_{\text{c,0,d}}} = \frac{6.04}{0.92 \times 15.68} = 0.42 \quad < 1 \quad \textbf{OK}$$

8.9 Verification of the diagonals

The most stressed diagonal is the one located closest to the support.



The diagonal shall be checked with regards to the net cross-section. Assume that the connection has 4 slotted-in steel plates fixed with 9 dowels, d = 12 mm, in a 3 × 3 grid. The design of the connection is shown in *example 21, page 211*.

a) Tension parallel to the grain

$$\sigma_{\text{t,0,d}} = \frac{T_{\text{Ed}}}{\left(b - 4 \times d_{\text{plate}}\right) \times \left(h - 3 \times d_{\text{dowel}}\right)} = \frac{581 \times 10^{3}}{\left(355 - 4 \times 8\right) \times \left(225 - 3 \times 12\right)} = 9.52 \text{ MPa}$$

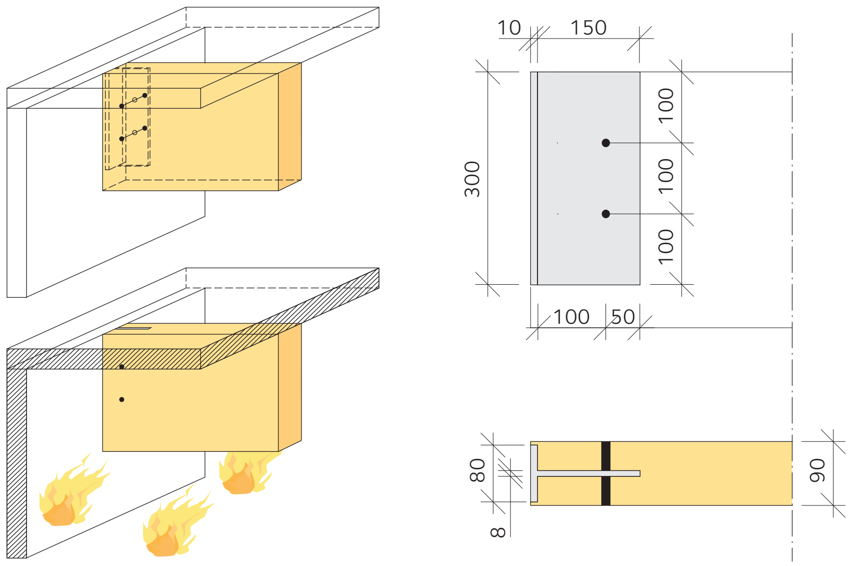
Tension parallel to the grain verification (*EN 1995-1-1, equation 6.1*):

$$\frac{\sigma_{\text{t,0,d}}}{f_{\text{t,0,d}}} = \frac{9.52}{12.48} = 0.76 \quad < 1 \quad \textbf{OK}$$

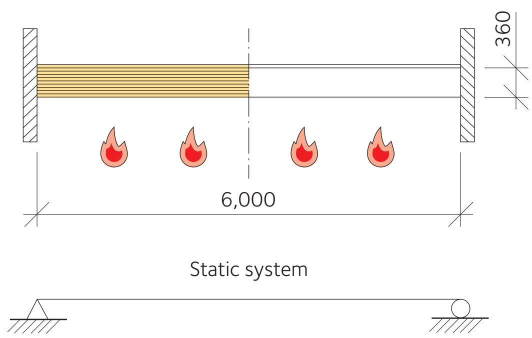
Example 9: Design of floor joist and connection with consideration to fire resistance class

9.1 System, dimensions and design parameters

In this example, the beam of *example 1, page 81* is checked for fire performance. It is assumed that the beam is connected to a concrete wall by means of a connection made with slotted-in plates and dowels as shown in the figure below.



The beam is made of glulam, strength class	GL30c
The slotted in plate is made of steel, structural steel grade	S355
The dowels (d = 12 mm) are made of steel, structural steel grade	S355
Safety class 3	$\gamma_d = 1$
Service class 1	
The beam and the connection must meet fire resistance class R60 (60 minutes)	



Use the following partial factors and ψ factors for design		
	ULS	**Fire**
Partial factor for permanent load	$\gamma_g = 1.2$	$\gamma_{g,fi} = 1.0$
Partial factor for variable load	$\gamma_q = 1.5$	$\gamma_{q,fi} = 1.0$
Material partial factor for glulam	$\gamma_M = 1.25$	$\gamma_{M,fi} = 1.0$
Partial factor for connections	$\gamma_C = 1.3$	$\gamma_{C,fi} = 1.0$
ψ factor in event of fire	–	$\psi_{fi} = 0.5$

9.2 Loads

The loads considered in the design are:

Structural

$$g_{k,1} = 0.2 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.5 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i = 0.50 \times 0.90 = 0.45 \text{ kN/m}$

Variable load

$Q_k = 2 \text{ kN/m}^2$ $\quad q_k = Q_k \times i = 2 \times 0.9 = 1.8 \text{ kN/m}$

9.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3*):

Combination 1 (self-load leading + variable load, medium term load, $k_{mod} = 0.8$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_q + q_k\right] = 1 \times \left[1.2 \times \left(0.2 + 0.5\right) + 1.5 \times 1.8\right] = 3.5 \text{ kN/m}$$

Combination 2 (self-load leading + variable load in event of fire, $k_{mod,fi} = 1.0$, $k_{fi} = 1.15$):

$$q_{dII} = \gamma_d \times \left[\gamma_{g,fi} \times \left(g_{k,1} + g_{k,2}\right) + \psi_{fi} \times \gamma_{q,fi} \times q_k\right] = 1 \times \left[1 \times \left(0.2 + 0.5\right) + 0.5 \times 1 \times 1.8\right] = 1.6 \text{ kN/m}$$

9.4 Design strengths

Load combination 1 (ULS without fire):

$$f_{m,d} = \frac{f_{m,k} \times k_{mod}}{\gamma_M} = \frac{30 \times 0.8}{1.25} = 19.2 \frac{\text{N}}{\text{mm}^2}$$

$$f_{v,d} = \frac{f_{v,k} \times k_{mod}}{\gamma_M} = \frac{3.5 \times 0.8}{1.25} = 2.24 \frac{\text{N}}{\text{mm}^2}$$

Load combination 2 (fire design):

$$f_{m,d,fi} = k_{fi} \times \frac{f_{m,k} \times k_{mod,fi}}{\gamma_{M,fi}} = 1.15 \times \frac{30 \times 1}{1} = 34.5 \frac{\text{N}}{\text{mm}^2}$$

$$f_{v,d,fi} = k_{fi} \times \frac{f_{v,k} \times k_{mod,fi}}{\gamma_{M,fi}} = 1.15 \times \frac{3.5 \times 1}{1} = 4.02 \frac{\text{N}}{\text{mm}^2}$$

9.5 ULS verifications (without fire)

a) Shear

$$V_{Ed} = q_{dI} \times \frac{l_{tot}}{2} = 3.48 \times \frac{6}{2} = 10.44 \text{ kN}$$

$$\tau = \frac{3 \times V_{Ed}}{2 \times (b - t_{bracket}) \times h} = \frac{3 \times 10.44 \times 10^3}{2 \times (90 - 8) \times 360} = 0.53 \text{ MPa}$$

Verify the figure for shear stress (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau}{f_{v,d} \times k_{cr}} = \frac{0.53}{2.24 \times 0.86} = 0.28 \quad < 1 \quad \textbf{OK}$$

b) Bending moment

$$M_{Ed} = q_{dI} \times \frac{l_{tot}}{8} = 3.48 \times \frac{6^2}{8} = 15.66 \text{ kNm}$$

$$\sigma_{m,d} = \frac{6 \times M_{Ed}}{b \times h^2} = \frac{6 \times 15.66 \times 10^6}{90 \times 360^2} = 80.6 \text{ MPa}$$

Lateral torsional buckling is prevented by the floor sheeting. Verification (*EN 1995-1-1, equation 6.11*):

$$k_h = \left(\frac{600}{360}\right)^{0.1} = 1.05 \qquad \frac{\sigma_{m,d}}{f_{m,d} \times k_h} = \frac{8.06}{19.2 \times 1.05} = 0.4 \quad < 1 \quad \textbf{OK}$$

c) Verification of the connection

$$d = 12 \text{ mm}$$

$$t_1 = \frac{b}{2} - \frac{s}{2} = \frac{90}{2} - \frac{8}{2} = 41 \text{ mm}$$

$$F_{Ed} = V_{Ed} = 10.4 \text{ kN}$$

Shear capacity of one dowel (*EN 1995-1-1, equation 8.11*):

$$f_{h,k} = 0.082 \times (1 - 0.01 \times d) \times \rho_k = 0.08 \times (1 - 0.01 \times 12) \times 390 = 28.14 \frac{\text{N}}{\text{mm}^2}$$

$$M_{y,Rk} = 0.3 \times f_{u,k} \times d^{2.6} = 0.3 \times 510 \times 12^{2.6} = 9.8 \times 10^4 \text{ Nmm}$$

$$F_{v,Rk,g} = 2 \times (f_{h,k} \times t_1 \times d) \times \left(\sqrt{2 + \frac{4 \times M_{y,Rk}}{f_{h,k} \times d \times t_1{}^2}} - 1\right) = 2 \times 28.14 \times 41 \times 12 \times \left(\sqrt{2 + \frac{4 \times 9.79 \times 10^4}{28.14 \times 12 \times 41^2}} - 1\right) = 1.77 \times 10^4 \text{ N}$$

Verification, two dowels d = 12 mm are used.

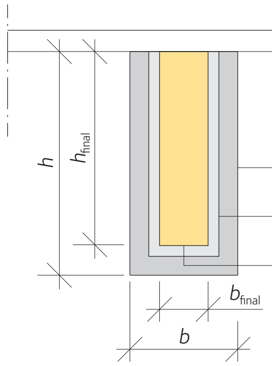
$$F_{v,Rd} = n \times k_{mod} \times \frac{F_{v,Rk,g}}{\gamma_C} = 2 \times 0.8 \times \frac{17.7}{1.3} = 22.7 \text{ kN}$$

$$\frac{F_{Ed}}{F_{v,Rd}} = \frac{10.44}{22.68} = 0.46 \quad < 1 \quad \textbf{OK}$$

For the SLS check, *see example 1, page 81*.
Note that the vibration check normally governs the design of a simply supported floor beams.

9.6 ULS verification with consideration to fire

a) Effective cross-section after 60 minutes of fire, three edges exposed to fire



Notional charring rate (*EN 1995-1-2, table 3.1*):

$$\beta_n = 0.7 \text{ mm/min}$$

Effective depth of charring (*EN 1995-1-2, equation 4.1*):

$$d_{ef} = d_{char,n} + k_0 \times d_0 = 42 + 1 \times 7 = 49 \text{ mm}$$

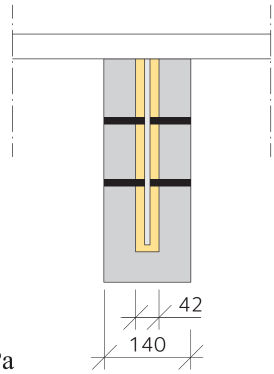
where:

$$d_{char,n} = \beta_n \times R \rightarrow 0.7 \times 60 = 42 \text{ mm}$$
$$k_0 = 1.0 d_0 = 7 \text{ mm}$$

Effective cross-section dimensions:

$$h_{final} = h - d_{ef} = 360 - 49 = 311 \quad > 300 \text{ mm} \quad \textbf{OK}$$
$$b_{final} = b - 2 \times d_{ef} = 90 - 2 \times 49 = -8 \quad \textbf{NOT OK}$$

The beam width must be increased.
The new beam width is:



$$b = 140 \text{ mm}$$
$$b_{final} = b - 2 \times d_{ef} = 140 - 2 \times 49 = 42 \text{ mm}$$

b) Shear

$$V_{Ed} = q_{dII} \times \frac{l_{tot}}{2} = 1.55 \times \frac{6}{2} = 4.65 \text{ kN}$$

$$\tau = \frac{3 \times V_{Ed}}{2 \times (b_{final} - t_{bracket}) \times h_{final}} = \frac{3 \times 4.65 \times 10^3}{2 \times (42 - 8) \times 311} = 0.66 \text{ MPa}$$

Verify the figure for shear stress (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau}{f_{vd,fi} \times k_{cr}} = \frac{0.66}{4.02 \times 0.85} = 0.19 \quad < 1 \quad \textbf{OK}$$

c) Bending moment

$$M_{Ed} = q_{dII} \times \frac{l_{tot}^2}{8} = 1.55 \times \frac{6^2}{8} = 6.98 \text{ kNm}$$

$$\sigma_{m,d} = \frac{6 \times M_{Ed}}{b_{final} \times h_{final}^2} = \frac{6 \times 6.98 \times 10^6}{42 \times 311^2} = 10.3 \text{ MPa}$$

Verify the figure for bending stress (*EN 1995-1-1, equation 6.11*):

$$k_h = (600/h_{final})^{0.1} = (600/311)^{0.1} = 1.07$$

$$f_{m,d,fi} = k_{mod,fi} \times \frac{f_{m,k} \times k_h \times k_{fi}}{\gamma_{M,fi}} = 30 \times 1.07 \times 1.15 = 36.8 \text{ MPa}$$

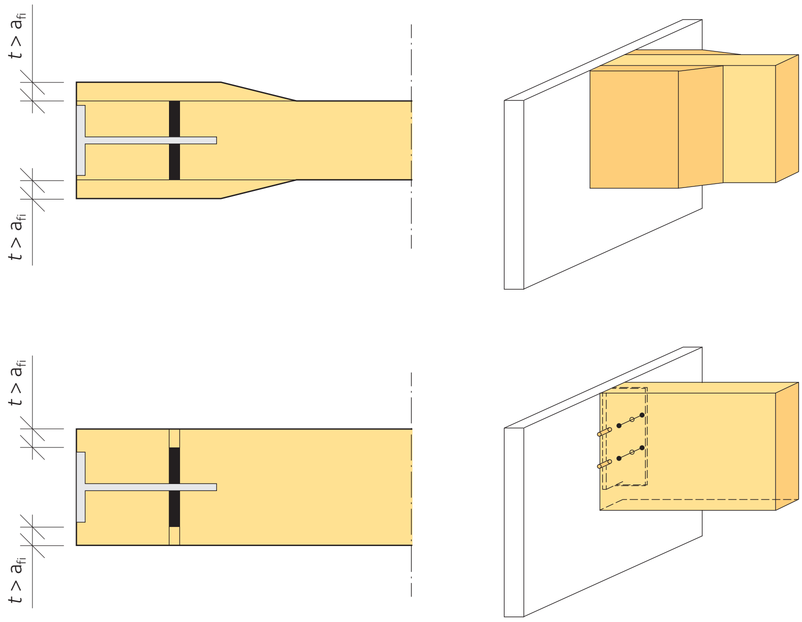
$$\frac{\sigma_{m,d}}{f_{m,d,fi}} = \frac{10.3}{36.8} = 0.28 \quad < 1 \quad \textbf{OK}$$

d) Connection

An unprotected connection is assumed to offer fire resistance $t_{d,fi}$ = 20 minutes, *see The Glulam Handbook Volume 2, table 16.3, page 244*. Verify the minimum required thickness, *t*, for bonded sheets or wood plugs, *see The Glulam Handbook Volume 2, equation 16.13, page 244*:

$$a_{fi} = \beta_n \times k_{flux} \times \left(t_{req} - t_{d,fi}\right) = 0.7 \times 1.5 \times \left(60 - 20\right) = 42 \text{ mm}$$

The dowels should never be exposed to fire. They shall therefore be protected by wood plugs or glued boards with a minimum thickness a_{fi} = 42 mm.

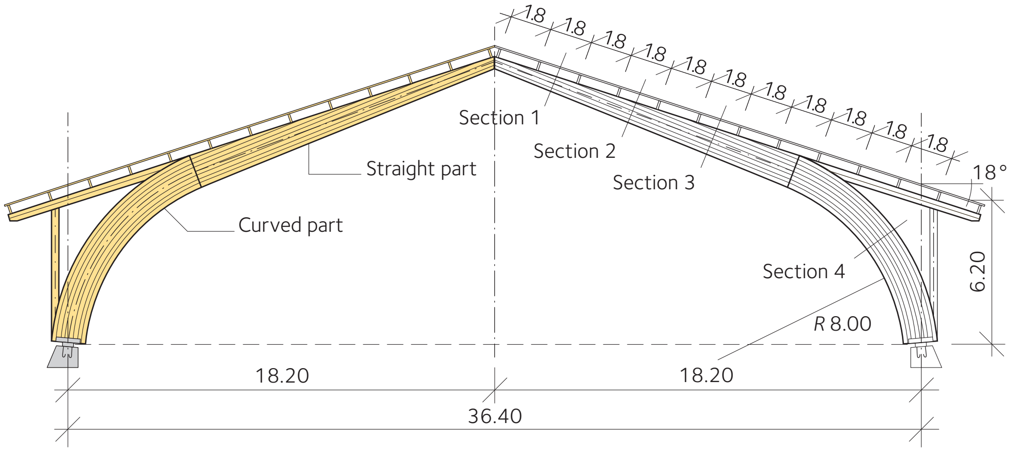


Example 10: Design of a frame with curved haunches

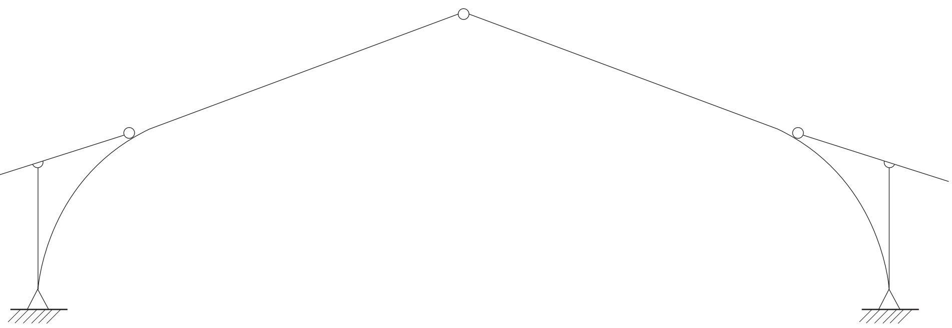
10.1 System, dimensions and design parameters

Design and verify the frame with curved haunches below.

Portal frame

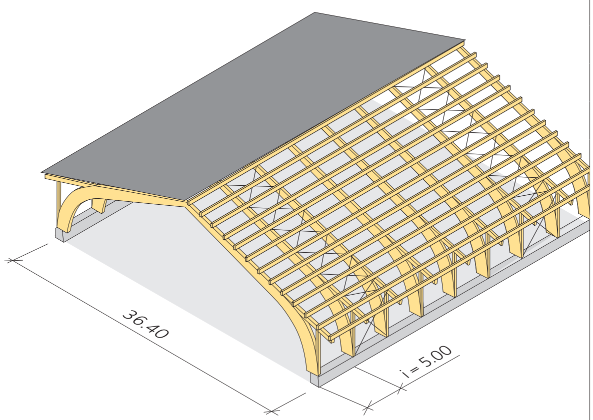


Static system



The cross-section dimensions are determined using the preliminary design method shown in *section 10.4, page 143*.

The arch is made of glulam, strength class	GL30c
Lamination thickness	t_{lam} = 33 mm
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$



10.2 Loads

The loads considered in the design of the portal frame are:

Structural

$$g_{k,1} = 2.3 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.6 \text{ kN/m}^2$ $\qquad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.6 \times 5 \times 1.1 = 3.3 \text{ kN/m}$

Snow load

$S_k = 1.5 \text{ kN/m}^2$ $\qquad s_k = S_k \times i \times \mu \times 1.1 = 1.5 \times 5 \times 1.07 \times 1.1 = 8.8 \text{ kN/m}$

Factor 1.1 used in the equations above accounts for the continuity of purlins over the frames.

10.3 Load combinations

Three different load combinations are considered (*EN 1990, clause 6.4.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times (2.3 + 3.3) = 6.72 \text{ kN/m}$$

Combination 2 (self-load leading + symmetric snow load, medium term load, $k_{mod} = 0.8$):

$$q_{dIIA} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_k\right] = 1 \times \left[1.2 \times (2.3 + 3.3) + 1.5 \times 8.8\right] = 19.96 \text{ kN/m}$$

Combination 3 (self-load leading + unsymmetric snow load, here given the lower value, medium term load, $k_{mod} = 0.8$):

$$q_{dIIB} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + 0.5\gamma_s \times s_k\right] = 1 \times \left[1.2 \times (2.3 + 3.3) + 0.5 \times 1.5 \times 6.6\right] = 11.67 \text{ kN/m}$$

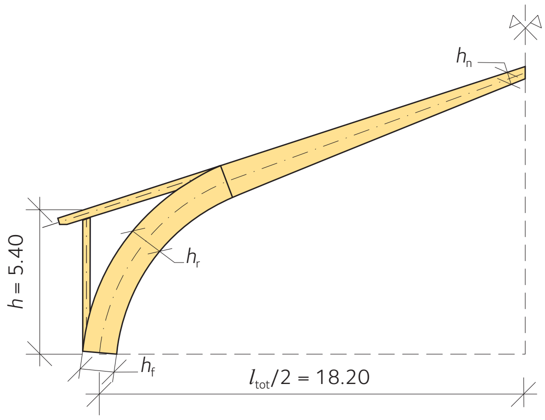
10.4 Preliminary design

The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, Chapter 10, page 140*:

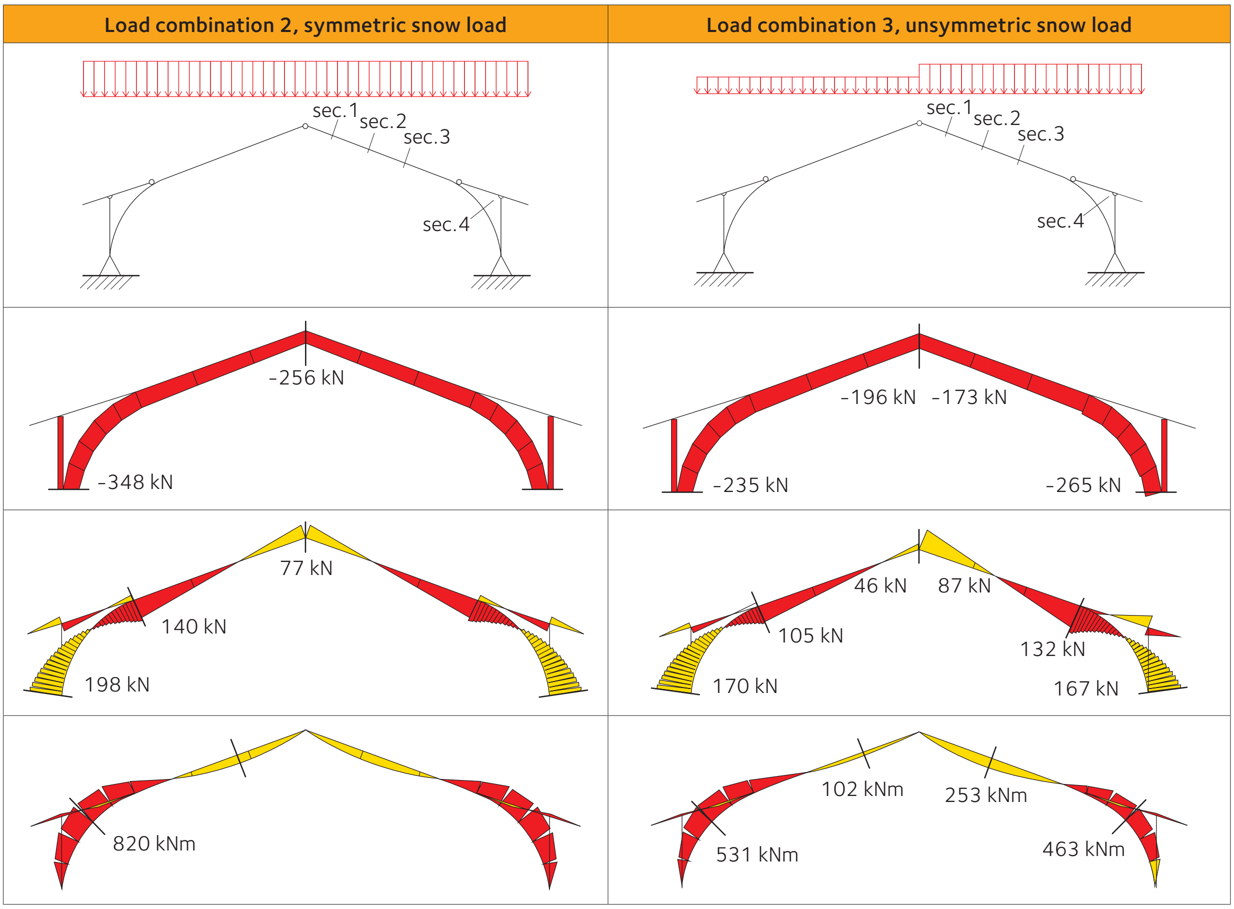
$$h_{r,min} = \frac{H}{15} + \frac{l_{tot}}{30} = \frac{5.4}{15} + \frac{36.4}{30} = 1.57 \text{ m} \quad \rightarrow \quad h_r = 1530 \text{ mm} \quad h_f = 1530 \text{ mm}$$

$$h_{n,min} = 0.3 \times h_{r,min} = 0.3 \times 1.57 = 0.47 \text{ m} \quad \rightarrow \quad h_n = 495 \text{ mm}$$

$$b_{min} = 0.15 \times h_{r,min} = 0.15 \times 1.57 = 0.24 \text{ m} \quad \rightarrow \quad b = 215 \text{ mm}$$



10.5 Internal forces and moments



10.6 ULS verification

a) Compression parallel to the grain at the support

The maximum stress occurs for load condition 2:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h_f} = \frac{348 \times 10^3}{215 \times 1530} = 1.06 \text{ MPa}$$

Compression parallel to the grain verification (*EN 1995-1-1, equation 6.2*):

$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} = \frac{1.06}{15.68} = 0.07 \quad < 1 \quad \textbf{OK}$$

b) Compression parallel to the grain at the ridge

The maximum stress occurs for load condition 2:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h_n} = \frac{256 \times 10^3}{215 \times 495} = 2.40 \text{ MPa}$$

Compression parallel to the grain verification (*EN 1995-1-1, equation 6.2*):

$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} = \frac{2.40}{15.68} = 0.15 \quad < 1 \quad \textbf{OK}$$

c) Shear verification at the support

The maximum stress occurs for load condition 2:

$$\tau_d = \frac{3 \times V_{Ed}}{2b \times h_f} = \frac{3 \times 198 \times 10^3}{2 \times 215 \times 1530} = 0.9 \text{ MPa}$$

Shear verification (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau_d}{f_{v,d} \times k_{cr}} = \frac{0.9}{2.24 \times 0.86} = 0.47 \quad < 1 \quad \textbf{OK}$$

d) Shear verification at the ridge

The maximum stress occurs for load condition 3:

$$\tau_d = \frac{3 \times V_{Ed}}{2b \times h_n} = \frac{3 \times 87 \times 10^3}{2 \times 215 \times 495} = 1.23 \text{ MPa}$$

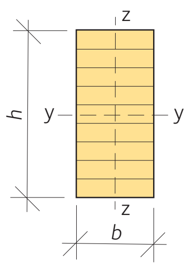
Shear verification (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau_d}{f_{v,d} \times k_{cr}} = \frac{1.23}{2.24 \times 0.86} = 0.64 \quad < 1 \quad \textbf{OK}$$

e) Stability check for combined bending and compression of the straight parts (combination 3)

The frame is laterally stiffened by means of a bracing system; braced points are 1.8 m apart.

Cross-sections 1, 2 and 3 are checked for load combination 3.
Corresponding values of axial force and bending moment are shown in the table below.



Section	Cross-section dimensions [mm]	Design axial force N_d [kN]	Design bending moment M_d [kNm]
1	215 × 670	-253	168
2	215 × 925	-263	218
3	215 × 1175	-274	210

Check for axial buckling about z-axis and bending about y-axis (*EN 1995-1-1, equation 6.24*):

Section	Effective buckling length, $l_{o,z}$ [mm]	Critical bending stress, $\sigma_{cr,z}$ [MPa]	Relative slenderness ratio, λ_{rel}	Reduction factor for buckling, $k_{c,z}$	Work ratio $\frac{N_d}{A \cdot k_{cz} \cdot f_{cd}} + \frac{M_{yd}}{W \cdot f_{myd}}$
1	$l_{0z,1} = 1.8$	$\sigma_{cr,z,1} = 126.73$	$\lambda_{rel,z,1} = 0.44$	$k_{cz,1} = 0.98$	$R_1 = 0.66$
2	$l_{0z,2} = 1.8$	$\sigma_{cr,z,2} = 126.73$	$\lambda_{rel,z,2} = 0.44$	$k_{cz,2} = 0.98$	$R_2 = 0.46$
3	$l_{0z,3} = 1.8$	$\sigma_{cr,z,3} = 126.73$	$\lambda_{rel,z,3} = 0.44$	$k_{cz,3} = 0.98$	$R_3 = 0.27$

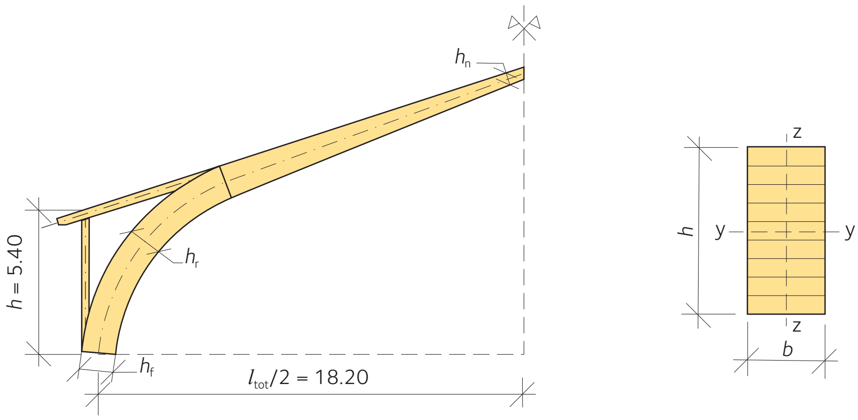
Check for lateral torsional buckling and axial buckling about z-axis (*EN 1995-1-1, equation 6.35*):

Section	Effective buckling length, $l_{o,z}$ [mm]	Critical bending stress, $\sigma_{cr,m}$ [MPa]	Relative slenderness ratio, λ_{rel}	Critical factor for lateral torsional buckling, $k_{c,z}$	Work ratio $\left(\frac{M_{yd}}{W \cdot k_{crit} \cdot f_{md}}\right)^2 + \frac{N_d}{A \cdot k_{cz} \cdot f_{cd}}$
1	$l_{0z,1} = 1.8$	$\sigma_{cr,m,1} = 291.33$	$\lambda_{rel,z,1} = 0.32$	$k_{crit,1} = 1$	$R_1 = 0.56$
2	$l_{0z,2} = 1.8$	$\sigma_{cr,m,2} = 211.02$	$\lambda_{rel,z,2} = 0.38$	$k_{crit,2} = 1$	$R_2 = 0.38$
3	$l_{0z,3} = 1.8$	$\sigma_{cr,m,3} = 211.02$	$\lambda_{rel,z,3} = 0.42$	$k_{crit,3} = 1$	$R_3 = 0.23$

f) Stability check for combined bending and compression of the curved part, section 4

$$\sigma_{m,d} = \frac{6 \times M_{Ed}}{b \times h^2} = \frac{6 \times 820 \times 10^6}{215 \times 1530^2} = 9.78 \text{ MPa}$$

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{408 \times 10^3}{215 \times 1530} = 1.24 \text{ MPa}$$



Stability about the z-axis, deflection in the y-direction, *see The Glulam Handbook Volume 2, clause 10.4.1, page 143*:
Buckling length:

$$l_{0,z} = 9.6 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,z} = \frac{\pi^2 \times E_{0.05} \times I_z}{A \times (l_{0,z})^2} = \frac{3.14^2 \times 10800 \times \frac{215^3 \times 1530}{12}}{215 \times 1530 \times (9.6 \times 10^3)^2} = 4.45 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,z}}} = \sqrt{\frac{24.5}{4.45}} = 2.35$$

k factor:

$$k_z = \frac{1}{2} \times \left[1 + \beta_c \times (\lambda_{rel,z} - 0.3) + \lambda_{rel,z}{}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times (2.35 - 0.3) + 2.35^2\right] = 3.35$$

Reduction factor for buckling:

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z{}^2 - \lambda_{rel,z}{}^2}} = \frac{1}{3.35 + \sqrt{3.35^2 - 2.35^2}} = 0.17$$

Lateral torsional buckling

Buckling length:

$$l_{0,z} = 9.6 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,m} = \frac{\left(\frac{\pi}{l_{0,z}} \times \sqrt{E_{0.05} \times I_z \times G_{05} \times k_v} + \frac{E_{0.05} \times I_z + G_{05} \times k_v}{2 \times R} \right)}{W_y} = 34.89 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{cr,m}}} = \sqrt{\frac{30}{34.89}} = 0.93$$

Reduction factor for buckling:

$$\text{for} \quad 0.75 < \lambda < 1.4 \quad \rightarrow \quad k_{crit} = 1.56 - 0.75 \times \lambda_{rel,m} = 0.86$$

The tensile bending strength shall be modified by factor k_r (*EN 1995-1-1, equation 6.49*):

$$\frac{R}{t_{lam}} = \frac{8 \times 10^3}{33.33} = 240.02$$

$$k_r = 1.0$$

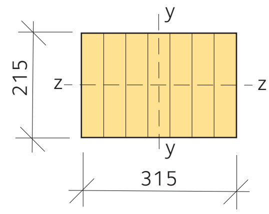
Check for axial buckling about z-axis and bending about y-axis (*EN 1995-1-1, equation 6.24*):

$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} + 0.7 \times \frac{\sigma_{m,d}}{k_r \times f_{m,d}} = \frac{1.24}{0.17 \times 15.68} + 0.7 \times \frac{9.78}{1 \times 19.2} = 0.82 \quad < 1 \quad \textbf{OK}$$

Check for lateral torsional buckling and axial buckling about z-axis (*EN 1995-1-1, equation 6.35*):

$$\left(\frac{\sigma_{m,d}}{k_r \times k_{crit} \times f_{m,d}} \right)^2 + \frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \left(\frac{9.78}{1 \times 0.86 \times 19.2} \right)^2 + \frac{1.24}{0.17 \times 15.68} = 0.82 \quad < 1 \quad \textbf{OK}$$

g) Verification of the frame leg



$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{128 \times 10^3}{315 \times 215} = 1.90 \text{ MPa}$$

Stability about the z-axis (deflection in the y-direction):
Buckling length:

$$l_{0,z} = 6.2 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,z} = \frac{\pi^2 \times E_{0.05} \times I_z}{(b \times h) \times l_{0,z}^2} = \frac{\pi^2 \times 10800 \times \frac{315 \times 215^3}{12}}{315 \times 215 \times (6.2 \times 10^3)^2} = 10.67 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,z}}} = \sqrt{\frac{24.5}{10.67}} = 1.52$$

k factor:

$$k_z = \frac{1}{2} \times \left[1 + \beta_c \times (\lambda_{rel,z} - 0.3) + \lambda_{rel,z}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times (1.52 - 0.3) + 1.52^2\right] = 1.71$$

Reduction factor for buckling:

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}} = \frac{1}{1.71 + \sqrt{1.71^2 - 1.52^2}} = 0.4$$

Check for axial buckling about z-axis (*EN 1995-1-1, equation 6.24*):

$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \frac{1.90}{0.4 \times 15.68} = 0.30 \quad < 1 \quad \textbf{OK}$$

h) Stability check for combined bending and compression in plane

The most stressed cross-section is section 4:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{408 \times 10^3}{215 \times 1530} = 1.24 \text{ MPa} \qquad \sigma_{m,d} = \frac{6M_{Ed}}{b \times h^2} = \frac{6 \times 820 \times 10^6}{215 \times 1530^2} = 9.78 \text{ MPa}$$

The corresponding buckling mode is shown below.

Stability about the y-axis (deflection in the z-direction)

The critical axial force is determined by means of a finite elements analysis.
Critical axial force:

$$N_{cr} = 4304.4 \text{ kN}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\frac{N_{cr}}{A}}} = \sqrt{\frac{24.5}{\frac{4304.4 \times 10^3}{328950}}} = 1.37$$

k factor:

$$k_y = 0.5 \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}{}^2\right] = 0.5 \times \left[1 + 0.1 \times (1.37 - 0.3) + 1.37^2\right] = 1.49$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y{}^2 - \lambda_{rel,y}{}^2}} = \frac{1}{1.49 + \sqrt{1.49^2 - 1.37^2}} = 0.48$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

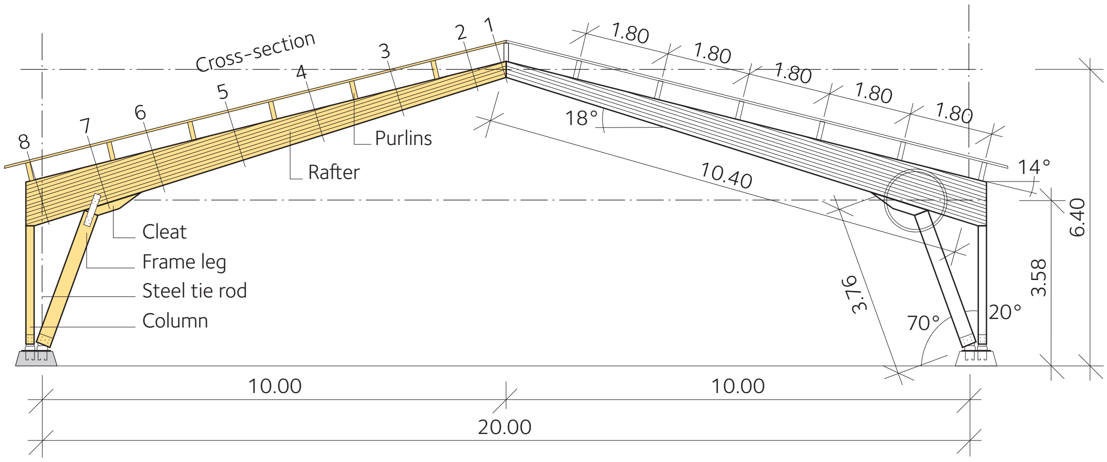
$$\frac{\sigma_{c,0,d}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} = \frac{1.90}{0.48 \times 15.68} + \frac{9.78}{19.2} = 0.76 \quad < 1 \quad \textbf{OK}$$

Example 11: Design of a knee-braced frame

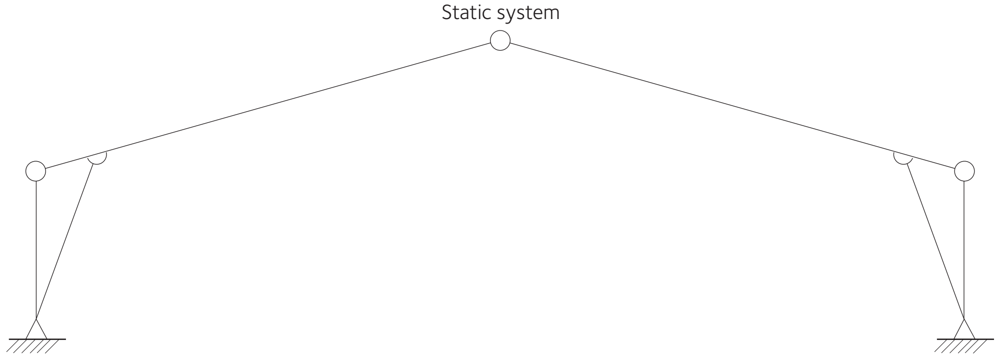
11.1 System, dimensions and design parameters

Design and verify the knee-braced frame below.

Portal frame

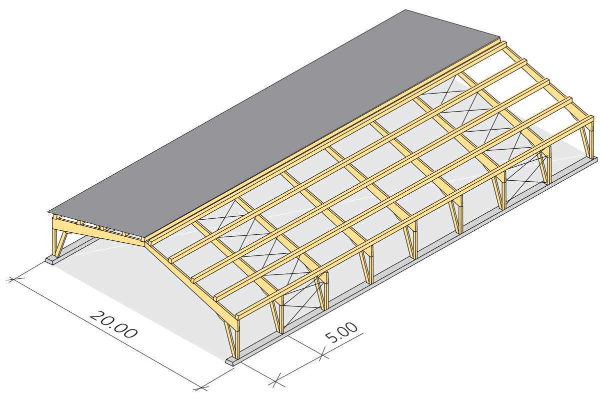


Static system



The cross-section dimensions are determined using the preliminary design method shown in *section 11.4, page 151*.

The arch is made of glulam, strength class	GL30c
Steel ties, structural steel grade	S355
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$
Partial factor for steel	$\gamma_{M2} = 1.2$



11.2 Loads

The loads considered in the design of the portal frame are:

Structural

$$g_{k,1} = 0.76 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.7 \text{ kN/m}^2$ $\qquad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.7 \times 5 \times 1.1 = 3.85 \text{ kN/m}$

Snow load

$S_k = 2.0 \text{ kN/m}^2$

Leeward $\quad s_{k,l} = S_k \times i \times \mu \times 1.1 = 2.0 \times 5 \times 1.01 \times 1.1 = 11.1 \text{ kN/m}$

Windward $\quad s_{k,r} = S_k \times i \times \mu \times 1.1 = 2.0 \times 5 \times 0.8 \times 1.1 = 8.8 \text{ kN/m}$

Factor 1.1 used in the equations above accounts for the continuity of purlins over the frames.

11.3 Load combinations

Three different load combinations are considered (*EN 1990, clause 6.4.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times \left(0.76 + 3.85\right) = 5.53 \text{ kN/m}$$

Combination 2 (self-load leading + symmetric snow load, medium term load, $k_{mod} = 0.8$):

Leeward

$$q_{dIIA,l} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,l}\right] = 1 \times \left[1.2 \times \left(0.76 + 3.85\right) + 1.5 \times 11.1\right] = 22.2 \text{ kN/m}$$

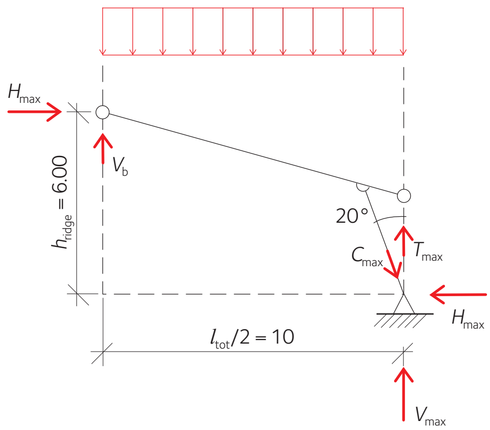
Windward

$$q_{dIIA,r} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,r}\right] = 1 \times \left[1.2 \times \left(0.76 + 3.85\right) + 1.5 \times 8.8\right] = 18.7 \text{ kN/m}$$

Combination 3 (self-load leading + unsymmetric snow load; here given the lower value, medium term load, $k_{mod} = 0.8$):

$$q_{dIIB,r} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + 0.5\gamma_s \times s_{k,r}\right] = 1 \times \left[1.2 \times \left(0.76 + 3.85\right) + 0.5 \times 1.5 \times 8.8\right] = 12.13 \text{ kN/m}$$

11.4 Preliminary design



The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, Chapter 10, page 140*:

$$V_{max} = \frac{\left(3 \times q_{dIIA,l} + q_{dIIA,r}\right) \times l_{tot}}{8} = \frac{(3 \times 22.2 + 18.7) \times 20}{8} = 213.30 \text{ kN}$$

$$H_{max} = \frac{\left(q_{dIIA,l} + q_{dIIA,r}\right) \times l_{tot}^2}{16 \times f} = \frac{(22.2 + 18.7) \times 20^2}{16 \times 6} = 170.54 \text{ kN}$$

$$C_{max} = \frac{H_{max}}{\cos(70°)} = \frac{170.54}{\cos(70°)} = 498.62 \text{ kN}$$

$$T_{max} = C_{max} \times \cos(20°) - V_{max} = 498.62 \times \cos(20°) - 213.30 = 255.24 \text{ kN}$$

$$V_b = \frac{\left(q_{dIIA,l} - q_{dIIA,r}\right) \times l_{tot}}{8} = \frac{(22.2 - 18.7) \times 20}{8} = 8.66 \text{ kN}$$

Rafter cross-section:

$$h_{\text{support}} = \frac{S_1 + S_2}{15} = \frac{10.41 \times 10^3 + 3.58 \times 10^3}{15} = 932.67 \text{ mm} \quad \rightarrow \quad h_{\text{support}} = 950 \text{ mm}$$

$$h_{\text{ridge}} = 0.3 \times h_{\text{support}} = 0.3 \times 950 = 285 \text{ mm} \quad \rightarrow \quad h_{\text{ridge}} = 318 \text{ mm}$$

$$b = \frac{h_{\text{support}}}{5} = \frac{950}{5} = 190 \text{ mm} \quad \rightarrow \quad b = 190 \text{ mm}$$

Frame leg cross-section:

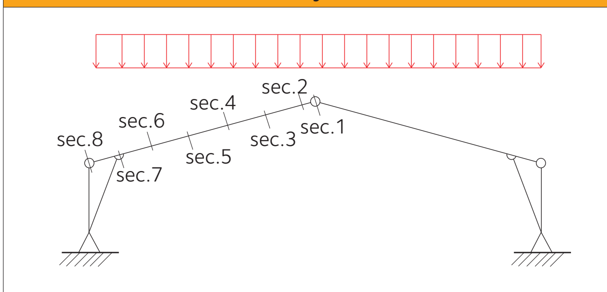
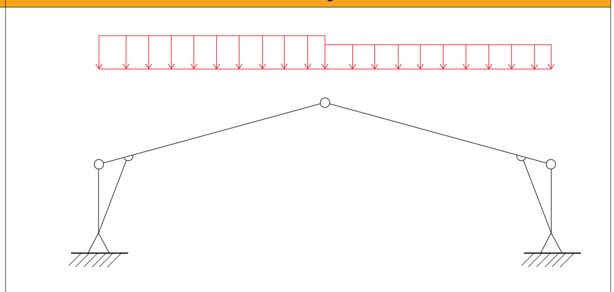
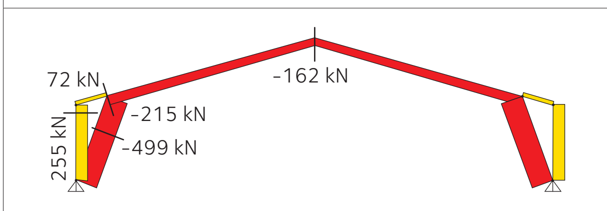
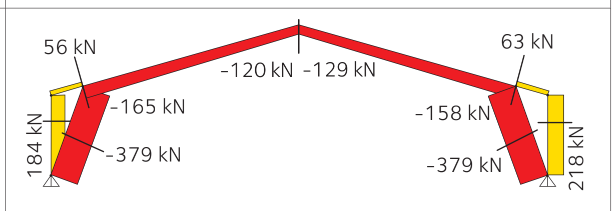
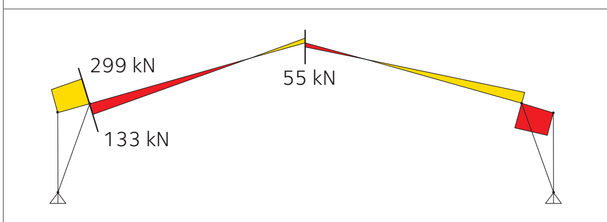
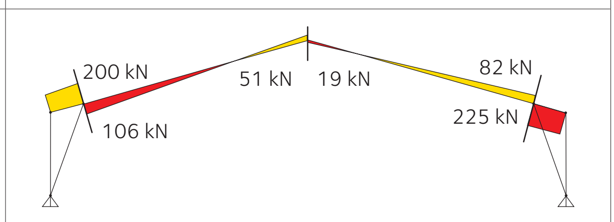
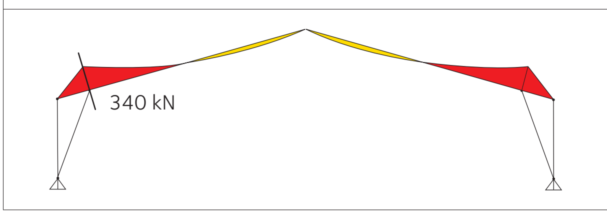
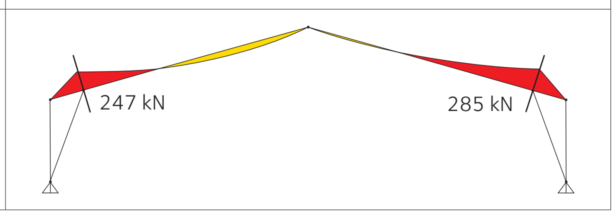
$$b = 190 \text{ mm}$$

$$h = \frac{C_{\max}}{k_r \times b \times f_{c,0,d}} = \frac{498.62 \times 10^3}{0.7 \times 190 \times 15.68} = 239.1 \text{ mm} \quad \rightarrow \quad h = 315 \text{ mm}$$

The factor k_r accounts for the strength reduction due to possible buckling.
Steel tie rod cross-section (It consists of two separate members side-by-side):

$$A_{\text{net,min}} = \frac{1}{2} \times \frac{T_{\max}}{0.9 \frac{f_{uk}}{\lambda_{M2}}} = \frac{1}{2} \times \frac{255.24 \times 10^3}{0.9 \times \frac{510}{1.2}} = 333.65 \text{ mm}^2 \quad \rightarrow \quad A = 452 \text{ mm}^2 \quad A_{\text{net}} = 353 \text{ mm}^2$$

11.5 Internal forces and moments

Load combination 2, symmetric snow load	Load combination 3, unsymmetric snow load
	
	
	
	

11.6 Calculations of upper frames in ultimate limit state

a) Compression parallel to the grain

The maximum stress occurs for load combination 2 and the most stressed cross-section is at the ridge, (section 1: 190 × 318 mm):

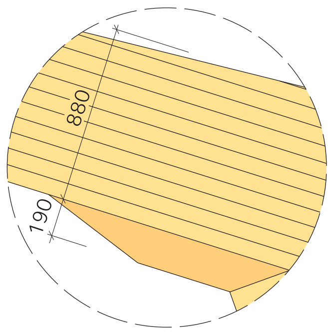
$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{161 \times 10^3}{190 \times 318} = 2.67 \text{ MPa}$$

Compression parallel to the grain (*EN 1995-1-1, equation 6.2*):

$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} = \frac{2.67}{15.68} = 0.17 \quad < 1 \quad \textbf{OK}$$

b) Shear

The maximum stress occurs for load combination 2 and the most stressed cross-section is the section 7, 190 × (880 + 190) mm:



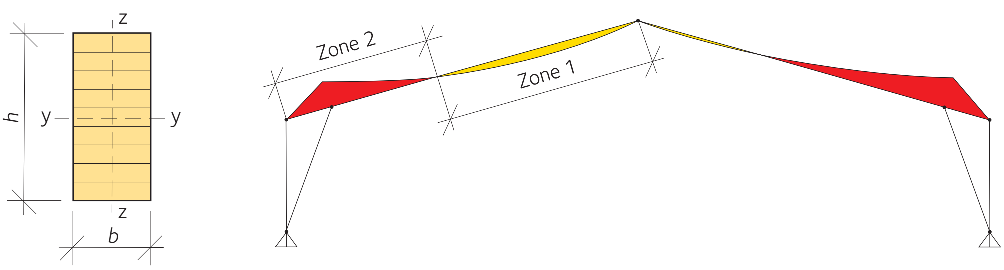
$$\tau_d = \frac{3 \times V_{Ed}}{2b \times h} = \frac{3 \times 299 \times 10^3}{2 \times 190 \times (880 + 190)} = 2.2 \text{ MPa}$$

Shear verification (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau_d}{f_{v,d} \times k_{cr}} = \frac{2.2}{2.24 \times 0.86} = 1.15 \quad > 1 \quad \textbf{NOT OK}$$

The condition is not met, increase dimensions to 190 × 1,040 or 215 × 900.

c) Stability check for combined bending and compression (load combination 2)



The rafter is laterally stiffened by means of a bracing system; braced points are 1.8 m apart. Two distinguished zones with different buckling length can be identified, namely:

- Zone 1 where the upper edge of rafter is in compression
- Zone 2 where the lower edge of rafter is in compression.

The cross-sectional depth is considered constant between two braced points.

Cross-sections 2, 3, 4 and 7 are checked. Corresponding values of axial force and bending moment are shown in the table below:

Section	Cross-section dimensions [mm]	Design axial force N_d [kN]	Design bending moment M_d [kNm]
2	190 × 357	-164	24
3	190 × 460	-175	68
4	190 × 571	-185	49
7	190 × 880	-216	340

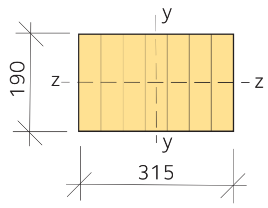
Check for axial buckling about z-axis and bending about y-axis (*EN 1995-1-1, equation 6.24*):

Section	Effective buckling length, $l_{o,z}$ [mm]	Critical bending stress, $\sigma_{cr,z}$ [MPa]	Relative slenderness ratio, λ_{rel}	Reduction factor for buckling, $k_{c,z}$	Work ratio $\frac{N_d}{A \cdot k_{cz} \cdot f_{cd}} + \frac{M_d}{W \cdot f_{md}}$
2	$l_{0z,2} = 1.8$	$\sigma_{cr,z,2} = 98.97$	$\lambda_{rel,z,2} = 0.5$	$k_{cz,2} = 0.97$	$R_2 = 0.46$
3	$l_{0z,3} = 1.8$	$\sigma_{cr,z,3} = 98.97$	$\lambda_{rel,z,3} = 0.5$	$k_{cz,3} = 0.97$	$R_3 = 0.66$
4	$l_{0z,4} = 1.8$	$\sigma_{cr,z,4} = 98.97$	$\lambda_{rel,z,4} = 0.5$	$k_{cz,4} = 0.97$	$R_4 = 0.36$

Check for lateral torsional buckling and axial buckling about z-axis (*EN 1995-1-1, equation 6.35*):

Section	Effective buckling length, $l_{o,z}$ [mm]	Critical bending stress, $\sigma_{cr,m}$ [MPa]	Relative slenderness ratio, λ_{rel}	Critical factor for lateral torsional buckling, k_{crit}	Work ratio $\left(\frac{M_d}{W \cdot k_{crit} \cdot f_{md}}\right)^2 + \frac{N_d}{A \cdot k_{cz} \cdot f_{cd}}$
2	$l_{0z,2} = 1.8$	$\sigma_{cr,m,2} = 427$	$\lambda_{rel,m2} = 0.27$	$k_{crit,2} = 1$	$R_2 = 0.25$
3	$l_{0z,3} = 1.8$	$\sigma_{cr,m,2} = 427$	$\lambda_{rel,m3} = 0.3$	$k_{crit,3} = 1$	$R_3 = 0.41$
4	$l_{0z,4} = 1.8$	$\sigma_{cr,m,2} = 427$	$\lambda_{rel,m4} = 0.34$	$k_{crit,4} = 1$	$R_4 = 0.17$
7	$l_{0z,7} = 5.5$	$\sigma_{cr,m,2} = 427$	$\lambda_{rel,m7} = 0.72$	$k_{crit,7} = 1$	$R_7 = 0.61$

11.7 Verification of the frame leg



$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{498.62 \times 10^3}{315 \times 190} = 8.33 \text{ MPa}$$

Stability about the z-axis (deflection in the y-direction):
Buckling length:

$$l_{0,z} = 3.76 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,z} = \frac{\pi^2 \times E_{0.05} \times I_z}{(b \times h) \times {l_{0,z}}^2} = \frac{\pi^2 \times 10800 \times \frac{315 \times 190^3}{12}}{315 \times 190 \times (3.76 \times 10^3)^2} = 22.68 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,z}}} = \sqrt{\frac{24.5}{22.68}} = 1.04$$

k factor:

$$k_z = \frac{1}{2} \times \left[1 + \beta_c \times (\lambda_{rel,z} - 0.3) + {\lambda_{rel,z}}^2\right] = \frac{1}{2} \times [1 + 0.1 \times (1.04 - 0.3) + 1.04^2] = 1.08$$

Reduction factor for buckling:

$$k_{c,z} = \frac{1}{k_z + \sqrt{{k_z}^2 - {\lambda_{rel,z}}^2}} = \frac{1}{1.08 + \sqrt{1.08^2 - 1.04^2}} = 0.74$$

Check for axial buckling about z-axis (*EN 1995-1-1, equation 6.24*):

$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \frac{8.33}{0.74 \times 15.68} = 0.72 \quad < 1 \quad \textbf{OK}$$

11.8 Verification of the tie rods of steel

Tie-rods with diameter d = 24 mm (A_{net} = 353 mm^2) are utilized. The verification is carried out following the rules for the design of bolts since the ends of the tie-rods are threaded:

$$T_{Ed} = 255.24 \text{ kN}$$

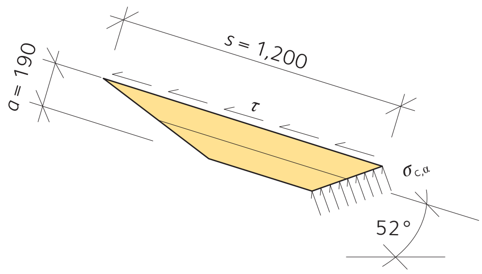
Tension capacity (*EN 1993-1-8, table 3.4*):

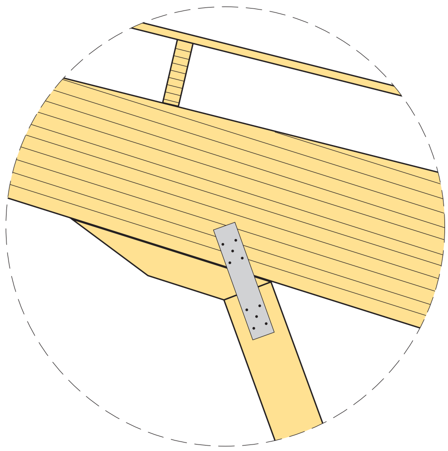
$$T_{Rd} = 2 \times \frac{A_{net} \times f_{uk} \times k_2}{\gamma_{M2}} \times 10^{-3} = 2 \times \frac{353 \times 510 \times 0.9}{1.2} \times 10^{-3} = 270.05 \text{ kN}$$

Verify the figure for tension (*EN 1993-1-1, equation 6.5*):

$$\frac{T_{Ed}}{T_{Rd}} = \frac{255.24}{270.05} = 0.95 \quad < 1 \quad \textbf{OK}$$

11.9 Verification of the cleat





a) Compression at an angle α to the grain

$$\sigma_{c,\alpha,d} = \frac{N_{Ed}}{b \times a} \cos(\alpha) = \frac{498.62 \times 10^3}{315 \times 190} \times \cos(52°) = 5.13 \text{ MPa}$$

Compression strength at an angle α to the grain:

$$f_{c,\alpha,d} = \frac{f_{c,0,d}}{\frac{f_{c,0,d}}{1.75 \times f_{c,90,k}} \times \sin(\alpha)^2 + \cos(\alpha)^2} = \frac{15.68}{\frac{15.68}{1.75 \times 2.5} \times \sin(52°)^2 + \cos(52°)^2} = 6.02 \text{ MPa}$$

$f_{c,90,d}$ can be replaced by $f_{c,90,k}$ due to the fact that g_k/s_k < 0.4, *see table 8.11, page 25, 8.12, page 25* and *8.13, page 26*.

Compression at an angle α to the grain verification (*EN 1995-1-1, equation 6.16*):

$$\frac{\sigma_{c,\alpha,d}}{f_{c,\alpha,d}} = \frac{5.13}{6.02} = 0.85 \quad < 1 \quad \textbf{OK}$$

b) Shear

$$\tau = \frac{N_{Ed} \times \cos(\alpha)}{b \times s} = \frac{498.62 \times 10^3 \times \cos(52°)}{315 \times 1200} = 0.81 \text{ MPa}$$

Verify the figure for shear stress, *see The Glulam Handbook Volume 2, section 10.5.3, page 149*:

$$\frac{\tau}{0.5 \times f_{v,d}} = \frac{0.81}{0.5 \times 2.24} = 0.73 \quad < 1 \quad \textbf{OK}$$

c) Geometrical verifications

See The Glulam Handbook Volume 2, section 10.5.3, page 149.
Cleat length:

$$s = 1200 \text{ mm} \quad > \quad 200 \text{ mm}$$
$$s = 1200 \text{ mm} \quad < \quad 8 \times a = 1520 \text{ mm} \quad \textbf{OK}$$

Ratio between length and depth:

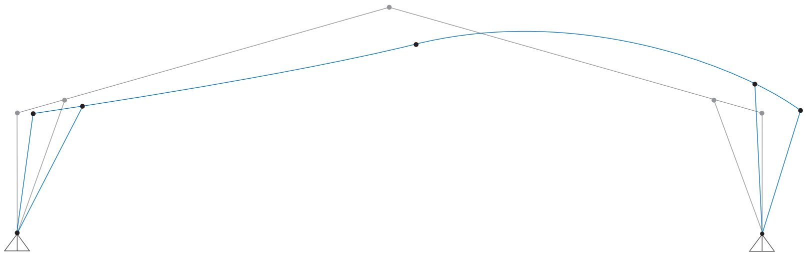
$$\frac{s}{a} = 6.32 > 6 \quad \textbf{OK}$$

11.10 Stability check for combined bending and compression in plane

The critical load combination for knee-braced frame is the number 2. The most stressed cross-section is section 7, 190 × 880 mm:

$$\sigma_{c,0,d} = \frac{216 \times 10^3}{b \times h} = \frac{216 \times 10^3}{190 \times 880} = 1.29 \text{ MPa} \qquad \sigma_{m,d} = \frac{340 \times 10^6}{b \times \frac{h^2}{6}} = \frac{340 \times 10^6}{190 \times \frac{880^2}{6}} = 13.87 \text{ MPa}$$

The corresponding buckling mode is shown below.



Stability about the y-axis (deflection in the z-direction)

The critical axial force is determined by means of a finite elements analysis.

Critical axial force:

$$N_{cr} = 2587.2 \text{ kN}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\frac{N_{cr}}{A}}} = \sqrt{\frac{24.5}{\frac{2587.2 \times 10^3}{167200}}} = 1.26$$

k factor:

$$k_y = 0.5 \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}{}^2\right] = 0.5 \times \left[1 + 0.1 \times \left(1.26 - 0.3\right) + 1.26^2\right] = 1.34$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y{}^2 - \lambda_{rel,y}{}^2}} = \frac{1}{1.34 + \sqrt{1.34^2 - 1.26^2}} = 0.56$$

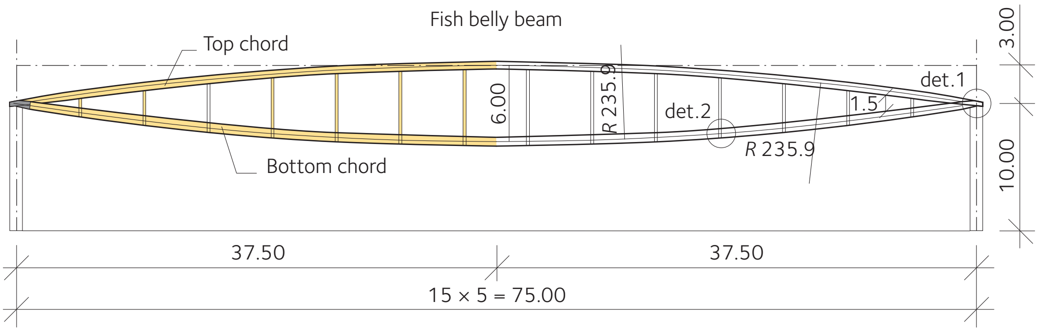
Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

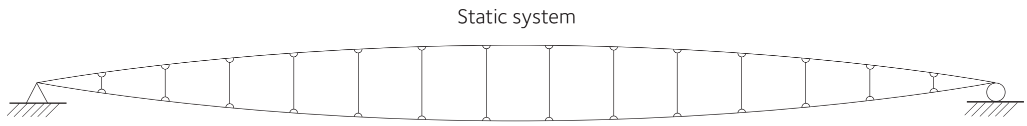
$$\frac{\sigma_{c,0,d}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} = \frac{1.29}{0.56 \times 15.68} + \frac{13.87}{19.2} = 0.87 \quad < 1 \quad \textbf{OK}$$

Example 12: Design of a fish belly beam

12.1 System, dimensions and design parameters

Design and verify the fish belly beam below.

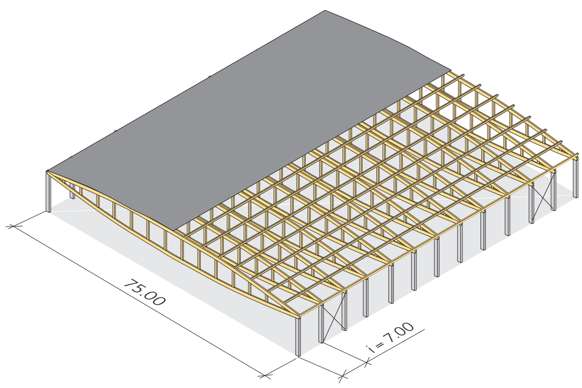




Three erection joints, both in the top chord and in the bottom chord, are needed for the transport of the beams. The joints are designed as moment stiff (completely rigid).

The cross-section dimensions are determined using the preliminary design method shown in *section 12.4, page 160*.

The truss is made of glulam, strength class	GL30c
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$

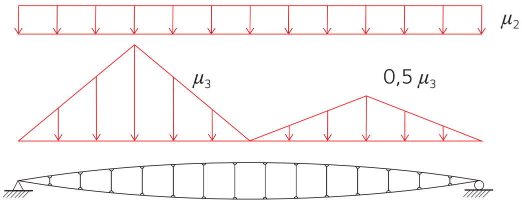


12.2 Loads

According to (*EN 1991-1-3, clause 6.3.8*), the following snow factors shall be considered.

$$\mu_2 = 0.8$$

$$\mu_3 = 0.2 + 10 \times \frac{h_{apex}}{l_{tot}} = 0.2 + 10 \times \frac{3}{75} = 0.6$$



The loads considered in the design are:

Structural

$$g_{k,1} = 2.8 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.7 \text{ kN/m}^2$ $\qquad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.7 \times 7 \times 1.1 = 5.4 \text{ kN/m}$

Snow load symmetric

$S_k = 1.5 \text{ kN/m}^2$ $\qquad s_{k,s} = S_k \times i \times \mu_2 \times 1.1 = 1.5 \times 7 \times 0.8 \times 1.1 = 9.2 \text{ kN/m}$

Snow load unsymmetric

$S_k = 1.5 \text{ kN/m}^2$ $\qquad s_{k,u} = S_k \times i \times \mu_3 \times 1.1 = 1.5 \times 7 \times 0.6 \times 1.1 = 6.9 \text{ kN/m}$

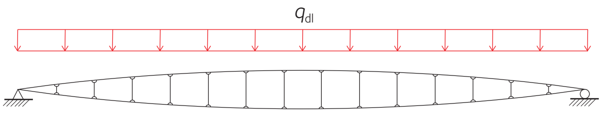
Factor 1.1 used in the equations above accounts for the continuity of purlins over fish belly beams. This example assumes that there is no snow guard.

12.3 Load combinations

Three different load combinations are considered (*EN 1990, clause 6.4.3*):

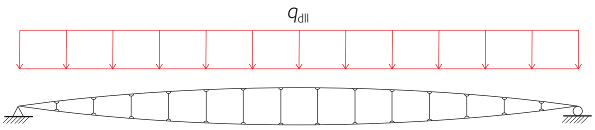
Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times \left(2.8 + 5.4\right) = 9.8 \text{ kN/m}$$



Combination 2 (self-load leading + symmetric snow load, medium term load, $k_{mod} = 0.8$):

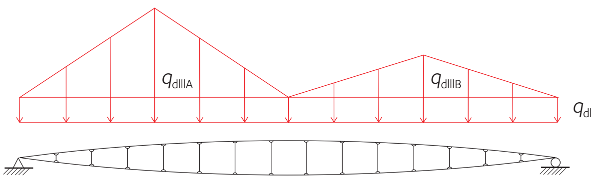
$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,s}\right] = 1 \times \left[1.2 \times \left(2.8 + 5.4\right) + 1.5 \times 9.2\right] = 23.7 \text{ kN/m}$$



Combination 3 (self-load leading + unsymmetric snow load, medium term load, $k_{mod} = 0.8$):

$$q_{dIIIA} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,u}\right] = 1 \times \left[1.2 \times \left(2.8 + 5.4\right) + 1.5 \times 6.9\right] = 20.2 \text{ kN/m}$$

$$q_{dIIIB} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times 0.5 \times s_{k,u}\right] = 1 \times \left[1.2 \times \left(2.8 + 5.4\right) + 1.5 \times 0.5 \times 6.9\right] = 15 \text{ kN/m}$$



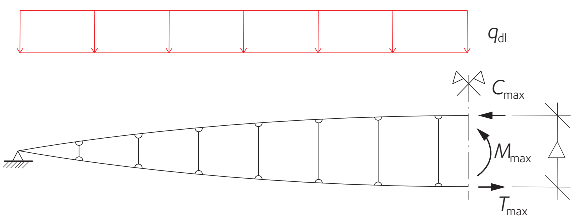
12.4 Preliminary design

The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, section 8.2, page 123*:

$$\Delta = \frac{l_{tot}}{12} = 6.25 \text{ m} \quad \rightarrow \quad \Delta = 6 \text{ m}$$

$$M_{max} = q_{dII} \times \frac{l_{tot}^{\,2}}{8} = 23.7 \times \frac{75^2}{8} = 16655.6$$

$$C = \frac{M_{max}}{\Delta} = \frac{16656}{6} = 2776 \text{ kN} \qquad T = C = 2776 \text{ kN}$$



Top chord

$$b_{\mathrm{min}} = \frac{l_{\mathrm{tot}}}{200} = 0.38 \text{ m} \quad \rightarrow \quad b = 430 \text{ mm}$$

$$h = \frac{C}{b \times f_{\mathrm{c,0,d}} \times k_{\mathrm{r}}} = \frac{2775.9 \times 10^3}{430 \times 15.7 \times 0.8} = 514.6 \text{ mm} \quad \rightarrow \quad h = 630 \text{ mm}$$

The width of the top chord is chosen as b = 430 mm (> 380 mm). The reasons for this are
- placing several slotted-in steel plates.
- increase lateral buckling resistance.

The factor k_{r} is to account for the strength reduction due to possible buckling.

Bottom chord

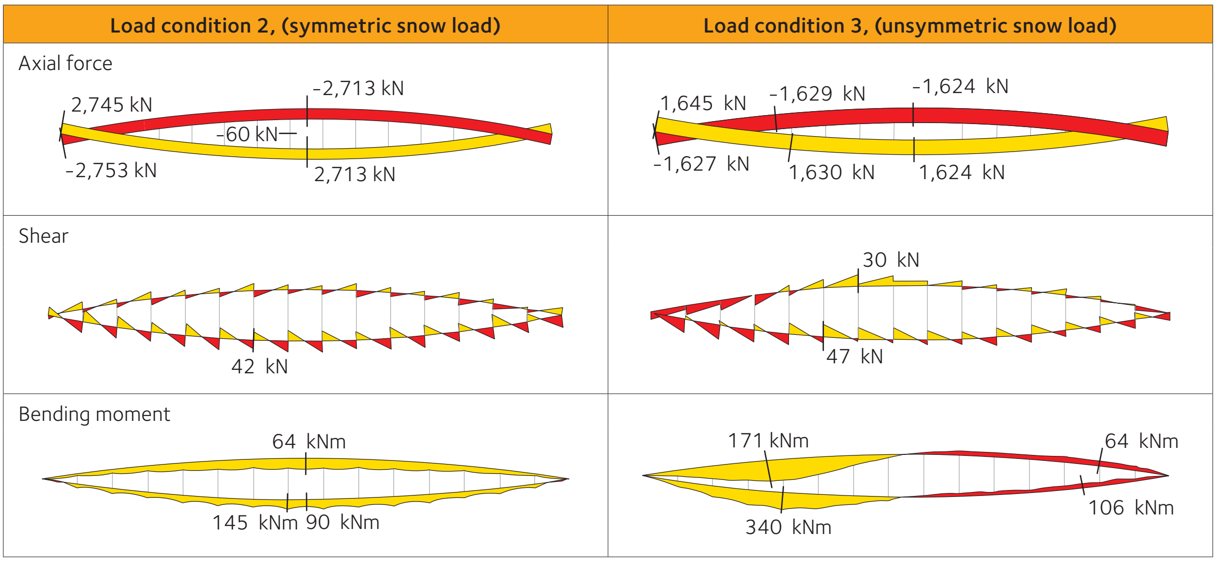
The bottom chord consists of two separate members side-by-side. Each member has a width of 190 mm.

$$b = 190 \times 2 = 380 \text{ mm}$$

$$h = \frac{T}{k_{\mathrm{net}} \times b \times f_{\mathrm{t,0,d}}} = \frac{2775.9 \times 10^3}{0.8 \times 380 \times 12.5} = 731.7 \text{ mm} \quad \rightarrow \quad h = 810 \text{ mm}$$

The factor k_{net} accounts for the reduction of the cross-section section due to the presence of fasteners.

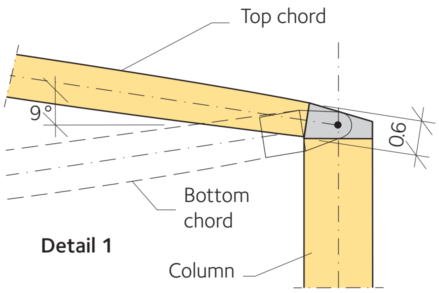
12.5. Internal forces and moments

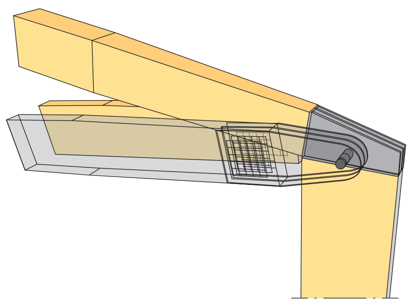


12.6 Verification of top chord

a) Compression parallel to the grain at the support

Notice that at the supports the top chord is double tapered in order to reduce the amount of steel of the connection; the cross-section at the support is 300 × 600 mm. A possible connection is shown below.





The maximum stress occurs for load condition 2:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{2753 \times 10^3}{300 \times 600} = 15.29 \text{ MPa}$$

Compression parallel to the grain verification (*EN 1995-1-1, equation 6.2*):

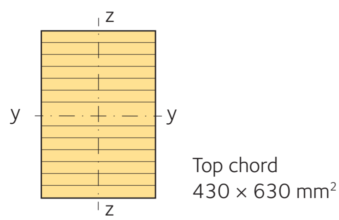
$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} = \frac{15.29}{15.68} = 0.98 \quad < 1 \quad \textbf{OK}$$

b) Stability check for combined bending and compression (out-of-plane buckling)

The beam is laterally stiffened by means of a bracing system; braced points are 5 m apart.
The maximum stress occurs for load condition 2:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{2713 \times 10^3}{430 \times 630} = 10.01 \text{ kN}$$

$$\sigma_{m,y,d} = \frac{6M_{Ed}}{b \times h^2} = \frac{6 \times 64 \times 10^6}{430 \times 630^2} = 2.25 \text{ MPa}$$



Stability about the z-axis (deflection along y-direction)

Buckling length:

$$l_{0,z} = 5 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,z} = \frac{\pi^2 \times E_{0.05} \times I_z}{(b \times h) \times l_{0,z}{}^2} = \frac{\pi^2 \times 10800 \times \frac{430^3 \times 630}{12}}{430 \times 630 \times (5 \times 10^3)^2} = 65.7 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,z}}} = \sqrt{\frac{24.5}{65.7}} = 0.61$$

k factor:

$$k_z = \frac{1}{2} \times \left[1 + \beta_c \times (\lambda_{rel,z} - 0.3) + \lambda_{rel,z}{}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times (0.61 - 0.3) + 0.61^2\right] = 0.7$$

Reduction factor for buckling:

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z{}^2 - \lambda_{rel,z}{}^2}} = \frac{1}{0.7 + \sqrt{0.7^2 - 0.61^2}} = 0.95$$

Lateral torsional buckling

Effective buckling length:

$$l_{0,z} = 5 \text{ m}$$

Critical bending stress:

$$\sigma_{cr,m} = \frac{0.78 \times b^2}{h \times l_{0,z}} \cdot E_{0.05} = \frac{0.78 \times 430^2}{630 \times 5 \times 10^3} \times 10800 = 494.48 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{cr,m}}} = \sqrt{\frac{30}{494.48}} = 0.25$$

Critical factor for lateral torsional buckling:

$$k_{crit} = 1$$

Check for axial buckling about z-axis and bending about y-axis (*EN 1995-1-1, equation 6.24*):

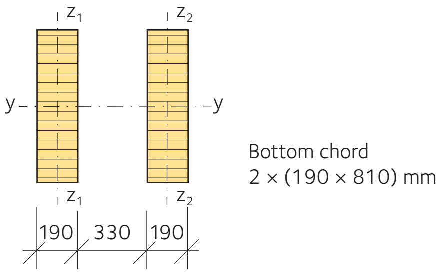
$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} + 0.7 \times \frac{\sigma_{m,y,d}}{f_{m,d}} = \frac{10.01}{0.95 \times 15.68} + 0.7 \times \frac{2.25}{19.2} = 0.75 \quad < 1 \quad \textbf{OK}$$

Check for lateral torsional buckling and axial buckling about z-axis (*EN 1995-1-1, equation 6.35*):

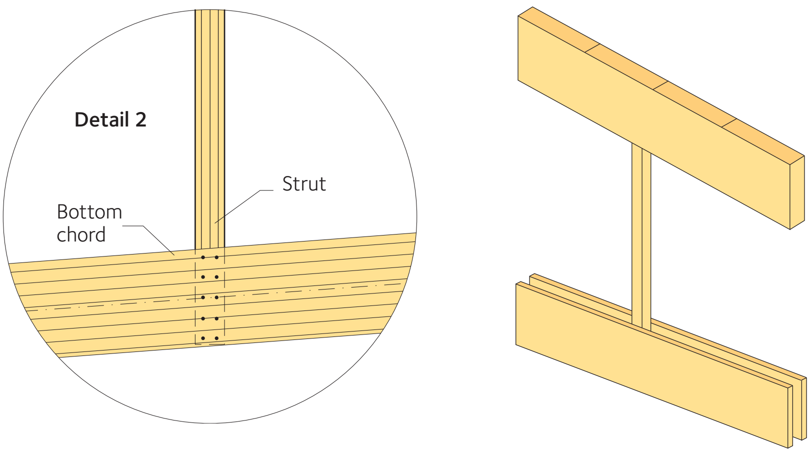
$$\left(\frac{\sigma_{m,y,d}}{k_{crit} \times f_{m,d}}\right)^2 + \frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \left(\frac{2.25}{19.2}\right)^2 + \frac{10.01}{0.95 \times 15.68} = 0.68 \quad < 1 \quad \textbf{OK}$$

12.7 Verification of bottom chord

The bottom chord consists of two separate members.



To connect the struts to the bottom chord, dowels d = 12, structural steel grade S355 are adopted.



a) Combined bending and tension

The maximum stress occurs for load condition 2:

$$\sigma_{m,y,d} = \frac{6 \times \frac{M_{Ed}}{2}}{b \times h^2} = \frac{6 \times \frac{145 \times 10^6}{2}}{190 \times 810^2} = 3.49 \text{ MPa}$$

$$\sigma_{t,0,d} = \frac{\frac{N_{Ed}}{2}}{b \times \left(h - 5 \times d_{dowel}\right)} = \frac{\frac{2713 \times 10^3}{2}}{190 \times \left(810 - 5 \times 12\right)} = 9.5 \text{ MPa}$$

Combined bending and tension verification (*EN 1995-1-1, equation 6.17*):

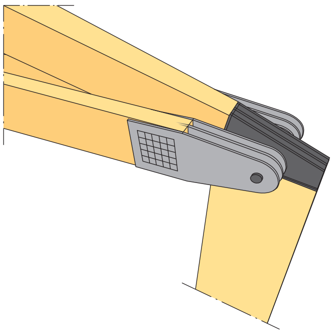
$$\frac{\sigma_{t,0,d}}{f_{t,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,d}} = \frac{9.52}{12.48} + \frac{3.49}{19.2} = 0.94 \quad < 1 \quad \textbf{OK}$$

b) Tension at the support

The maximum stress occurs for load condition 2.

Notice that at the supports the bottom chord is tapered; the cross-section at the support is 2 × 190 × 750 mm. Dowels with diameter d = 20 mm are adopted for the connection between the bottom chord and steel plates. The steel plate thickness is t_s = 10 mm.

$$\sigma_{t,0,d} = \frac{\frac{N_{Ed}}{2}}{(b - t_s) \times (h - 6 \times d_{dowel})} = \frac{\frac{2745 \times 10^3}{2}}{(190 - 10) \times (750 - 6 \times 20)} = 12.1 \text{ MPa}$$



Tension verification (*EN 1995-1-1, equation 6.1*):

$$\frac{\sigma_{t,0,d}}{f_{t,0,d}} = \frac{12.1}{12.48} = 0.97 \quad < 1 \quad \textbf{OK}$$

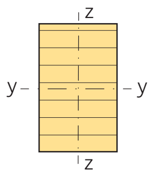
Block-shear failure and load caring capacity of the connection shall be checked in the final design of the structure. However, these verifications are not shown in this example.

12.8 Verification of the struts

a) Axial buckling

The maximum stress occurs for the load condition 2.
The buckling lengths are the same about both axes, the worst case therefore is about the z-axis:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{60 \times 10^3}{215 \times 330} = 0.8 \text{ MPa}$$



Stability about the z-axis (deflection in the y-direction)

Buckling length:

$$l_{0,z} = 6 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,z} = \frac{\pi^2 \times E_{0.05} \times I_z}{(b \times h) \times l_{0,z}^2} = \frac{\pi^2 \times 10800 \times \frac{215^3 \times 330}{12}}{215 \times 330 \times (6 \times 10^3)^2} = 11.41 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,z}}} = \sqrt{\frac{24.5}{11.41}} = 1.47$$

k factor:

$$k_z = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,z} - 0.3\right) + \lambda_{rel,z}{}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times (1.47 - 0.3) + 1.47^2\right] = 1.63$$

Reduction factor for buckling:

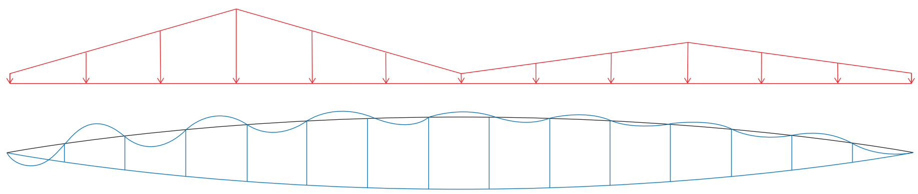
$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z{}^2 - \lambda_{rel,z}{}^2}} = \frac{1}{1.63 + \sqrt{1.63^2 - 1.47^2}} = 0.43$$

Check for axial buckling about z-axis and bending about y-axis (*EN 1995-1-1, equation 6.24*):

$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \frac{0.85}{0.43 \times 15.68} = 0.13 \quad < 1 \quad \textbf{OK}$$

12.9 In-plane buckling of the structure

The critical load combination for the fish belly beam is the number 2.
The corresponding buckling mode is shown below.



Thus, only buckling between nodes of the structure is likely to occur. Global buckling will occur only at higher buckling modes, therefore it is not relevant. In this example, the axial stress and the bending stress are computed at the quarter point of the beam, in the top chord, i.e. where the bending moment is maximum:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{1629 \times 10^3}{430 \times 630} = 6.01 \text{ kN} \qquad \sigma_{m,y,d} = \frac{6M_{Ed}}{b \times h^2} = \frac{6 \times 171 \times 10^6}{430 \times 630^2} = 6.01 \text{ MPa}$$

The critical axial force is determined by means of a finite elements analysis.

Stability about the y-axis (deflection in the z-direction)

Critical axial force:

$$N_{cr} = 36143 \text{ kN}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\frac{N_{cr}}{b \times h}}} = \sqrt{\frac{24.5}{\frac{36143 \times 10^3}{430 \times 630}}} = 0.43$$

k factor:

$$k_y = 0.5 \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}^2\right] = 0.5 \times \left[1 + 0.1 \times (0.43 - 0.3) + 0.43^2\right] = 0.6$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} = \frac{1}{0.6 + \sqrt{0.6^2 - 0.43^2}} = 0.98$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

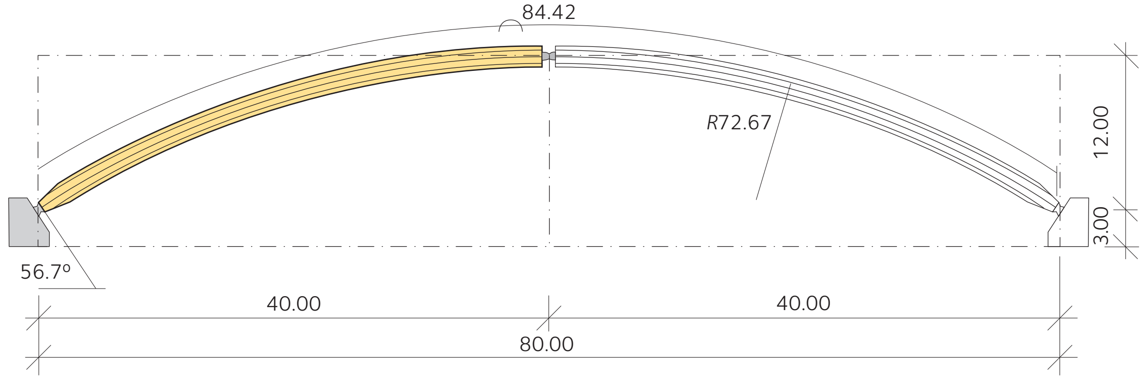
$$\frac{\sigma_{c,0,d}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,y,d}}{f_{m,d}} = \frac{6.01}{0.98 \times 15.68} + \frac{6.01}{19.2} = 0.7 \quad < 1 \quad \textbf{OK}$$

Example 13: Design of a three-hinged arch

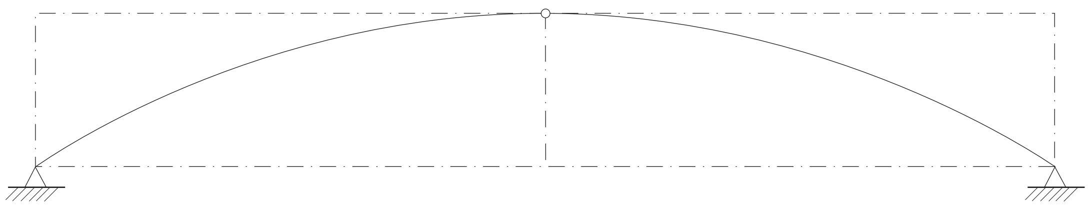
13.1 System, dimensions and design parameters

Design and verify the arch below.

Three-pin arch

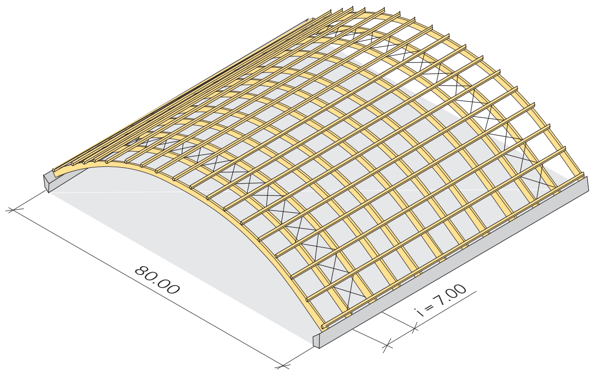


Static system



The cross-section dimensions are determined using the preliminary design method shown in *section 13.4, page 170*.

The arch is made of glulam, strength class	GL30c
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$

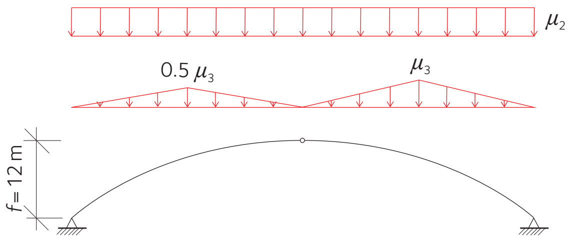


13.2 Loads

Use the following form factors for snow load (*EN 1991-1-3, clause 6.3.8*):

$$\mu_2 = 0.8$$

$$\mu_3 = 0.2 + 10 \times \frac{f}{l_{tot}} = 0.2 + 10 \times \frac{12}{80} = 1.7$$



The loads considered in the design of the arch are:

Structural

$$g_{k,1} = 3 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.75 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.75 \times 7 \times 1.1 = 5.78 \text{ kN/m}$

Snow load symmetric

$S_k = 2.0 \text{ kN/m}^2$ $\quad s_{k,2} = S_k \times \mu_2 \times i \times 1.1 = 2.0 \times 0.8 \times 7 \times 1.1 = 12.32 \text{ kN/m}$

Snow load unsymmetric

$$s_{k,3} = S_k \times \mu_3 \times i \times 1.1 = 2.0 \times 1.7 \times 7 \times 1.1 = 26.18 \text{ kN/m}$$

The self-weight considered in the equations above is the load projection onto the horizontal plane.
Factor 1.1 used in the equations above accounts for the continuity of purlins over arches.
This example assumes that there is no snow guard.

13.3 Load combinations

Three different load combinations are considered (*EN 1990, clause 6.4.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1.2 \times \left(3 + 5.78\right) = 10.53 \text{ kN/m}$$

Combination 2 (self-load leading + symmetric snow load, medium term load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,2}\right] = \left[1.2 \times \left(3 + 5.78\right) + 1.5 \times 12.32\right] = 29.01 \text{ kN/m}$$

Combination 3 (self-load leading + unsymmetric snow load, medium term load, $k_{mod} = 0.8$):

$$q_{dIIIA} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,3}\right] = \left[1.2 \times \left(3 + 5.78\right) + 1.5 \times 26.18\right] = 49.8 \text{ kN/m}$$

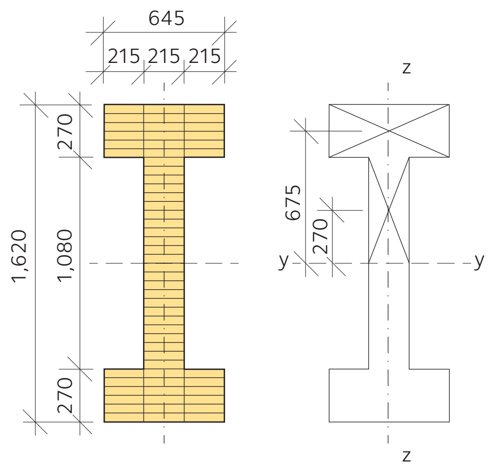
$$q_{dIIIB} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times 0.5 \times s_{k,3}\right] = \left[1.2 \times \left(3 + 5.78\right) + 1.5 \times 0.5 \times 26.18\right] = 30.16 \text{ kN/m}$$

13.4 Preliminary design

The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, section 11.2, page 155*:

$$f = 0.15 l_{tot} = 0.15 \times 80 = 12 \text{ m}$$

$$h_{min} = \frac{l_{tot}}{50} = \frac{80 \times 10^3}{50} = 1600 \text{ mm} \quad \rightarrow \quad h = 1620 \text{ mm}$$



The width of the cross-section is chosen large in order to increase the laterally buckling resistance, especially during erection:

$$b_{min} = \frac{h_{min}}{3} = \frac{1600}{3} = 533 \text{ mm} \quad \rightarrow \quad b = 645 \text{ mm}$$

I-shaped cross-section is adopted in order to reduce the amount of glulam and optimize the mechanical properties.

Area:

$$A = 215 \times 1620 + 4 \times 215 \times 270 = 580500 \text{ mm}^2$$

First moment of area about y-axis:

$$S_y = (645 \times 270) \times 675 + \left(215 \times \frac{1080}{2}\right) \times 270 = 1.49 \times 10^8 \text{ mm}^3$$

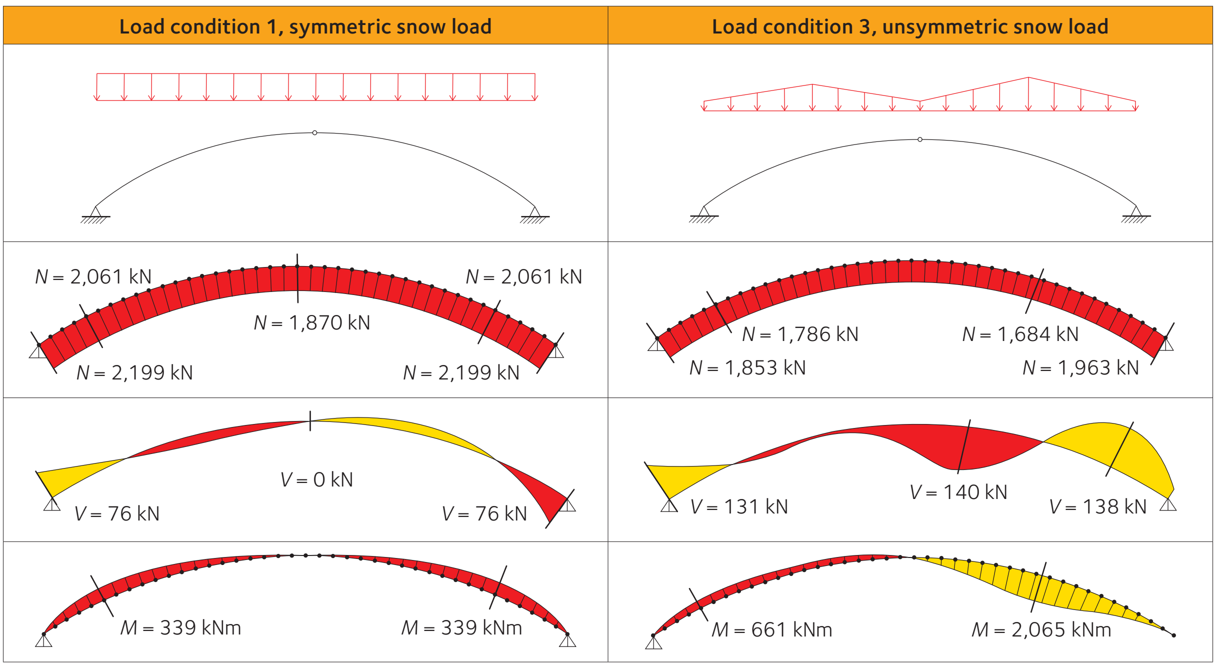
Second moment of area about y-axis:

$$I_y = \frac{645 \times (1620)^3}{12} - 2 \times \frac{215 \times 1080^3}{12} = 1.83 \times 10^{11} \text{ mm}^4$$

Second moment of area about z-axis:

$$I_z = \frac{1620 \times 645^3}{12} - 2 \times \left(\frac{1080 \times 215^3}{12} + 1080 \times 215 \times 215^2\right) = 1.3 \times 10^{10} \text{ mm}^4$$

13.5 Internal forces and moments



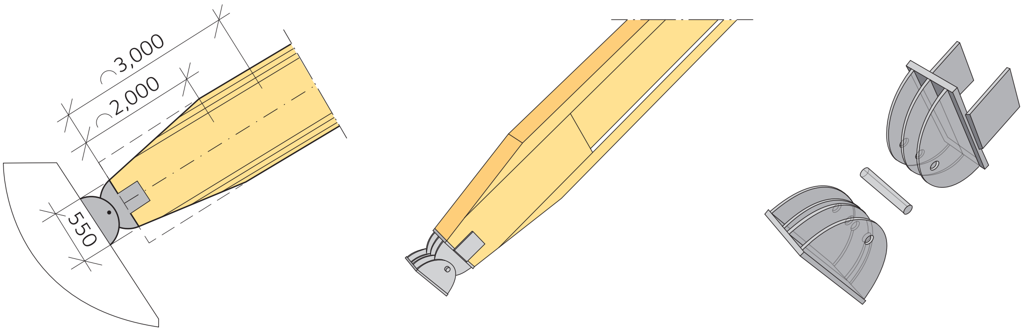
13.6. Calculations in ultimate limit state, arch

a) Compression parallel to the grain

The maximum stress occurs for the load condition 1 and the most stressed cross-section is at the supports.

Notice that at the supports the cross-section of the arch is rectangular rather than I-shaped. In order to reduce the amount of steel of the connection, the depth of the arch is tapered.

A possible connection at the supports is shown below.



Compression parallel to the grain (*EN 1995-1-1, equation 6.2*):

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{2199 \times 10^3}{645 \times 550} = 6.2 \text{ MPa}$$

$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} = \frac{6.2}{15.68} = 0.4 \quad < 1 \quad \textbf{OK}$$

b) Shear verification

The unsymmetric load condition generates the highest shear stress:

$$\tau_d = \frac{V_{Ed} \times S_y}{b_I \times I_y} = \frac{140 \times 10^3 \times 1.49 \times 10^8}{215 \times 1.83 \times 10^{11}} = 0.53 \text{ MPa}$$

Verify the figure for shear stress (*EN 1995-1-1, equation 6.13*):

$$\frac{\tau_d}{f_{v,d} \times k_{cr}} = \frac{0.53}{2.24 \times 0.86} = 0.28 \quad < 1 \quad \textbf{OK}$$

c) Tension perpendicular to the grain

The curvature of the beam gives arise to stresses perpendicular to the grain.
Such stresses are maximum where the bending moment is maximum:

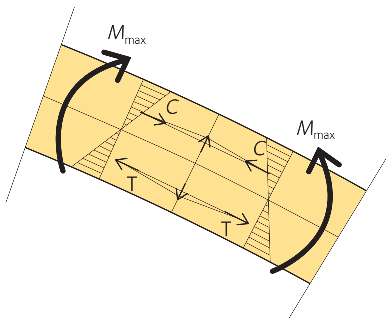
$$\sigma_{t,90,d} = \frac{M_{Ed}}{I_y} \times \frac{h}{2} \times \left(\frac{h}{4 \times r_{curve}}\right) = \frac{2065 \times 10^6}{1.83 \times 10^{11}} \times \frac{1620}{2} \times \frac{1620}{4 \times 72.67 \times 10^3} = 0.05 \text{ MPa}$$

Reference volume:

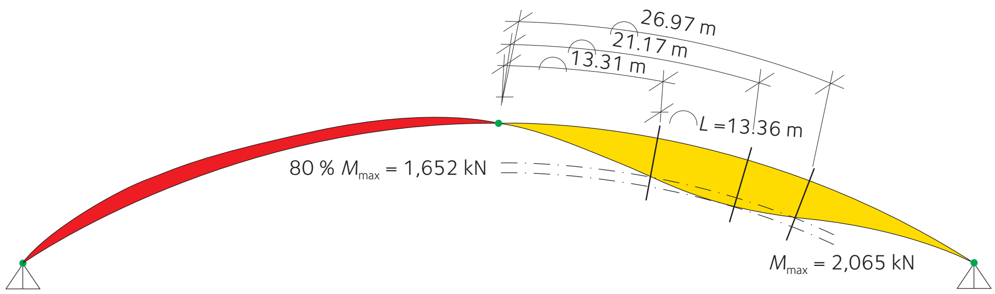
$$V_0 = 0.01 \text{ m}^3$$

Stressed volume:

$$V_{arch} = l_{curve} \times A = 13.36 \times 0.58 = 7.76 \text{ m}^3$$



In accordance with the Australian building code AS 1720.1—1997, only the volume of timber subject to a bending moment that is 80 – 100 percent of the maximum bending moment is taken into account for the determination of k_{vol}, *see The Glulam Handbook Volume 2, Chapter 11, page 153*.



Tension perpendicular to the grain verification (*EN 1995-1-1, equation 6.50*):

$$\frac{\sigma_{t,90,d}}{k_{dis} \times \left(\frac{V_0}{V_{arch}}\right)^{0.2} \times f_{t,90,d}} = \frac{0.05}{1.4 \times \left(\frac{0.01}{7.76}\right)^{0.2} \times 0.32} = 0.43 \quad < 1 \quad \textbf{OK}$$

d) Stability check for combined bending and compression out-of-plane

The arch is laterally stiffened by means of a bracing system; braced points are 6 m apart. Combination 3 is dimensioned as follows:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{A} = \frac{1684 \times 10^3}{580500} = 2.9 \text{ MPa} \qquad \sigma_{m,y,d} = \frac{M_{Ed}}{I_y} \times \frac{h}{2} = \frac{2065 \times 10^6}{1.83 \times 10^{11}} \times \frac{1620}{2} = 9.12 \text{ MPa}$$

Stability about the z-axis (deflection in the y-direction).

Buckling length:

$$l_{0,z} = 6 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,z} = \frac{\pi^2 \times E_{0.05} \times I_z}{A \times {l_{0,z}}^2} = \frac{\pi^2 \times 10800 \times 1.3 \times 10^{10}}{580500 \times \left(6 \times 10^3\right)^2} = 66.15 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,z}}} = \sqrt{\frac{24.5}{66.15}} = 0.61$$

k factor:

$$k_z = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,z} - 0.3\right) + {\lambda_{rel,z}}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times \left(0.61 - 0.3\right) + 0.61^2\right] = 0.7$$

Reduction factor for buckling:

$$k_{c,z} = \frac{1}{k_z + \sqrt{{k_z}^2 - {\lambda_{rel,z}}^2}} = \frac{1}{0.7 + \sqrt{0.7^2 - 0.61^2}} = 0.95$$

Check for lateral torsional buckling

Effective buckling length:

$$l_{0,z} = 6 \text{ m}$$

Critical bending stress:

$$\sigma_{cr,m} = \frac{\pi \times \sqrt{E_{0.05} \times I_z \times G_{05} \times I_{tor}}}{l_{0,z} \times I_y \times \frac{2}{h}} = \frac{\pi \times \sqrt{10800 \times 1.3 \times 10^{10} \times 540 \times 1.2 \times 10^{10}}}{6 \times 10^3 \times 1.83 \times 10^{11} \times \frac{2}{1620}} = 69.8 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,m} = \sqrt{\frac{f_{m,k}}{\sigma_{cr,m}}} = 0.66$$

Critical factor for lateral torsional buckling:

$$k_{crit} = 1$$

Bending stress must be increased by factor k_l, in order to account for the effect of the curvature (*EN 1995-1-1, equation 6.43*):

$$k_l = 1 + 0.35 \times \left(\frac{h}{r_{curve}}\right) + 0.6 \times \left(\frac{h}{r_{curve}}\right)^2 = 1 + 0.35 \times \frac{1620}{72.67 \times 10^3} + 0.6 \times \left(\frac{1620}{72.67 \times 10^3}\right)^2 = 1.01$$

Check for lateral torsional buckling and axial buckling about z-axis (*EN 1995-1-1, equation 6.35*):

$$\left(\frac{\sigma_{m,y,d} \times k_l}{k_{crit} \times f_{m,d}}\right)^2 + \frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \left(\frac{9.12 \times 1.01}{19.2}\right)^2 + \frac{2.9}{0.95 \times 15.68} = 0.42 \quad < 1 \quad \textbf{OK}$$

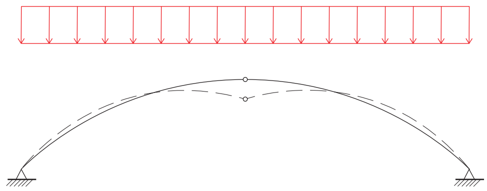
Check for axial buckling about z-axis and bending about y-axis (*EN 1995-1-1, equation 6.24*):

$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} + 0.7 \times \frac{\sigma_{m,y,d} \times k_l}{f_{m,d}} = \frac{2.9}{0.95 \times 15.68} + 0.7 \times \frac{9.12 \times 1.01}{19.2} = 0.53 \quad < 1 \quad \textbf{OK}$$

e) Stability check for combined bending and compression in plane

Two verifications are performed for each load condition.

Load condition 2, symmetric snow load:



$$\sigma_{c,0,d} = \frac{N_{Ed}}{A} = \frac{2061 \times 10^3}{580500} = 3.55 \text{ MPa}$$

$$\sigma_{m,y,d} = \frac{M_{Ed}}{I_y} \times \frac{h}{2} = \frac{339 \times 10^6}{1.83 \times 10^{11}} \times \frac{1620}{2} = 1.5 \text{ MPa}$$

Verification by hand calculation method
Stability about the y-axis (deflection in the z-direction)

Buckling length, *see The Glulam Handbook Volume 2, equation 11.23, page 163*:

$$l_{0,y} = 1.25 \times \frac{l_{curve}}{2} = 1.25 \times \frac{84.42}{2} = 52.76 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,y} = \frac{\pi^2 \times E_{0.05} \times I_y}{A \times l_{0,y}^2} = \frac{\pi^2 \times 10800 \times 1.83 \times 10^{11}}{580500 \times (52.76 \times 10^3)^2} = 12.1 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,y}}} = \sqrt{\frac{24.5}{12.1}} = 1.42$$

k factor:

$$k_y = 0.5 \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}^2\right] = 0.5 \times \left[1 + 0.1 \times (1.42 - 0.3) + 1.42^2\right] = 1.57$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}} = \frac{1}{1.57 + \sqrt{1.57^2 - 1.42^2}} = 0.45$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

$$\frac{\sigma_{c,0,d}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,y,d} \times k_l}{f_{m,d}} = \frac{3.55}{0.45 \times 15.68} + \frac{1.5 \times 1.01}{19.2} = 0.58 \quad < 1 \quad \textbf{OK}$$

Check of the results by finite elements analysis
A numerical analysis is performed in order to check the validity of the above verification by hand calculation.

Critical axial force:

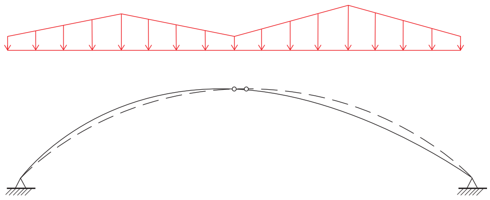
$$N_{cr} = 8743 \text{ kN}$$

Approximate buckling length:

$$l_{cr,y} = \sqrt{\frac{\pi^2 \times E_{0.05} \times I_y}{N_{cr}}} = \sqrt{\frac{\pi^2 \times E_{0.05} \times 1.83 \times 10^{11}}{8743 \times 10^3}} = 47283.26 \text{ mm}$$

The buckling length obtained by means of finite elements analysis is smaller than that obtained by hand calculation method. Therefore, the stability check is performed only on the basis of hand calculation method – which is on the safe side.

Load condition 3, unsymmetric snow load:



$$\sigma_{c,0,d} = \frac{N_{Ed}}{A} = \frac{1684 \times 10^3}{580500} = 2.9 \text{ MPa}$$

$$\sigma_{m,y,d} = \frac{M_{Ed}}{I_y} \times \frac{h}{2} = \frac{2065 \times 10^6}{1.83 \times 10^{11}} \times \frac{1620}{2} = 9.12 \text{ MPa}$$

Verification by hand calculation method

Stability about the y-axis (deflection in the z-direction)

Buckling length, *see The Glulam Handbook Volume 2, equation 11.23, page 163*:

$$l_{0,y} = 1.25 \times \frac{l_{curve}}{2} = 1.25 \times \frac{84.42}{2} = 52.76 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,y} = \frac{\pi^2 \times E_{0.05} \times I_y}{A \times l_{0,y}{}^2} = \frac{\pi^2 \times 10800 \times 1.83 \times 10^{11}}{580500 \times \left(52.76 \times 10^3\right)^2} = 12.1 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,y}}} = \sqrt{\frac{24.5}{12.1}} = 1.42$$

k factor:

$$k_y = 0.5 \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}{}^2\right] = 0.5 \times \left[1 + 0.1 \times (1.42 - 0.3) + 1.42^2\right] = 1.57$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y{}^2 - \lambda_{rel,y}{}^2}} = \frac{1}{1.57 + \sqrt{1.57^2 - 1.42^2}} = 0.45$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

$$\frac{\sigma_{c,0,d}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,y,d} \times k_l}{f_{m,d}} = \frac{2.9}{0.45 \times 15.68} + \frac{9.12 \times 1.01}{19.2} = 0.89 \quad < 1 \quad \textbf{OK}$$

Check of the results by finite elements analysis

A numerical analysis is performed in order to check the validity of the above verification by hand calculation.

Critical axial force:

$$N_{cr} = 8433.02$$

Approximate buckling length:

$$l_{cr,y} = \sqrt{\frac{\pi^2 \times E_{0.05} \times I_y}{N_{cr}}} = \sqrt{\frac{\pi^2 \times 10800 \times 1.83 \times 10^{11}}{8433.02 \times 10^3}} = 48144.44 \text{ mm}$$

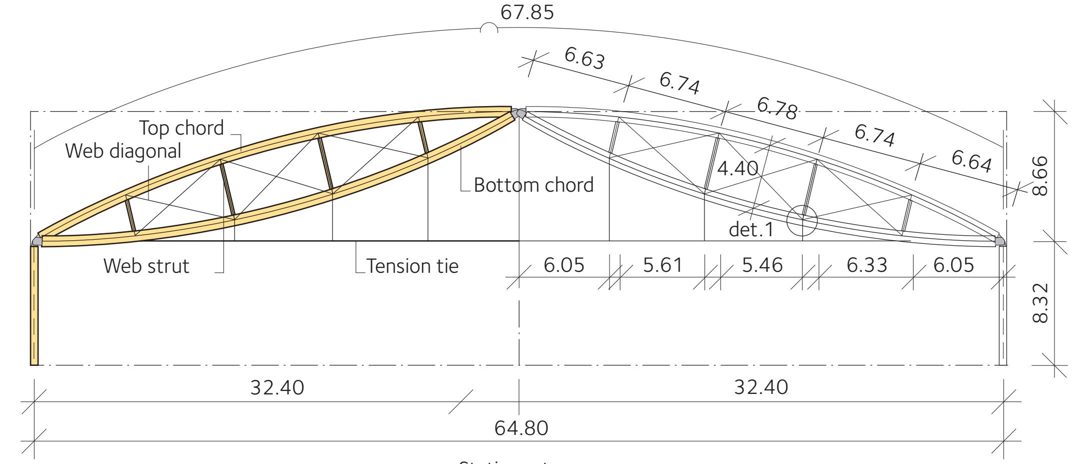
The buckling length obtained by means of finite elements analysis is smaller than that obtained by hand calculation method. Therefore, the stability check is performed only on the basis of hand calculation method – which is on the safe side.

Example 14: Design of a trussed arch

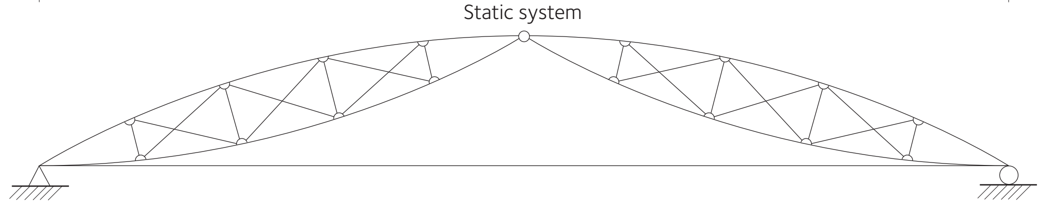
14.1 System, dimensions and design parameters

Design and verify the trussed arch below.

Trussed arch

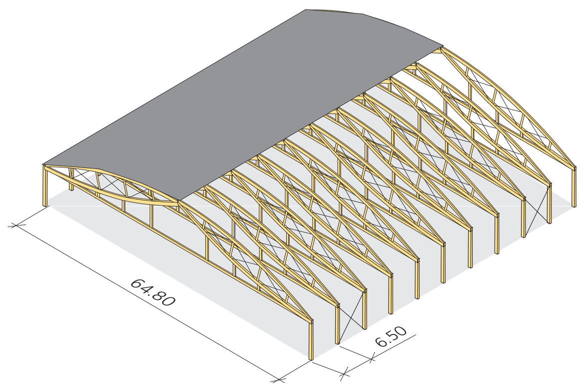


Static system



The cross-section dimensions are determined using the preliminary design method shown in *section 14.4, page 178*.

The truss is made of glulam, strength class	GL30c
Steel diagonals, structural steel grade	S355
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$
Partial factor for steel	$\gamma_{M2} = 1.2$

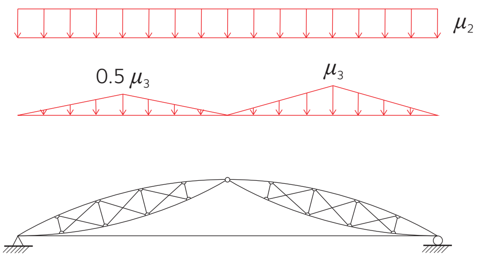


14.2 Loads

According to (*EN 1991-1-3, clause 6.3.8*), the following form factors for snow load shall be considered.

$$\mu_2 = 0.8$$

$$\mu_3 = 0.2 + 10 \times \frac{h_{\text{ridge}}}{l_{\text{tot}}} = 0.2 + 10 \times \frac{8.66}{64.81} = 1.54$$



The loads considered in the design are:

Structural

$$g_{k,1} = 1.85 \text{ kN/m}$$

Non-structural

$$G_{k,2} = 0.7 \text{ kN/m}^2 \qquad g_{k,2} = G_{k,2} \times i \times 1.1 = 0.7 \times 6.5 \times 1.1 = 5 \text{ kN/m}$$

Snow load symmetric

$$S_k = 2.0 \text{ kN/m}^2 \qquad s_{k,s} = S_k \times i \times \mu_2 \times 1.1 = 2.0 \times 6.5 \times 0.8 \times 1.1 = 11.44 \text{ kN/m}$$

Snow load unsymmetric

$$s_{k,u} = S_k \times i \times \mu_3 \times 1.1 = 2 \times 6.5 \times 1.54 \times 1.1 = 21.97 \text{ kN/m}$$

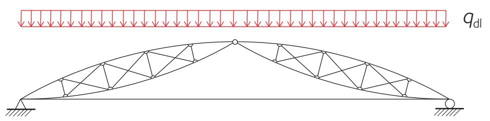
Factor 1.1 used in the equations above accounts for the continuity of purlins over the trusses.
This example assumes that there is no snow guard.

14.3 Load combinations

Three different load combinations are considered (*EN 1990, clause 6.4.3*):

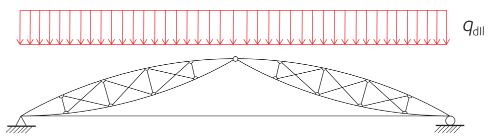
Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1 \times 1.2 \times \left(1.85 + 5\right) = 8.23 \text{ kN/m}$$



Combination 2 (self-load leading + symmetric snow load, medium term load, $k_{mod} = 0.8$):

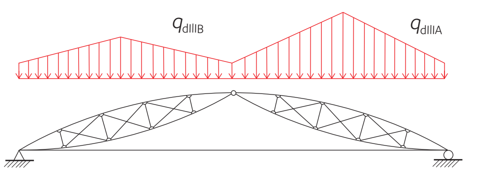
$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,s}\right] = 1 \times \left[1.2 \times \left(1.85 + 5\right) + 1.5 \times 11.44\right] = 25.39 \text{ kN/m}$$



Combination 3 (self-load leading + unsymmetric snow load, medium term load, $k_{mod} = 0.8$):

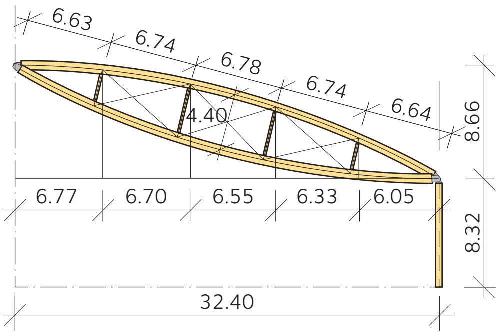
$$q_{dIIIA} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_{k,u}\right] = 1 \times \left[1.2 \times \left(1.85 + 5\right) + 1.5 \times 21.97\right] = 41.18 \text{ kN/m}$$

$$q_{dIIIB} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times 0.5 \times s_{k,u}\right] = 1 \times \left[1.2 \times \left(1.85 + 5\right) + 1.5 \times 0.5 \times 21.97\right] = 24.7 \text{ kN/m}$$



14.4 Preliminary design

The preliminary design is based on the recommendations given in *The Glulam Handbook Volume 2, section 8.2, page 123*.



Top chord:

$$h = \frac{\Delta}{7} = \frac{4400}{7} = 628.57 \text{ mm} \quad \rightarrow \quad h = 630 \text{ mm}$$

$$b = \frac{h}{3} = \frac{630}{3} = 210 \text{ mm} \quad \rightarrow \quad b = 215 \text{ mm}$$

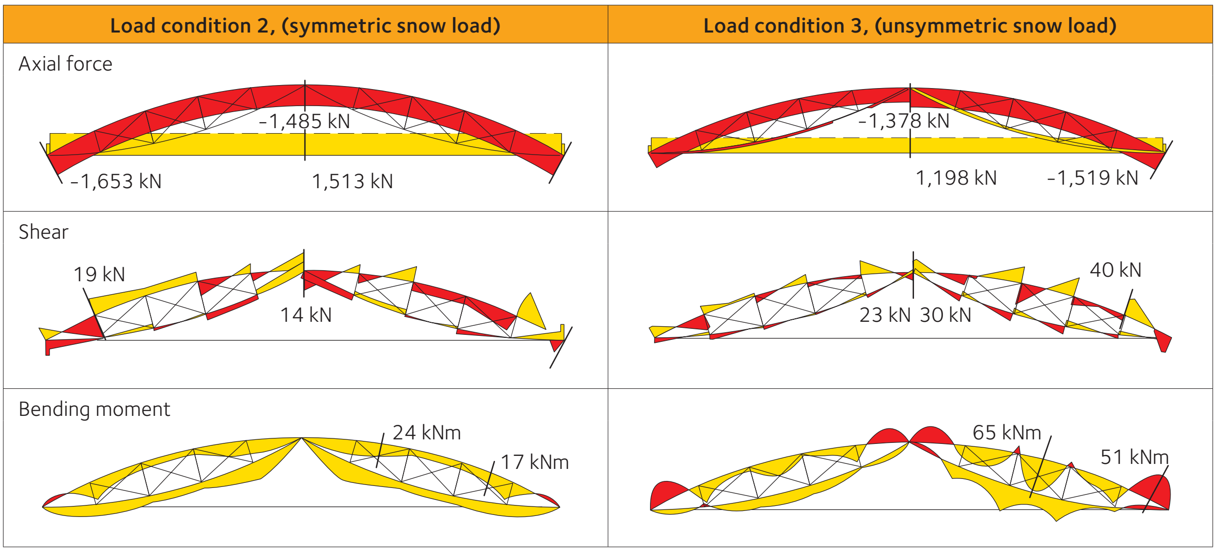
Tension tie of steel (it consists of eight separate steel bars):

$$N_{max} = \frac{q_{dII} \times l_{tot}}{8 \times h_{ridge}} = \frac{25.39 \times 64.80^2}{8 \times 8.66} = 1539.11 \text{ kN}$$

$$A_{min} = \frac{1}{num} \times \frac{N_{max} \times \gamma_{M2}}{0.9 \times f_{uk}} = \frac{1}{8} \times \frac{1539.11 \times 10^3}{0.9 \times 510} = 502.98 \text{ mm}^2$$

$\rightarrow$ tension tie of steel is chosen; $A_s = 561 \text{ mm}^2$

14.5 Internal forces and moments



14.6 Calculations in ultimate limit state, upper frame

a) Compression parallel to the grain at the support

The maximum stress occurs for the condition 2:

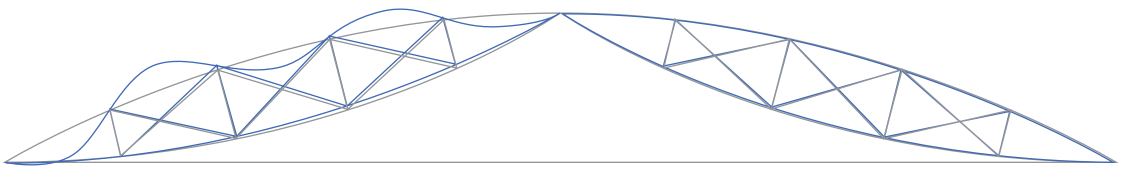
$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{1653 \times 10^3}{215 \times 630} = 12.2 \text{ MPa}$$

Compression parallel to the grain verification (*EN 1995-1-1, equation 6.2*):

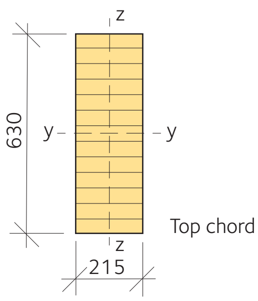
$$\frac{\sigma_{c,0,d}}{f_{c,0,d}} = 0.78 \quad < 1 \quad \textbf{OK}$$

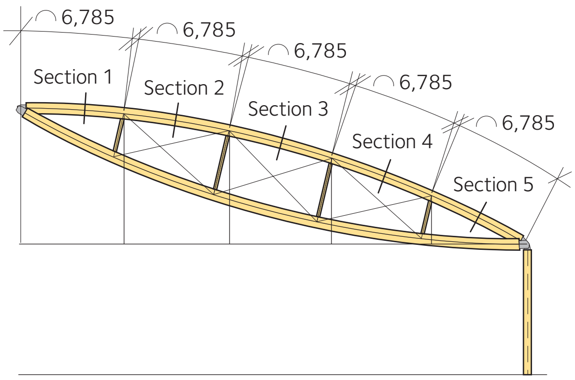
b) Stability check for combined bending and compression

Load condition 2 and 3 give the same buckling load. The first critical buckling mode is shown below.



The upper edge of the top chord is assumed to be completely braced by means of a rigid corrugate steel roof plate. Hence, lateral buckling is not an option. Section 1, section 2 and section 4 are checked.





Section 1:

$$\sigma_{c,0,d,1} = \frac{N_{Ed,1}}{b \times h} = \frac{1487 \times 10^3}{215 \times 630} = 10.98 \text{ MPa} \qquad \sigma_{m,d,1} = \frac{6M_{Ed,1}}{b \times h^2} = \frac{6 \times 22 \times 10^6}{215 \times 630^2} = 1.55 \text{ MPa}$$

Section 2:

$$\sigma_{c,0,d,2} = \frac{N_{Ed,2}}{b \times h} = \frac{1490 \times 10^3}{215 \times 630} = 11 \text{ MPa} \qquad \sigma_{m,d,2} = \frac{6M_{Ed,2}}{b \times h^2} = \frac{6 \times 24 \times 10^6}{215 \times 630^2} = 1.69 \text{ MPa}$$

Section 4:

$$\sigma_{c,0,d,4} = \frac{N_{Ed,4}}{b \times h} = \frac{1546 \times 10^3}{215 \times 630} = 11.41 \text{ MPa} \qquad \sigma_{m,d,4} = \frac{6M_{Ed,4}}{b \times h^2} = \frac{6 \times 17 \times 10^6}{215 \times 630^2} = 1.2 \text{ MPa}$$

Stability about the y-axis (deflection in the z-direction):

Buckling length:

$$l_{0,y} = 6.78 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,y} = \frac{\pi^2 \times E_{0.05} \times I_y}{(b \times h) \times l_{0,y}^2} = \frac{\pi^2 \times 10800 \times \frac{215 \times 630^3}{12}}{215 \times 630 \times (6.78 \times 10^3)} = 76.69 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,y} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,y}}} = \sqrt{\frac{24.5}{76.69}} = 0.57$$

k factor:

$$k_y = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,y} - 0.3\right) + \lambda_{rel,y}{}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times \left(0.57 - 0.3\right) + 0.57^2\right] = 0.67$$

Reduction factor for buckling:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y{}^2 - \lambda_{rel,y}{}^2}} = \frac{1}{0.67 + \sqrt{0.67^2 - 0.57^2}} = 0.96$$

Check for axial buckling about y-axis and bending about y-axis (*EN 1995-1-1, equation 6.23*):

Section 1:

$$\frac{\sigma_{c,0,d,1}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,d,1}}{f_{m,d}} = \frac{10.98}{0.96 \times 15.68} + \frac{1.55}{19.2} = 0.81 \quad < 1 \quad \textbf{OK}$$

Section 2:

$$\frac{\sigma_{c,0,d,2}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,d,2}}{f_{m,d}} = \frac{11}{0.96 \times 15.68} + \frac{1.69}{19.2} = 0.82 \quad < 1 \quad \textbf{OK}$$

Section 4:

$$\frac{\sigma_{c,0,d,4}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,d,4}}{f_{m,d}} = \frac{11.41}{0.96 \times 15.68} + \frac{1.2}{19.2} = 0.82 \quad < 1 \quad \textbf{OK}$$

c) Stability check for combined bending and compression, load condition 3

Section 3 and section 5 are checked:
Section 3:

$$\sigma_{c,0,d,3} = \frac{N_{Ed,3}}{b \times h} = \frac{1477 \times 10^3}{215 \times 630} = 10.9 \text{ MPa} \qquad \sigma_{m,d,3} = \frac{6M_{Ed,3}}{b \times h^2} = \frac{6 \times 65 \times 10^6}{215 \times 630^2} = 4.57 \text{ MPa}$$

Section 5:

$$\sigma_{c,0,d,5} = \frac{N_{Ed,5}}{b \times h} = \frac{1506 \times 10^3}{215 \times 630} = 11.12 \text{ MPa} \qquad \sigma_{m,d,5} = \frac{6M_{Ed,5}}{b \times h^2} = \frac{6 \times 51 \times 10^6}{215 \times 630^2} = 3.59 \text{ MPa}$$

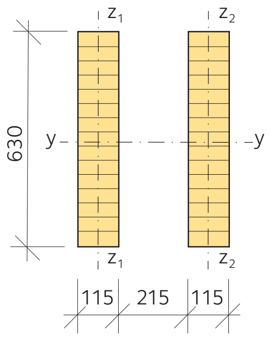
Check for axial buckling about y-axis and bending about y-axis *(EN 1995-1-1, equation 6.23)*:

Section 3:

$$\frac{\sigma_{c,0,d,3}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,d,3}}{f_{m,d}} = \frac{10.9}{0.96 \times 15.68} + \frac{4.57}{19.2} = 0.96 \quad < 1 \quad \textbf{OK}$$

Section 5:

$$\frac{\sigma_{c,0,d,5}}{k_{c,y} \times f_{c,0,d}} + \frac{\sigma_{m,d,5}}{f_{m,d}} = \frac{11.12}{0.96 \times 15.68} + \frac{3.59}{19.2} = 0.92 \quad < 1 \quad \textbf{OK}$$



Bottom chord

14.7 Verification of the bottom chord

a) Combined bending and tension

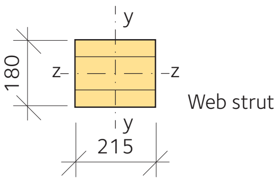
The maximum stress occurs for the condition 3:

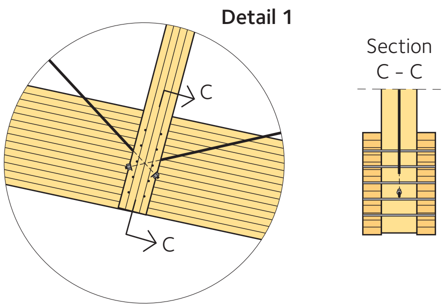
$$\sigma_{t,0,d} = \frac{\frac{N_{Ed}}{2}}{b \times h} = \frac{\frac{255 \times 10^3}{2}}{115 \times 630} = 1.76 \text{ MPa} \qquad \sigma_{m,d} = \frac{6 \times \frac{M_{Ed}}{2}}{b \times h^2} = \frac{6 \times \frac{50 \times 10^6}{2}}{115 \times 630^2} = 3.29 \text{ MPa}$$

Combined tension and bending verification *(EN 1995-1-1, equation 6.17)*:

$$\frac{\sigma_{t,0,d}}{f_{t,0,d}} + \frac{\sigma_{m,d}}{f_{m,d}} = \frac{1.76}{12.48} + \frac{3.29}{19.2} = 0.31 \quad < 1 \quad \textbf{OK}$$

14.8. Verification of the web strut





a) Axial buckling

The load condition 3 generates the highest stress:

$$\sigma_{c,0,d} = \frac{N_{Ed}}{b \times h} = \frac{28 \times 10^3}{180 \times 215} = 0.72 \text{ MPa}$$

Stability about the z-axis (deflection in the y-direction)

Buckling length:

$$l_{0,z} = 4.4 \text{ m}$$

Euler critical stress:

$$\sigma_{cr,z} = \frac{\pi^2 \times E_{0.05} \times I_z}{(b \times h) \times {l_{0,z}}^2} = \frac{\pi^2 \times 10800 \times \frac{180^3 \times 215}{12}}{180 \times 215 \times \left(4.4 \times 10^3\right)^2} = 14.87 \text{ MPa}$$

Relative slenderness ratio:

$$\lambda_{rel,z} = \sqrt{\frac{f_{c,0,k}}{\sigma_{cr,z}}} = \sqrt{\frac{24.5}{14.87}} = 1.28$$

k factor:

$$k_z = \frac{1}{2} \times \left[1 + \beta_c \times \left(\lambda_{rel,z} - 0.3\right) + {\lambda_{rel,z}}^2\right] = \frac{1}{2} \times \left[1 + 0.1 \times (1.28 - 0.3) + 1.28^2\right] = 1.37$$

Reduction factor for buckling:

$$k_{c,z} = \frac{1}{k_z + \sqrt{{k_z}^2 - {\lambda_{rel,z}}^2}} = \frac{1}{1.37 + \sqrt{1.37^2 - 1.28^2}} = 0.54$$

Check for axial buckling about z-axis and bending about y-axis *(EN 1995-1-1, equation 6.24)*:

$$\frac{\sigma_{c,0,d}}{k_{c,z} \times f_{c,0,d}} = \frac{0.72}{0.54 \times 15.68} = 0.09 \quad < 1 \quad \textbf{OK}$$

14.9 Verification of the steel tension tie

Load condition 2 generates the worst condition.
Design tensile strength *(EN 1993-1-8, table 3.4)*:

$$T_{Rd} = \frac{A_s \times f_{uk} \times 0.9}{\gamma_{M2}} = \frac{561 \times 510 \times 0.9}{1.2} = 214582.5 \text{ N}$$

Tensile capacity verification *(EN 1993-1-1, equation 6.5)*:

$$\frac{T_{Ed}}{num \times T_{Rd}} = \frac{1513}{8 \times 214.58} = 0.88 \quad < 1 \quad \textbf{OK}$$

The verification of the connections are not performed in this example.
It is however required by the *EN 1995-1-1*.

14.10 Verification of the web diagonal

The bracing system is made with M16 steel rods, grade S355.
Load condition 3 generates the worst condition.

Design tensile strength *(EN 1993-1-8, table 3.4)*:

$$T_{Rd} = \frac{A_s \times f_{uk} \times 0.9}{\gamma_{M2}} = \frac{157 \times 510 \times 0.9}{1.2} = 60052 \text{ N}$$

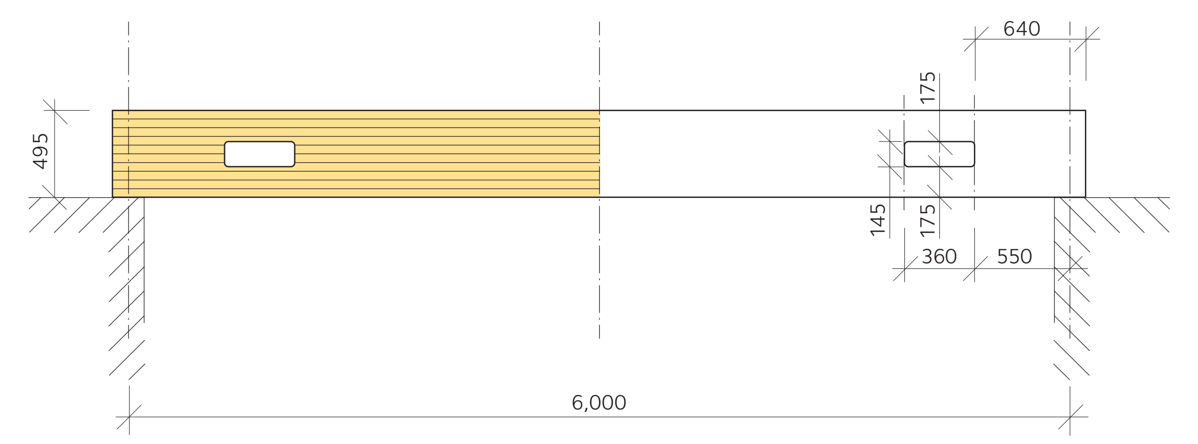
Tensile capacity verification *(EN 1993-1-1, equation 6.5)*:

$$\frac{T_{Ed}}{T_{Rd}} = \frac{40}{60.05} = 0.67 \quad < 1 \quad \textbf{OK}$$

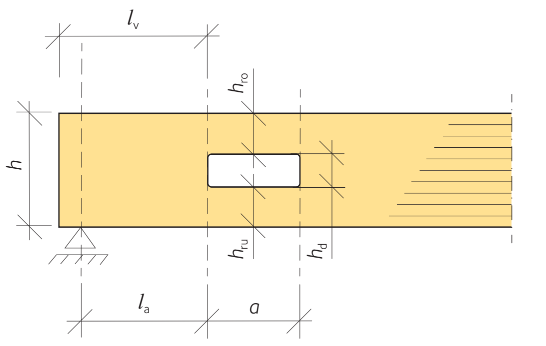
Example 15: Beam with holes

15.1 System, dimensions and design parameters

Verify the beam with holes below.



The beam is made of glulam, strength class	GL30c
Threaded bonded-in rods, strength class	4.8
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_q = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$
Partial factor for steel	$\gamma_{M2} = 1.2$



15.2 Loads

The loads considered in the design are:

Structural

$$g_{k,1} = 0.3 \text{ kN/m}$$

Non-structural

$G_{k,2} = 1.0 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i = 1.0 \times 2.0 = 2.0 \text{ kN/m}$

Variable load

$Q_k = 2.5 \text{ kN/m}^2$ $\quad q_k = Q_k \times i = 2.5 \times 2 = 5 \text{ kN/m}$

15.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1.0 \times 1.2 \times (0.3 + 2.0) = 2.8 \text{ kN/m}$$

Combination 2 (self-load leading + variable load, medium term load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_q \times q_k\right] = 1.0 \times \left[1.2 \times (0.3 + 2.0) + 1.5 \times 5.0\right] = 10.3 \text{ kN/m}$$

Leading design combinations at ULS:

$$\frac{q_{dI}}{k_{mod,1}} = \frac{2.8}{0.6} = 4.6 \qquad \frac{q_{dII}}{k_{mod,2}} = \frac{10.3}{0.8} = 12.8$$

Thus combination 2 is leading.

15.4 Geometrical rules

See table 10.8, page 47:

$l_v = 640 \text{ mm} > h = 495 \text{ mm}$ **OK**

$l_a = 550 \text{ mm} > \frac{h}{2} = 247.5 \text{ mm}$ **OK**

$h_{ro} = 175 \text{ mm} > 0.25 \times h = 123.8 \text{ mm}$ **OK**

$h_{ru} = 175 \text{ mm} > 0.25 \times h = 123.8 \text{ mm}$ **OK**

$h_d = 145 \text{ mm} < 0.3 \times h = 149 \text{ mm}$ **OK**

$r = 30 \text{ mm} > r_{min} = 25 \text{ mm}$ **OK**

$a = 360 \text{ mm} < h = 495 \text{ mm}$ **OK**

$a = 360 \text{ mm} < 2.5 \times h_d = 362.5 \text{ mm}$ **OK**

15.5 Internal forces at the hole edge closest to the support

Shear:

$$V_{\text{hole}} = q_{\text{dII}} \times \left(\frac{l_{\text{tot}}}{2} - l_{\text{a}}\right) = 10.26 \times \left(\frac{6}{2} - 0.55\right) = 25.14 \text{ kN}$$

Bending moment t:

$$M_{\text{hole}} = q_{\text{dII}} \times \frac{l_{\text{tot}}}{2} \times l_{\text{a}} - q_{\text{dII}} \times \frac{l_{\text{a}}^2}{2} = 10.26 \times \frac{6}{2} \times 0.55 - 10.26 \times \frac{0.55^2}{2} = 15.38 \text{ kNm}$$

15.6 ULS verifications

a) Tensile force perpendicular to the grain at the edge of the hole

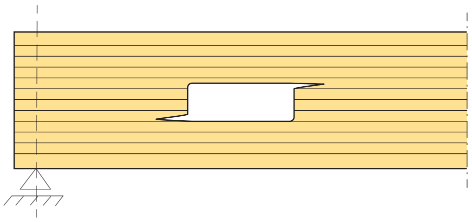
$$h_{\text{r}} = \min\left(h_{\text{ro}}, h_{\text{ru}}\right) = 175 \text{ mm}$$

$$F_{\text{t,V,d}} = V_{\text{hole}} \times \frac{h_{\text{d}}}{4 \times h} \times \left(3 - \frac{h_{\text{d}}^2}{h^2}\right) = 25.1 \times \frac{145}{4 \times 495} \times \left(3 - \frac{145^2}{495^2}\right) = 5.4 \text{ kN}$$

$$F_{\text{t,M,d}} = 0.008 \times \frac{M_{\text{hole}}}{h_{\text{r}}} = 0.008 \times \frac{12.274}{0.175} = 0.561 \text{ kN}$$

$$F_{\text{t,90,d}} = F_{\text{t,V,d}} + F_{\text{t,M,d}} = 5.9 \text{ kN}$$

Possible crack pattern:



Verification for tension perpendicular to the grain, *see table 9.16, page 40*:

$$l_{\text{t,90,d}} = 0.5 \times \left(h_{\text{d}} + h\right) = 0.5 \times \left(145 + 495\right) = 320 \text{ mm}$$

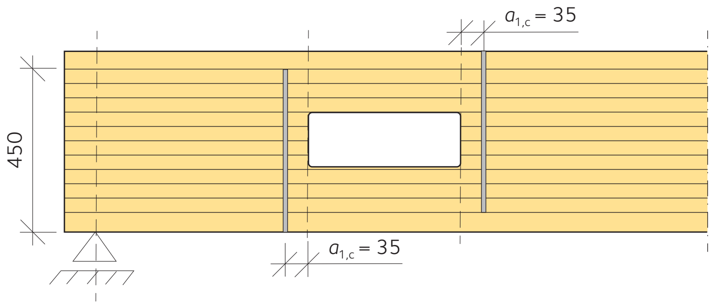
$$F_{\text{t,90,R}} = 0.5 \times l_{\text{t,90,d}} \times f_{\text{t,90,d}} \times b = 0.5 \times 320 \times 0.3 \times 90 = 4608 \text{ N}$$

$$\frac{F_{\text{t,90,d}}}{F_{\text{t,90,R}}} = \frac{5.9}{4.6} = 1.3 \quad > 1$$ **NOT OK**

The verification is not satisfied, reinforcement is needed.
Two reinforcement methods are proposed:

- threaded bonded-in rods.
- fully threaded self-tapping screws.

15.7 Reinforcement with threaded bonded-in rods



Use threaded bonded-in rods M10, strength class 4.8:

$d = 10$ mm

$f_{uk} = 400$ MPa

$A_s = 58$ mm²

Withdrawal capacity of a steel rod, *see table 13.23, page 74 and 13.24, page 75*:

$\kappa_1 = 1 \quad k_1 = 0.84$

$l_i = h_{ru} = 175$ mm

$f_{ax,k} = 5.5$ MPa

$$R_{t,k,timber} = \pi \times (d + 1) \times l_i \times f_{ax,k} \times k_1 \times \kappa_1 = \pi \times (10 + 1) \times 175 \times 5.5 \times 0.8 = 27939.8 \text{ N}$$

Tensile capacity of a steel rod, *see table 13.23, page 74*:

$$R_{t,k,rod} = 0.6 \times f_{uk} \times A_s = 1 \times 400 \times 58 = 13920 \text{ N}$$

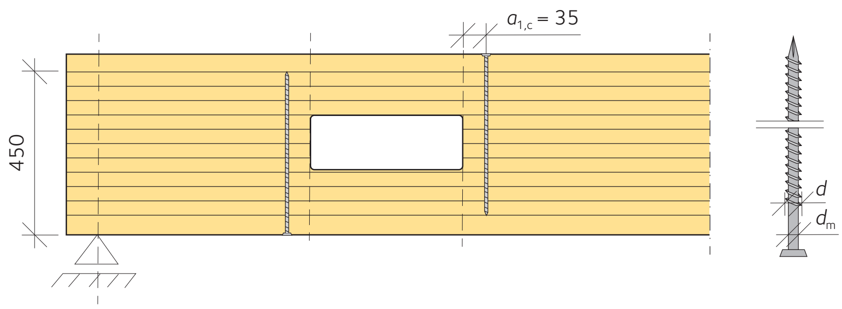
Axial capacity of a steel rod:

$$R_t = \min\left(\frac{R_{t,k,rod}}{\gamma_{M2}}, \frac{k_{mod} \times R_{t,k,timber}}{\gamma_M}\right) = \min\left(\frac{13.92}{1.2}, \frac{0.8 \times 27.94}{1.25}\right) = 11.6 \text{ kN}$$

Verification for tension perpendicular to the grain:

$$\frac{F_{t,90,d}}{R_t} = \frac{5.9}{11.6} = 0.5 \quad \textbf{OK}$$

15.8 Reinforcement with full-threaded self-tapping screws



Fully threaded screws 9 × 450 mm are used:

$$f_u = 1000 \text{ MPa}$$

$$d = 9 \text{ mm} \quad d_m = 5.9 \text{ mm}$$

$$l_{ef} = h_{ru} = 175 \text{ mm}$$

Withdrawal capacity of a screw with an angle of 90° between screw axis and grain direction (*EN 1995-1-1, equation 8.38*):

$$f_{ax,k,s} = 0.52 \times d^{-0.5} \times l_{ef}{}^{-0.1} \times \rho_k{}^{0.8} = 0.5 \times 9^{-0.5} \times 175^{-0.1} \times 390^{0.8} = 12.2 \text{ MPa}$$

$$k_d = \min\left(1.0, \frac{d}{8}\right) = 1$$

$$F_{ax,k,rk} = \frac{f_{ax,k,s} \times d \times l_{ef} \times k_d}{1.2 \times \cos(\alpha)^2 + \sin(\alpha)^2} = \frac{12.2 \times 9 \times 175}{1.2 \times \cos(90°)^2 + \sin(90°)^2} = 19262.4 \text{ N}$$

Tensile capacity of screw, *see table 6.10, page 15*:

$$F_{t,Rk} = 0.9 f_u \times \pi \times \frac{d_m{}^2}{4} = 0.9 \times 1000 \times \pi \times \frac{5.9^2}{4} = 24605.7 \text{ N}$$

Design axial capacity of one screw:

$$F_{t,d} = \min\left(\frac{F_{ax,k,rk} \times k_{mod}}{\gamma_M}, \frac{F_{t,Rk}}{\gamma_{M2}}\right) = \min\left(\frac{19.26 \times 0.8}{1.25}, \frac{24.61}{1.2}\right) = 12.33 \text{ kN}$$

Verification for tension perpendicular to the grain:

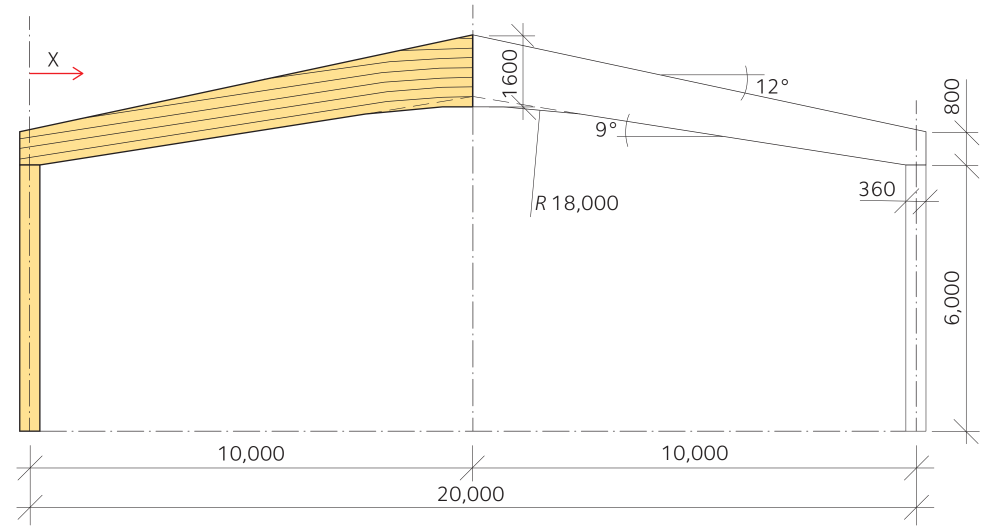
$$\frac{F_{t,90,d}}{F_{t,d}} = \frac{5.9}{12.3} = 0.5 \quad \textbf{OK}$$

Example 16: Reinforcement for tension perpendicular to the grain at the apex zone

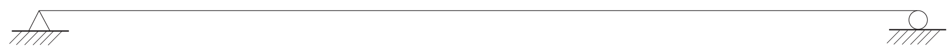
16.1 System, dimensions and design parameters

Design and verify the reinforcement of the apex zone for the pitched cambered beam analysed in *example 3, page 94*.

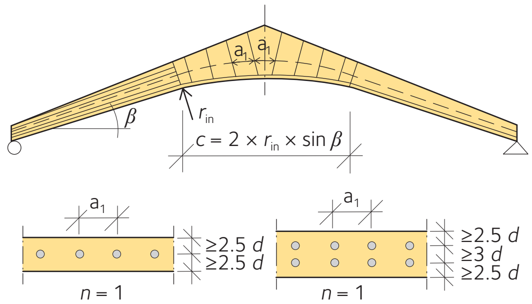
Pitched cambered beam



Static system



The beam is made of glulam, strength class	GL30c
Steel rods, strength class	5.8
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for snow load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$
Partial factor for connection	$\gamma_{MC} = 1.3$
Partial factor for steel	$\gamma_{M2} = 1.2$



16.2 Loads

The loads considered in the design are:

Structural

$$g_{k,1} = 1.2 \text{ kN/m}$$

Non-structural

$G_{k,2} = 0.60 \text{ kN/m}^2$

$$g_{k,2} = G_{k,2} \times i \times 1.1 = 0.60 \times 6 \times 1.1 = 4 \text{ kN/m}$$

Snow load

$S_k = 1.5 \text{ kN/m}^2$

$$s_k = S_k \times \mu \times i \times 1.1 = 1.5 \times 0.98 \times 6 \times 1.1 = 9.7 \text{ kN/m}$$

Factor 1.1 used in the equations above accounts for the continuity of purlins over trusses.

16.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1.0 \times 1.2 \times \left(1.2 + 3.96\right) = 6.2 \text{ kN/m}$$

Combination 2 (self-load leading + symmetric snow load, medium term load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_k\right] = 1 \times \left[1.2 \times (1.2 + 4) + 1.5 \times 9.7\right] = 20.8 \text{ kN/m}$$

Leading design combinations at ULS:

$$\frac{q_{dI}}{k_{mod,1}} = \frac{6.2}{0.6} = 10.3 \quad < \quad \frac{q_{dII}}{k_{mod,2}} = \frac{20.8}{0.8} = 26.0$$

Thus combination 2 is leading.

16.4 ULS verifications

See example 3, section 3.6, page 94:

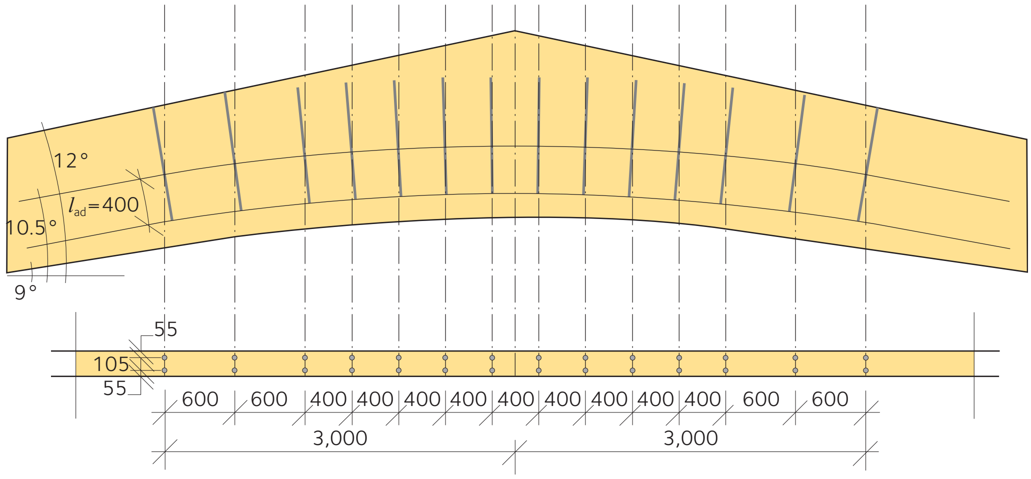
$$\sigma_{t,90,d} = k_p \times \frac{6 \times M_{max}}{b \times {h_{apex}}^2} - 0.6 \times \frac{q_{dII}}{b} = 0.05 \times \frac{6 \times 1040 \times 10^6}{215 \times 1600^2} - 0.6 \times \frac{20.8}{215} = 0.51 \text{ MPa}$$

Verify the figure for tension perpendicular to the grain (*EN 1995-1-1, equation 6.50*):

$$\frac{\sigma_{t,90,d}}{k_{dis} \times k_{vol} \times f_{t,90,d}} = \frac{0.51}{1.7 \times 0.36 \times 0.32} = 2.6 \quad > 1 \quad \textbf{NOT OK} \quad \text{Risk of failure}$$

The condition is not met, the beam apex needs to be reinforced.

16.5. Reinforcement with threaded bonded-in rods



Bonded-in rods M12, strength class 5.8 are used:

$$d = 12 \text{ mm}$$
$$f_{uk} = 500 \text{ MPa}$$
$$A_s = 84 \text{ mm}^2$$
$$l_{ad} = 400 \text{ mm}$$

Length of the reinforced zone:

$$c_{min} = 2 \times r_{in} \times \sin(\beta) = 5.6 \text{ m}$$

Spacing between roads, see figure above:

$$a_{1,min} = 250 \text{ mm} \quad < \quad \begin{matrix} a_{1,apex} = 400 \text{ mm} \\ a_{1,outer} = 600 \text{ mm} \end{matrix} \quad < \quad a_{1,max} = 0.75 \times h_{apex} = 1200 \text{ mm}$$

Withdrawal capacity of a steel rod, *see table 13.23, page 74*, and *13.24, page 75*:

$$\kappa_1 = 1 \quad k_1 = 0.52$$
$$f_{ax,k} = 5.5 \text{ MPa}$$
$$R_{k,timber} = \pi \times (d+1) \times l_{ad} \times f_{ax,k} \times k_1 \times \kappa_1 = \pi \times (12+1) \times 400 \times 5.5 \times 0.52 = 46722 \text{ N}$$

Tensile capacity of a steel rod, *see table 13.23, page 74*:

$$R_{t,k,rod} = 0.5 \times f_{uk} \times A_s = 0.6 \times 500 \times 84 = 25200 \text{ N}$$

Axial capacity of a steel rod:

$$R_t = \min\left(\frac{R_{t,k,rod}}{\gamma_{M2}}, \frac{k_{mod} \times R_{t,k,timber}}{\gamma_{MC}}\right) = \min\left(\frac{25.2}{1.2}, \frac{0.8 \times 46.7}{1.3}\right) = 21.0 \text{ kN}$$

Design tensile force perpendicular to the grain in the central part of the apex area:

$$F_{t,90,d,apex} = \frac{\sigma_{t,90,d} \times b \times a_{1,apex}}{n} = \frac{0.51 \times 215 \times 400}{2} = 21930 \text{ N}$$

Design tensile force perpendicular to the grain in the outer quarters of the apex area:

$$F_{t,90,d,outer} = \frac{2}{3} \times \frac{\sigma_{t,90,d} \times b \times a_{1,outer}}{n} = \frac{2}{3} \times \frac{0.51 \times 215 \times 600}{2} = 21930 \text{ N}$$

Verification for tension perpendicular to the grain:

$$\frac{F_{t,90,d,apex}}{R_t} = \frac{21.93}{21} = 1.04 \quad > 1 \quad \textbf{NOT OK}$$

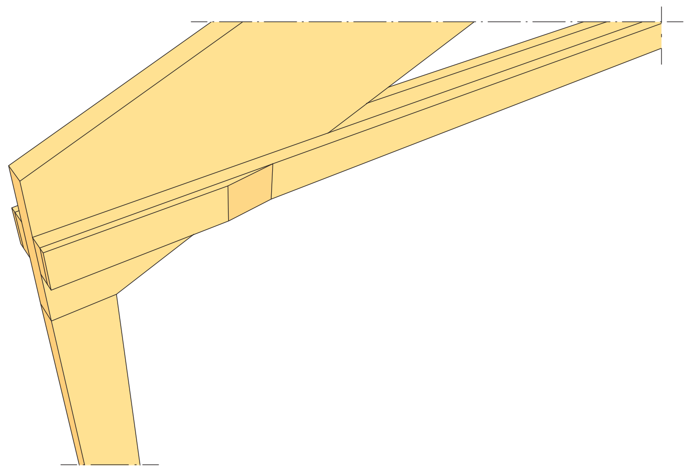
$$\frac{F_{t,90,d,outer}}{R_t} = \frac{21.93}{21} = 1.04 \quad > 1 \quad \textbf{NOT OK}$$

The condition is not met, closer spacing of rods required.

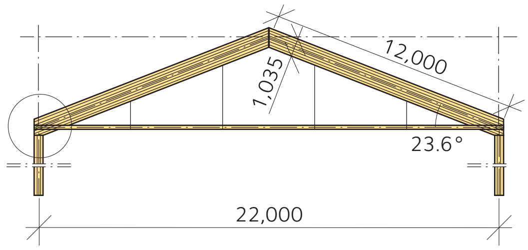
Example 17: Design of a tie fixing

17.1 System, dimensions and design parameters

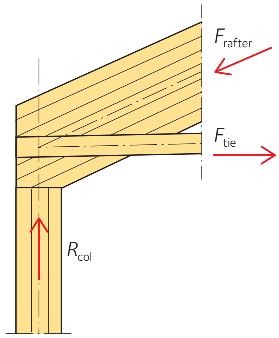
Design and verify the tie fixing below. The tie fixing is part of the structure dimensioned in *example 5, page 107*.



The structure is made of glulam, strength class	GL30c
Steel screws, strength class	10.9
Safety class 3	$\gamma_d = 1$
Service class 1	
Material partial factor for glulam	$\gamma_M = 1.25$
Partial factor for connections	$\gamma_{MC} = 1.3$



17.2 Forces



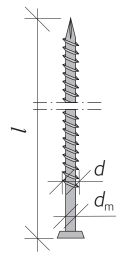
The forces acting in the connection system are, *see example 5, page 107*:

$$R_{col} = 277 \text{ kN}$$
$$F_{tie} = 301 \text{ kN}$$
$$F_{raf} = 340 \text{ kN}$$

17.3 Design of the connection

Fully threaded screws are adopted:

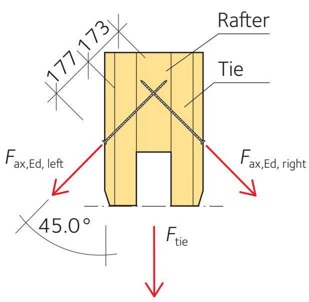
$$l = 350 \text{ mm}$$
$$d = 11 \text{ mm}$$
$$d_m = 7.5 \text{ mm}$$
$$f_u = 1000 \text{ MPa}$$

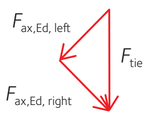


Axial force $F_{ax,Ed}$ acting in the screws:

$$F_{ax,Ed} = F_{tie} \times \cos\left(45°\right) = 301 \times \cos\left(45°\right) = 213 \text{ kN}$$
$$F_{ax,Ed,left} = F_{ax,Ed,right} = F_{ax,Ed}$$

Withdrawal capacity of a screw where the angle between the wood screw axis and the grain is arc tan (1/cos 23.6°) = 47.5° (*EN 1995-1-1, equation 8.38*):





$$l_{ad} = 173 \text{ mm}$$

$$f_{ax,k,s} = 0.52 \times d^{-0.5} l_{ad}{}^{-0.1} \times \rho_k{}^{0.8} = 0.52 \times 11^{-0.5} \times 173^{-0.1} \times 390^{0.8} = 11.08 \text{ MPa}$$

$$k_d = \min\left(1, \frac{d}{8}\right) = 1.0$$

$$F_{ax,k,rk} = \frac{f_{ax,k,s} \times d \times l_{ad} \times k_d}{1.2 \times \cos(\alpha)^2 + \sin(\alpha)^2} = \frac{11.1 \times 11 \times 173}{1.2 \times \cos\left(47.5°\right)^2 + \sin\left(47.5°\right)^2} = 19321 \text{ N}$$

Tensile capacity of a screw, *see table 6.10, page 15*:

$$F_{t,Rk} = 0.9 f_u \times \pi \times \frac{d_m{}^2}{4} = 0.9 \times 1000 \times 3.14 \times \frac{7.5^2}{4} = 39740.63 \text{ N}$$

Design axial capacity of one screw:

$$F_{t,d} = \min\left(\frac{F_{ax,k,rk} \times k_{mod}}{\gamma_{MC}}, \frac{F_{t,s,k}}{\gamma_{M2}}\right) = \min\left(\frac{19.32 \times 0.8}{1.3}, \frac{39.74}{1.2}\right) = 11.89 \text{ kN}$$

The number of horizontal screw rows which can fit in the tension tie is $n_{rows} = 4$.
Thus, the required number of vertical rows is:

$$n_{min} = \frac{F_{ax,Ed}}{n_{rows} \times F_{t,d}} = \frac{213}{4 \times 11.89} = 5$$

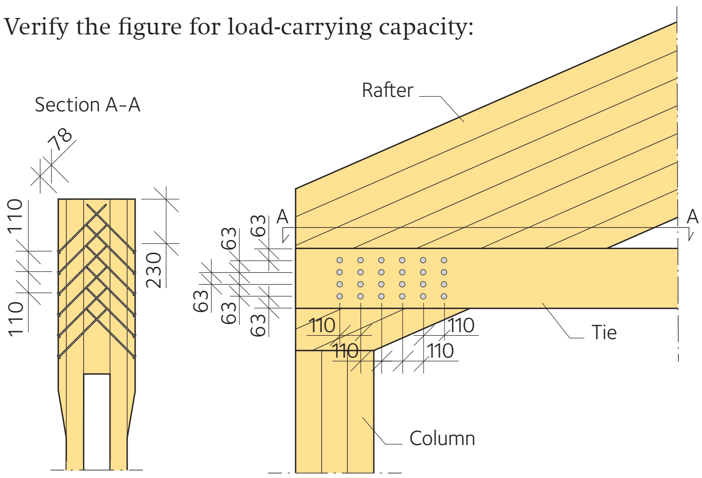
Taking into consideration that the effective number of fasteners is less than the real number, the chosen number of screws in the grain direction for this example is:

$$n = 6$$
$$n_{ef} = n^{0.9} = 5$$

Spacing (*EN 1995-1-1, clause 8.7.2*):

$$a_{1,min} = 7 \times d = 7 \times 11 = 77 \text{ mm}$$
$$a_{2,min} = 5 \times d = 5 \times 11 = 55 \text{ mm}$$
$$a_{1CG,min} = 10 \times d = 110 \text{ mm}$$
$$a_{2CG,min} = 4 \times d = 44 \text{ mm}$$

Verify the figure for load-carrying capacity:

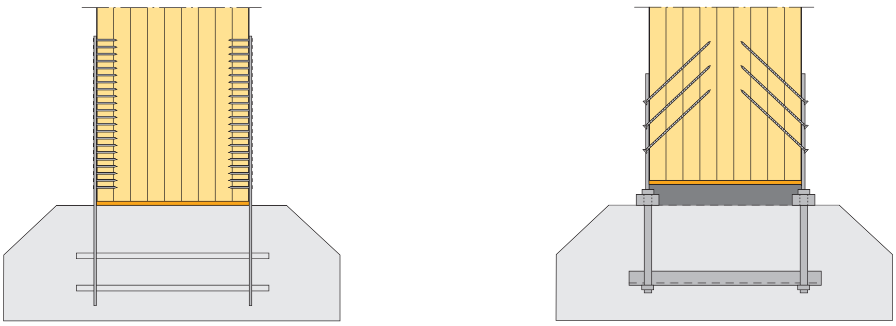


$$\frac{F_{tie} \times \cos(45°)}{n_{rows} \times n_{ef} \times F_{td}} = \frac{301 \times \cos(45°)}{4.5 \times 11.89} = 0.89 \quad < 1 \quad \textbf{OK}$$

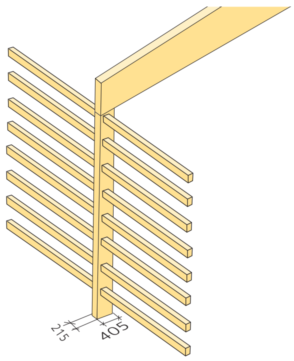
Example 18: Column with fixed support

18.1 System, dimensions and design parameters

Design and verify the fixed support of the column below.
The fixing relates to the column dimensioned in *example 6, page 117*.



The column is made of glulam, strength class	GL30c
Steel screws, strength class	10.9
Safety class 3	$\gamma_d = 1$
Service class 1	
Material partial factor for glulam	$\gamma_M = 1.25$
Material partial factor for steel:	$\gamma_{M0} = 1.0$
Partial factor for steel ties	$\gamma_{M2} = 1.2$
Partial factor for connection	$\gamma_{MC} = 1.3$



18.2 Normal force, shear and moment

The connection is designed on the basis of the internal forces and the bending moment evaluated in *example 6, page 117*.

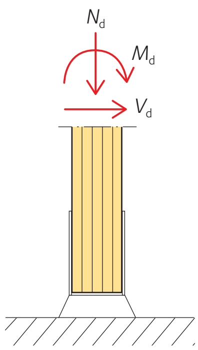
Combination 2 (wind load leading, $k_{mod} = 0.9$):

$$q_{ver,2} = \gamma_d \times \left[\gamma_g \times \left(g_{k,beam} + g_{k,2}\right) + 0 \times \psi_{0,s} \times s_k\right] = 1 \times \left[1 \times (1.2 + 4) + 0 \times 0.6 \times 7.9\right] = 5.2 \text{ kN/m}$$

$$q_{w,p,2} = \gamma_d \times \gamma_q \times q_{w,k,pos} = 1 \times 1.5 \times 2.7 = 4.1 \text{ kN/m}$$

$$q_{w,n,2} = \gamma_d \times \gamma_q \times q_{w,k,neg} = 1 \times 1.5 \times 1.44 = 2.16 \text{ kN/m}$$

$$q_{w,i,2} = \gamma_d \times \gamma_q \times q_{w,k,int} = 1 \times 1.5 \times 1.26 = 1.89 \text{ kN/m}$$



The maximum tensile force in the connection occurs when the vertical load is minimum:

$$M_d = 78.4 \text{ kN}$$

$$N_d = 51.6 \text{ kN}$$

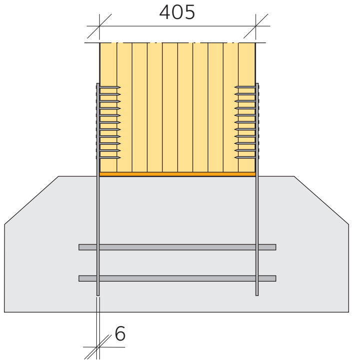
$$V_d = 30.9 \text{ kN}$$

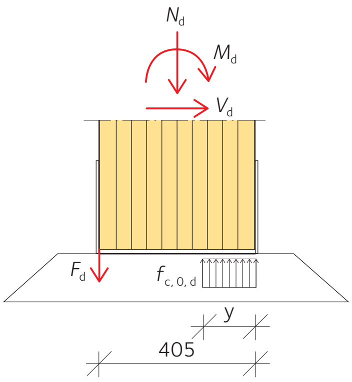
18.3 Design of the connection

Two possible fixed supports are proposed.

a) Fixed support with nails

Nails 60 × 4 mm², f_u = 800 MPa;
Steel plate thickness t_s = 6 mm, structural steel grade S355.





Length of the compression zone y and tensile force F_d:

$$y = h \times \left(1 - \sqrt{1 - \frac{2 \times M_d + N_d \times h}{b \times h^2 \times f_{c,0,d}}}\right) = 405 \times \left(1 - \sqrt{1 - \frac{2 \times 78.4 \times 10^6 + 51.6 \times 10^3 \times 405}{215 \times 405^2 \times 15.68}}\right) = 71.36 \text{ mm}$$

$$F_d = b \times y \times f_{c,0,d} - N_d = 215 \times 71.4 \times 15.7 - 51.6 \times 10^3 = 188975.2 \text{ N}$$

Load-carrying capacity of one nail, *see table 13.19, page 71*:

$$t_{pen,min} = 41 \text{ mm} \quad < \quad t_{pen} = 52 \text{ mm}$$

$$R_k = 1.72 \text{ kN}$$

$$R_d = R_k \times \frac{k_{mod}}{\gamma_{MC}} = 1.72 \times \frac{0.9}{1.3} = 1.19 \text{ kN}$$

The number of vertical nail rows which can fit in the plate is $n_{rows} = 19$ (the nails are placed in a staggered manner, see the figure below). Thus, the required number of horizontal rows is:

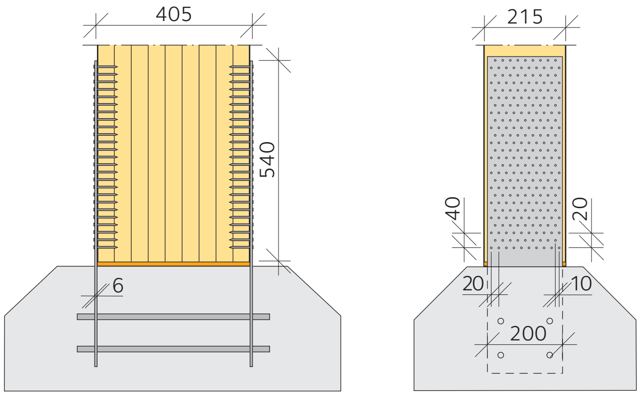
$$n_{ef,min} = \frac{F_d}{n_{rows} \times R_d} = \frac{188.98}{19 \times 1.19} = 8.35$$

Taking into consideration that the effective number of fasteners is less than the real number. Therefore choose 13 nails in a row, parallel with the grain:

$$n = 13$$

$$n_{ef} = n^{k_{ef}} = 8.8$$

Steel plates with standard holes pattern are adopted.



Verification of the nailed connection:

$$F_{v,rd} = n_{rows} \times n_{ef} \times R_d = 19 \times 8.8 \times 1.2 = 200.2 \text{ kN}$$

$$\frac{F_d}{F_{v,rd}} = \frac{188.975}{200.186} = 0.944 \quad < 1 \quad \textbf{OK}$$

Verification of the steel plate:

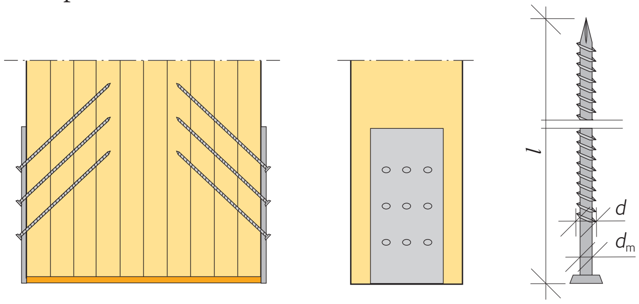
$$A_{net} = t_s \times \left(b_1 - n_{rows} \times d_{hole}\right) = 6 \times \left(200 - 19 \times 5\right) = 630 \text{ mm}^2$$

$$N_{Rd,steel} = 0.9 \times \frac{f_{uk} \times A_{net}}{1.2} = 0.9 \times \frac{430 \times 630}{1.2} = 2.03 \times 10^5 \text{ N}$$

$$\frac{F_d}{N_{Rd,steel}} = \frac{189}{203.2} = 0.9 \quad < 1 \quad \textbf{OK}$$

b) Fixed support with screws

Screws 300 × 11 mm², f_u = 1000 MPa;
Steel plate thickness t = 6 mm.



Fully threaded screws are utilized:

$$l = 300 \text{ mm}$$
$$d = 11 \text{ mm}$$
$$d_m = 7.5 \text{ mm}$$
$$f_u = 1000 \text{ MPa}$$

Axial capacity of screw with an angle of 45° between screw axis and grain direction (*EN 1995-1-1, equation 8.38*):

$$l_{ad} = 300 - t_s \times \sqrt{2} = 291.5 \text{ mm}$$

$$f_{ax,k,s} = 0.52 \times d^{-0.5} l_{ad}{}^{-0.1} \times \rho_k{}^{0.8} = 0.52 \times 11^{-0.5} \times 291.51^{-0.1} \times 390^{0.8} = 10.51 \text{ MPa}$$

$$k_d = \min\left(1, \frac{d}{8}\right) = 1.0$$

$$F_{ax,k,rk} = \frac{f_{ax,k,s} \times d \times l_{ad} \times k_d}{1.2 \times \cos(\alpha)^2 + \sin(\alpha)^2} = \frac{10.5 \times 11 \times 291.5}{1.2 \times \cos\left(45°\right)^2 + \sin\left(45°\right)^2} = 30644.5 \text{ N}$$

Tensile capacity of screw, *se table 6.10, page 15*:

$$F_{t,Rk} = 0.9 f_u \times \pi \times \frac{d_m{}^2}{4} = 3.98 \times 10^4 \text{ N}$$

Design value for a wood screw's axial load-carrying capacity:

$$F_{td} = \min\left(\frac{F_{ax,k,rk} \times k_{mod}}{\gamma_{MC}}, \frac{F_{t,s,k}}{\gamma_{M2}}\right) \times \cos\left(45°\right) = \min\left(\frac{30.64 \times 0.9}{1.3}, \frac{39.76}{1.2}\right) \times \cos\left(45°\right) = 15 \text{ kN}$$

The number of vertical screw rows which can fit in the plate is n_{rows} = 3.
Thus, the required number of horizontal rows is:

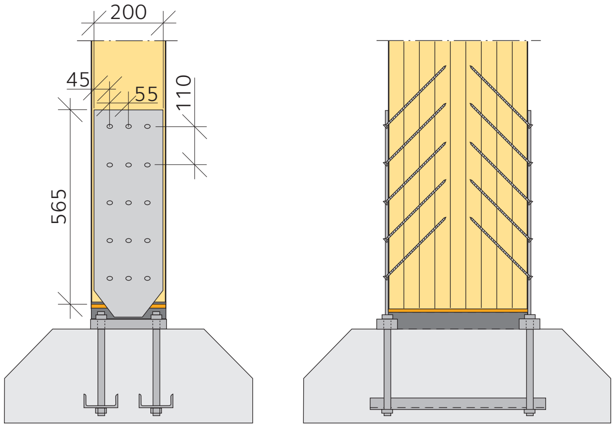
$$n_{min} = \frac{F_d}{n_{rows} \times F_{td}} = \frac{188.98}{3 \times 15} = 4.2$$

Taking into consideration that the effective number of fasteners is less than the real number. Therefore choose 5 wood screws, parallel with the grain:

$$n = 5$$
$$n_{ef} = n^{0.9} = 4.3$$

Spacing (*EN 1995-1-1, clause 8.7.2*):

$$a_{1,min} = 7 \times d = 7 \times 11 = 77 \text{ mm}$$
$$a_{2,min} = 5 \times d = 5 \times 11 = 55 \text{ mm}$$
$$a_{1CG,min} = 10 \times d = 110 \text{ mm}$$
$$a_{2CG,min} = 4 \times d = 44 \text{ mm}$$



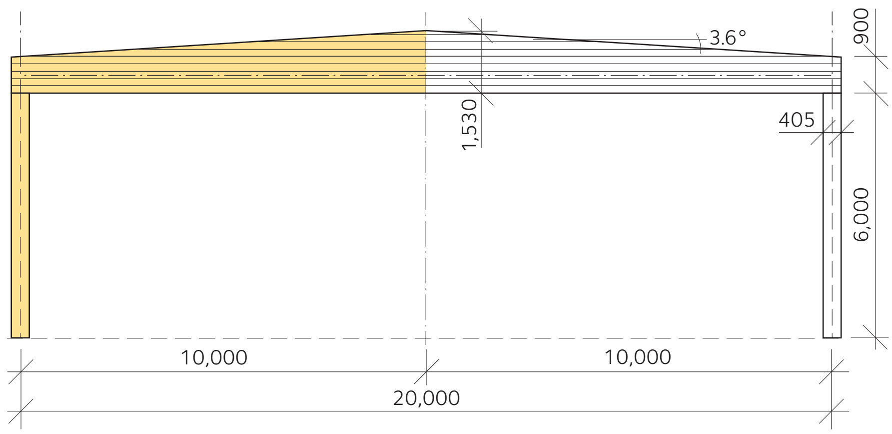
Verify the figure for load-carrying capacity:

$$\frac{F_d}{n_{rows} \times n_{ef} \times F_{td}} = \frac{188.98}{3 \times 4.26 \times 15} = 0.99 \quad < 1 \quad \textbf{OK}$$

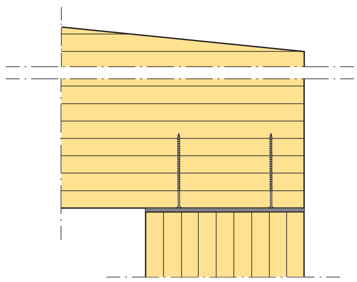
Example 19: Design of a reinforcement at the supports

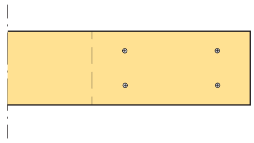
19.1 System, dimensions and design parameters

Design the reinforcement at the support for the tapered beam shown below.



The beam is made of glulam, strength class	GL30c
Steel plate, structural steel grade	S275
Steel screws, strength class	10.9
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for snow load	$\gamma_s = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$
Partial factor for connection	$\gamma_{MC} = 1.3$
Partial factor for steel	$\gamma_{M2} = 1.2$





19.2 Loads

The loads considered in the design are:

Structural

$$g_{k,1} = 1.1 \text{ kN/m}$$

Non-structural

$G_{k,2} = 1.0 \text{ kN/m}^2$ $\quad g_{k,2} = G_{k,2} \times i \times 1.1 = 1.5 \times 1.1 = 5.5 \text{ kN/m}$

Snow load

$S_k = 3 \text{ kN/m}^2$ $\quad s_k = S_k \times \mu \times i \times 1.1 = 3 \times 0.854 \times 5 \times 1.1 = 14.1 \text{ kN/m}$

Factor 1.1 used in the equations above accounts for the continuity of purlins over beams.

19.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1.2 \times \left(1.1 + 5.5\right) = 7.9 \text{ kN/m}$$

Combination 2 (self-load leading + symmetric snow load, medium term load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_s \times s_k\right] = \left[1.2 \times \left(1.1 + 5.5\right) + 1.5 \times 14.1\right] = 29.1 \text{ kN/m}$$

19.4 ULS verifications

a) Compression perpendicular to the grain at the support

The verification method shown in *table 8.12, page 25*:

$$N_{Ed} = q_{dII} \times \frac{l_{tot}}{2} = 296 \text{ kN} \qquad \sigma_{c,90,d} = \frac{N_{Ed} \times 10^3}{b \times \left(h_{ef} + 30\right)} = \frac{296 \times 10^3}{190 \times (400 + 30)} = 3.63 \text{ MPa}$$

Compression perpendicular to the grain verification (*EN 1995-1-1, equation 6.3*):

$$\frac{\sigma_{c,90,d}}{k_{c,90} \times f_{c,90,d}} = \frac{3.63}{1.75 \times 1.6} = 1.3 \quad > 1$$ **NOT OK**

The verification is not satisfied. Thus, the beam needs to be reinforced at the supports.

19.5 Reinforcement of the support

The reinforcement is realized by means of 4 self-tapping fully threaded screws and a steel plate.

$d = 11$ mm

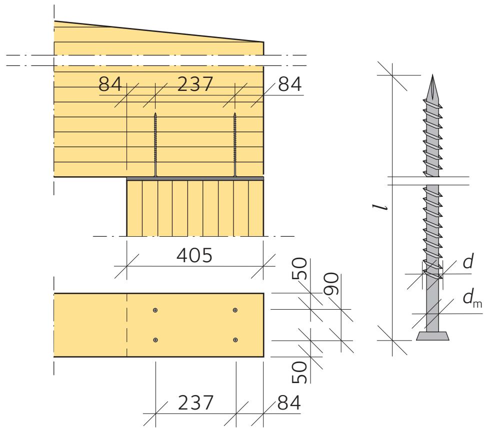
$d_m = 7.5$ mm

$f_{uk} = 1000$ MPa

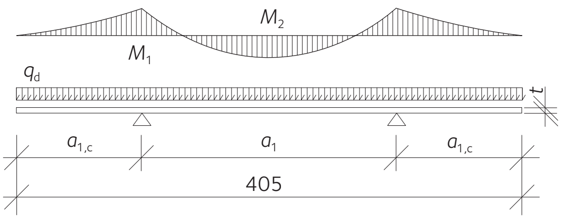
$f_{yk} = 900$ MPa

$l = 450$ mm

$l_{ef} = l - d = 450 - 11 = 439$ mm



Spacing and distances between the screws are chosen in order to minimize the thickness of the steel plate. The static system of the reinforcement is shown below.



Spacing parallel to grain:

$$a_{1,min} = 5 \times d = 55 \text{ mm}$$

Spacing perpendicular to grain (between rows of screws):

$$a_{2,min} = 5 \times d = 55 \text{ mm} \quad \rightarrow \quad a_2 = 90 \text{ mm}$$

Distance between screws and beam end:

$$a_{1c,min} = 5 \times d = 55 \text{ mm}$$

Distance between screws and beam edge:

$$a_{2c,min} = 3 \times d = 33 \text{ mm} \quad \rightarrow \quad a_2 = 50 \text{ mm}$$

The distance between the screws in the beam grain direction is chosen so as to produce equal values of sagging and hogging moment (M_1 and M_2):

$$M_1 = q_d \times \frac{a_{1,c}^2}{2} \qquad a_{1,c} = \frac{a_1}{2 \times \sqrt{2}}$$

$$M_2 = q_d \times \frac{a_1^2}{8} - M_1 \quad \rightarrow \quad a_1 = h_{col} \times \frac{\sqrt{2}}{\sqrt{2}+1}$$

$$h_{col} = 2 \times a_{1,c} + a_1$$

$$q_d = \frac{N_{Ed}}{h_{col}} = \frac{277200}{405} = 684 \text{ N/mm}$$

$$a_1 = h_{col} \times \frac{\sqrt{2}}{\sqrt{2}+1} = 405 \times \frac{\sqrt{2}}{\sqrt{2}+1} = 237 \text{ mm}$$

$$a_{1,c} = \frac{a_1}{2 \times \sqrt{2}} = \frac{237}{2 \times \sqrt{2}} = 84 \text{ mm}$$

$$M_{el} = W_{el} \times f_{y,d} = \frac{b_{col} \times t^2}{6} \times \frac{f_{y,k}}{\gamma_M} \quad \rightarrow \quad M_1 = M_{el} \quad \rightarrow \quad t_{min} = \sqrt{\frac{6 \times M_1}{b_{col} \times f_{y,d}}}$$

$$M_1 = q_d \times \frac{a_{1,c}^2}{2} = 684 \times \frac{84^2}{2} = 2407725 \text{ Nmm}$$

$$t_{min} = \sqrt{\frac{6M_1}{b_{col} \times f_{y,d}}} = \sqrt{\frac{6 \times 2407725.052}{190 \times 229.167}} = 18.21 \text{ mm} \quad \rightarrow \quad t = 20 \text{ mm}$$

a) Load bearing capacity of the screw

Push-in capacity of a screw in the perpendicular to the grain direction (*EN 1995-1-1, equation 8.38*):

$$f_{ax,k} = 0.52 \times d^{-0.5} l_{ef}^{-0.1} \times \rho_k^{0.8} = 0.52 \times 11^{-0.5} \times 439^{-0.1} \times 390^{0.8} = 10.09 \text{ MPa} \qquad k_d = \min\left(1, \frac{d}{8}\right) = 1$$

$$F_{ax,k,rk} = \frac{f_{ax,k} \times d \times l_{ef} \times k_d}{1.2 \times \left(\cos(\alpha)\right)^2 + \sin(\alpha)} = \frac{10 \times 11 \times 439}{1 \times \cos(90°)^2 + \sin(90°)} = 48727 \text{ N}$$

Buckling capacity of the screw, *see table 8.15, page 28*:
Horizontal stiffness constant:

$$c_h = \left(0.19 + 0.012 \times d\right) \times \rho_k = (0.19 + 0.01 \times 11) \times 390 = 125.58$$

Moment of inertia of screw:

$$I_s = \frac{d_m^{\ 4} \times \pi}{64} = \frac{7.5^4 \times \pi}{64} = 155.3 \text{ mm}^4$$

Screw buckling load:

$$N_{cr} = \sqrt{c_h \times E_s \times I_s} \quad \rightarrow \quad \sqrt{125.6 \times 210000 \times 155.3} = 63999.6 \text{ N}$$

Screw yield strength:

$$N_{pl} = \frac{\pi \times d_m^{\ 2} \times f_{yk}}{4} = \frac{\pi \times 8^2 \times 900}{4} = 39761 \text{ N}$$

Relative slenderness ratio:

$$\lambda_{\text{rel}} = \sqrt{\frac{N_{\text{pl}}}{N_{\text{cr}}}} = \sqrt{\frac{39760.8}{63999.6}} = 0.8$$

k factor:

$$k = 0.5 \times \left[1 + 0.49 \times \left(\lambda_{\text{rel}} - 0.2\right) + {\lambda_{\text{rel}}}^2\right] = 0.5 \times \left[1 + 0.49 \times (0.79 - 0.2) + 0.79^2\right] = 0.95$$

Reduction factor for buckling:

$$k_{\text{c}} = \left(k + \sqrt{k - {\lambda_{\text{rel}}}^2}\right)^{-1} = \left(0.95 + \sqrt{0.95 - 0.79^2}\right)^{-1} = 0.65$$

Buckling capacity of the screw:

$$R_{\text{kl,k}} = k_{\text{c}} \times \left(\frac{\pi \times {d_{\text{m}}}^2}{4}\right) \times f_{\text{yk}} = k_{\text{c}} \times \frac{\pi \times 8^2}{4} \times 900 = 25950 \text{ N}$$

The load bearing capacity of the screw is the minimum between withdrawal and buckling capacity:

$$R_{\text{k}} = \min\left(f_{\text{ax,k,rk}} \, , R_{\text{kl,k}}\right) = \min\left(48.7 \, , 25.9\right) = 25.9 \text{ kN}$$

b) Compressive capacity perpendicular to grain of the reinforced support

See table 8.14, page 27:

$$l_{\text{ef,1}} = h_{\text{ef}} + 30 = 400 + 30 = 430 \text{ mm}$$

$$l_{\text{ef,2}} = h_{\text{col}} + 0.25 l_{\text{ef}} \times e^{\frac{3.3 l_{\text{ef}}}{h_0}} = 405 + 0.25 \times 439 \times e^{\frac{3.3 \times 439}{900}} = 953.88 \text{ mm}$$

$$R_{90,\text{k}} = \min\left(k_{\text{c},90} \times b \times l_{\text{ef,1}} \times f_{\text{c,90,k}} + n \times R_{\text{k}} \, ; b \times l_{\text{ef,2}} \times f_{\text{c,90,k}}\right) = \min\left(1.75 \times 190 \times 430 \times 2.5 + 4 \times 25.95 \times 10^3 \, ; 190 \times 953.88 \times 2.5\right) = 453090.88 \text{ N}$$

$$R_{90,\text{d}} = \frac{R_{90,\text{k}} \times k_{\text{mod}}}{\gamma_{\text{MC}}} = \frac{453.1 \times 0.8}{1.3} = 278.8 \text{ kN}$$

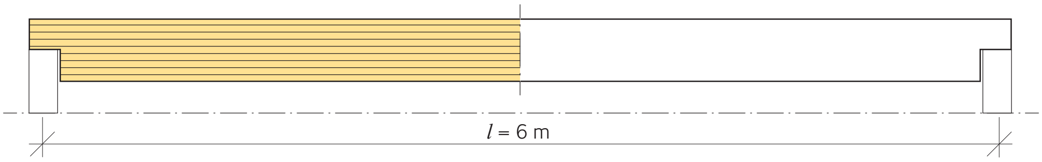
Verification of the compression perpendicular to grain of the reinforced support:

$$\frac{N_{\text{Ed}}}{R_{90,\text{d}}} = \frac{277.2}{278.83} = 0.99 \quad < 1 \quad \textbf{OK}$$

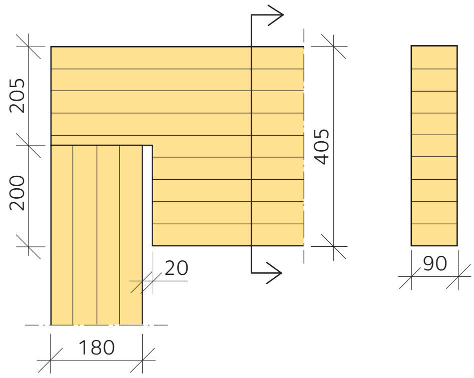
Example 20: Beam with notched supports

20.1 System, dimensions and design parameters

Verify the notched beam below.



The beam is made of glulam, strength class	GL30c
Threaded bonded-in rods, strength class	4.8
Safety class 3	$\gamma_d = 1$
Service class 1	
Partial factor for permanent load	$\gamma_g = 1.2$
Partial factor for variable load	$\gamma_q = 1.5$
Material partial factor for glulam	$\gamma_M = 1.25$
Partial factor for connection	$\gamma_{MC} = 1.3$
Partial factor for steel	$\gamma_{M2} = 1.2$
Load width 2.0 m	



20.2 Loads

The loads considered in the design are:

Structural

$$g_{k,1} = 0.2 \text{ kN/m}$$

Non-structural

$$G_{k,2} = 0.6 \text{ kN/m}^2 \qquad g_{k,2} = G_{k,2} \times i = 0.6 \times 2.0 = 1.2 \text{ kN/m}$$

Variable load

$$Q_k = 2 \text{ kN/m}^2 \qquad q_k = Q_k \times i = 2 \times 2 = 4 \text{ kN/m}$$

20.3 Load combinations

Two different load combinations are considered (*EN 1990, clause 6.4.3* and *EN 1991-1-3, clause 5.3.3*):

Combination 1 (self-load leading, permanent load, $k_{mod} = 0.6$):

$$q_{dI} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right)\right] = 1.0 \times 1.2 \times (0.2 + 1.2) = 1.7 \text{ kN/m}$$

Combination 2 (self-load leading + variable load, medium term symmetric load, $k_{mod} = 0.8$):

$$q_{dII} = \gamma_d \times \left[\gamma_g \times \left(g_{k,1} + g_{k,2}\right) + \gamma_q \times q_k\right] = 1.0 \times \left[1.2 \times (0.2 + 1.2) + 1.5 \times 4.0\right] = 7.7 \text{ kN/m}$$

Leading design combinations at ULS:

$$\frac{q_{dI}}{k_{mod,1}} = \frac{1.7}{0.6} = 2.8 \quad < \quad \frac{q_{dII}}{k_{mod,2}} = \frac{7.7}{0.8} = 9.6$$

Thus combination 2 is leading.

20.4 ULS verifications

Shear force at the support:

$$V_{Ed} = q_{dII} \times \frac{l}{2} = 7.7 \times \frac{6}{2} = 23 \text{ kN}$$

$$\tau_d = \frac{1.5 \times V_{Ed}}{b \times h_{ef}} = \frac{1.5 \times 23040}{90 \times 205} = 1.87 \text{ MPa}$$

Shear verification (*EN 1995-1-1, equation 6.60*):

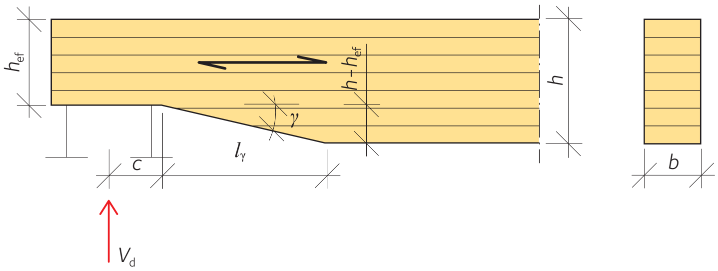
$$\frac{\tau_d}{k_{cr} \times f_{v,d}} = \frac{1.87}{0.86 \times 2.24} = 0.98 \quad < 1 \quad \textbf{OK}$$

Verification of the notch at the support according to *table 9.10, page 37*:

$$\alpha = \frac{h_{ef}}{h} = \frac{205}{405} = 0.5 \qquad c = 110 \text{ mm}$$

$$l_\gamma = 0 \qquad i = \frac{l_\gamma}{h - h_{ef}} = 0 \qquad \gamma = 90°$$

$$k_v = \min\left[1, \frac{6.5\left(1 + 1.1 \times \frac{i^{1.5}}{\sqrt{h}}\right)}{\sqrt{h} \times \left[\alpha \times (1-\alpha) + \sqrt{\frac{1}{\alpha} - \alpha^2 \times 0.8 \times \frac{c}{h}}\right]}\right] = 0.41$$

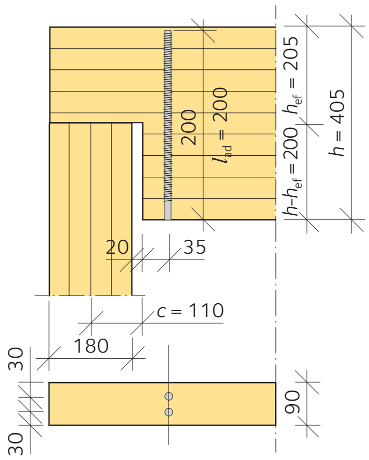


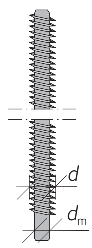
$$\frac{\tau_d}{k_v \times f_{v,d}} = \frac{1.87}{0.41 \times 2.24} = 2.03 \quad > 1 \quad \textbf{NOT OK} \quad \text{Risk of failure}$$

The verification is not satisfied, reinforcement is needed. Two reinforcement methods are proposed:

- threaded bonded-in rods.
- fully threaded screws.

20.5 Reinforcement with threaded bonded-in rods





Rods M10, 4.8 are used:

$$d = 10 \text{ mm}$$
$$f_{uk} = 400 \text{ MPa}$$
$$A_s = 58 \text{ mm}^2$$
$$l_{ad} = 200$$

Withdrawal capacity of a steel rod, *see table 13.23, page 74*:

$$\kappa_1 = 1 \quad k_1 = 0.78$$

$$f_{ax,k} = 5.5 \text{ MPa}$$

$$R_{t,k,timber} = \pi \times (d+1) \times l_{ad} \times f_{ax,k} \times k_1 \times \kappa_1 = \pi \times (10+1) \times 200 \times 5.5 \times 0.8 = 29635.3 \text{ N}$$

Tensile capacity of a steel rod, *see table 13.23, page 74*:

$$R_{t,k,rod} = 0.6 \times f_{uk} \times A_s = 1 \times 400 \times 58 = 13920 \text{ N}$$

Design axial capacity of one rod:

$$R_t = \min\left(\frac{R_{t,k,rod}}{\gamma_{M2}}, \frac{k_{mod} \times R_{t,k,timber}}{\gamma_{MC}}\right) = \min\left(\frac{13.9}{1.2}, \frac{0.8 \times 29.6}{1.3}\right) = 11.6 \text{ kN}$$

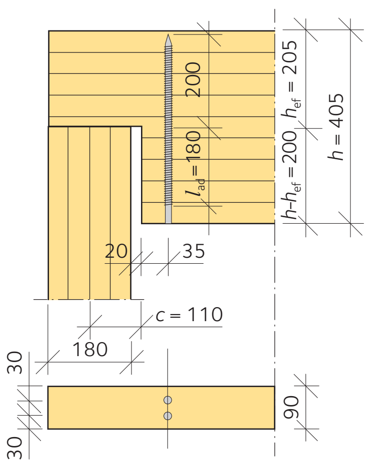
Design tensile force perpendicular to the grain, *see table 9.11, page 37*:

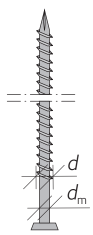
$$F_{t,90,d} = 1.3 \times V_{Ed} \times \left[3 \times (1-\alpha)^2 - 2 \times (1-\alpha)^3\right] = 1.3 \times 23 \times \left[3 \times (1-0.5)^2 - 2 \times (1-0.5)^3\right] = 14.7 \text{ kN}$$

Verification for tension perpendicular to the grain:

$$\frac{F_{t,90,d}}{n_r \times R_t} = \frac{14.7}{2 \times 11.6} = 0.63 \quad \textbf{OK}$$

20.6 Reinforcement with fully threaded screws





Fully threaded screws 9 × 400 mm are used:

$$f_u = 1000 \text{ MPa}$$

$$d = 9 \text{ mm} \qquad d_m = 5.9 \text{ mm}$$

$$l_{ad} = 180 \text{ mm}$$

Withdrawal capacity of a screw with an angle of 90° between screw axis and grain direction. (*EN 1995-1-1, equation 8.38*):

$$f_{ax,k,s} = 0.52 \times d^{-0.5} l_{ad}{}^{-0.1} \times \rho_k{}^{0.8} = 0.52 \times 9^{-0.5} \times 180^{-0.1} \times 390^{0.8} = 12.2 \text{ MPa}$$

$$k_d = \min\left(1, \frac{d}{8}\right) = 1$$

$$F_{ax,k,rk} = \frac{f_{ax,k,s} \times d \times l_{ad} \times k_d}{1.2 \times \cos(\alpha)^2 + \sin(\alpha)^2} = \frac{12.2 \times 9 \times 180}{1.2 \times \cos(90°)^2 + \sin(90°)^2} = 19757 \text{ N}$$

Tensile capacity of a screw, *see table 6.10, page 15*:

$$F_{t,Rk} = 0.9 f_u \times \pi \times \frac{d_m{}^2}{4} = 2.46 \times 10^4 \text{ N}$$

Design axial capacity of one screw:

$$F_{t,d} = \min\left(\frac{F_{ax,k,rk} \times k_{mod}}{\gamma_{MC}}, \frac{F_{t,s,k}}{\gamma_{M2}}\right) = \min\left(\frac{19.76 \times 0.8}{1.3}, \frac{24.59}{1.2}\right) = 12.16 \text{ kN}$$

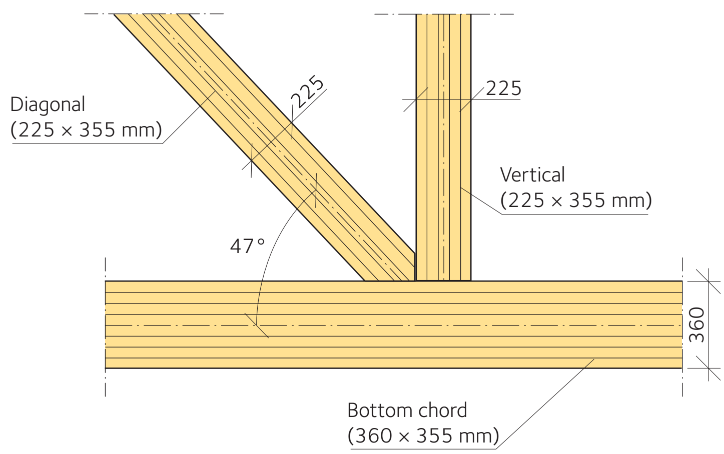
Verification for tension perpendicular to the grain:

$$\frac{F_{t,90,d}}{n_r \times F_{t,d}} = \frac{14.7}{2 \times 12.16} = 0.6 \quad < 1 \quad \textbf{OK}$$

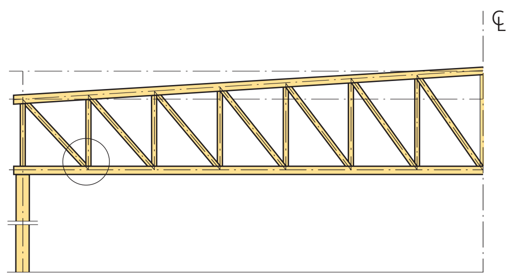
Example 21: Design of a truss node with multiple steel-to-timber dowelled connection

21.1 System, dimensions and design parameters

Design and verify the truss joint below using a dowelled connection. The joint is taken from *example 8, page 131*.



The truss is made of glulam, strength class	GL30c
Steel dowels, structural steel grade	S355
Safety class 3	$\gamma_d = 1$
Service class 1	
Material partial factor for glulam	$\gamma_M = 1.25$
Material partial factor for steel	$\gamma_{M2} = 1.2$
Connections partial factor	$\gamma_{MC} = 1.3$



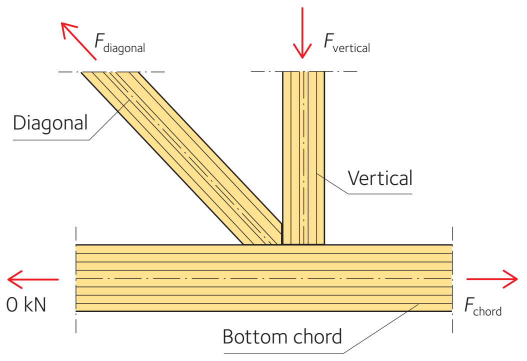
21.2 Internal forces

The joint forces are, *see example 8, page 131*:

$$F_{\text{vertical}} = 424 \text{ kN}$$

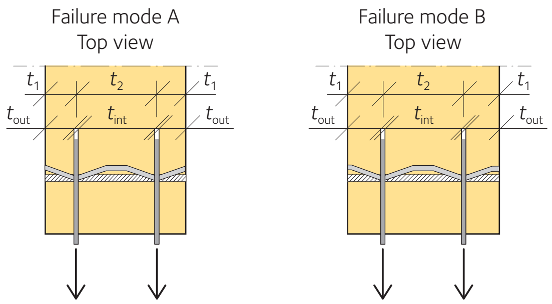
$$F_{\text{diagonal}} = 581 \text{ kN}$$

$$F_{\text{chord}} = 397 \text{ kN}$$



21.3 Design of the connection

The spacing between the slotted-in plates should be chosen so as to prevent brittle failure.
In other words, one of the failure modes shown in the figure below should decide the capacity.



Slotted-in steel plates and dowels, both with steel grade S355 are used.
Plates:

$$t = 8 \text{ mm}$$

$$f_{\text{yd}} = 355 \text{ MPa}$$

$$f_{\text{uk}} = 510 \text{ MPa}$$

Dowels:

$$d = 12 \text{ mm}$$

$$f_{\text{yd}} = 355 \text{ MPa}$$

$$f_{\text{uk}} = 510 \text{ MPa}$$

Embedment strength (*EN 1995-1-1, equation 8.32*):

$$f_{\text{h,0,k}} = 0.082 \times \left(1 - 0.01 \times d\right) \times \rho_{\text{k}} = 0.08 \times (1 - 0.01 \times 12) \times 390 = 28.14 \text{ MPa}$$

Yield moment of dowels (*EN 1995-1-1, equation 8.30*):

$$M_{\text{y,Rk}} = 0.3 \times f_{\text{uk}} \times \left(d\right)^{2.6} = 0.3 \times 510 \times 12^{2.6} = 97850.4 \text{ N} \times \text{mm}$$

Minimum thickness of the internal timber member, *see table 13.10, page 63*:

$$t_{int} = 1.15 \times 4 \times \sqrt{\frac{M_{y,Rk}}{f_{h,0,k} \times d}} = 1.15 \times 4 \times \sqrt{\frac{97850.41}{28.14 \times 12}} = 78.3 \text{ mm}$$

Minimum thickness of the outer timber member, *see table 13.10, page 63*:

$$t_{out,modeA} = \sqrt{2 \times \frac{M_{y,Rk}}{f_{h,0,k} \times d}} = \sqrt{2 \times \frac{97850.41}{28.14 \times 12}} = 24.07 \text{ mm}$$

$$t_{out,modeB} = 1.15 \times 4 \times \sqrt{\frac{M_{y,Rk}}{f_{h,0,k} \times d}} = 1.15 \times 4 \times \sqrt{\frac{97850.41}{28.14 \times 12}} = 78.3 \text{ mm}$$

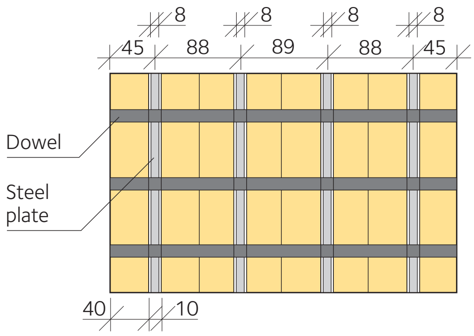
A number of 4 slotted-in plates is chosen, *see figure 13.3, page 65*. The geometry of the connection is shown below:

$$t_2 = 88 \text{ mm}$$

$$t_1 = 45 \text{ mm}$$

$$n_{max} = 1 + \frac{b - 2 \times t_1}{t_2} = 1 + \frac{355 - 2.45}{88} = 4.01 \quad \rightarrow \quad n = 4$$

$$2 \times t_1 + 3 \times t_2 = 354 \text{ mm} \quad < \quad 355 \text{ mm} \quad \textbf{OK}$$



Since 24 mm < t_1 = 45 mm < 78 mm failure mode A will occur, *see table 13.10, page 63*.

Load-carrying capacity for one dowel, *see table 13.10, page 63*:

$$R_{k,out} = f_{h,0,k} \times d \times t_1 \times \left(\sqrt{2 + \frac{4 \times M_{y,Rk}}{f_{h,0,k} \times d \times t_1^2}} - 1 \right) = 28.14 \times 12 \times 45 \times \left(\sqrt{2 + \frac{4 \times 97850.41}{28.14 \times 12 \times 45^2}} - 1 \right) = 9176.68 \text{ N}$$

$$R_{k,int} = 2 \times \left(1.15 \times 2 \times \sqrt{M_{y,Rk} \times f_{h,0,k} \times d} \right) = 2 \times 1.15 \times 2 \times \sqrt{97850.41 \times 28.14 \times 12} = 26442.98 \text{ N}$$

$$R_k = 2 \times R_{k,out} + (n - 1) \times R_{k,int} = 2 \times 9.2 + (4 - 1) \times 26.4 = 97.7 \text{ kN}$$

$$R_d = \frac{R_k \times k_{mod}}{\gamma_{MC}} = \frac{97.7 \times 0.8}{1.3} = 60.1 \text{ kN}$$

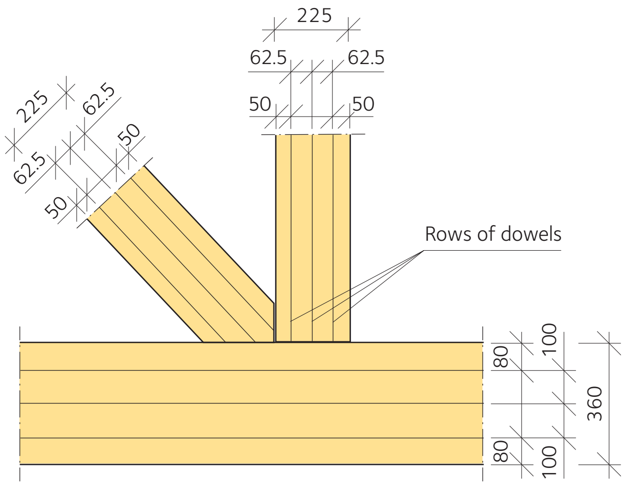
Rows of dowels in each member
Minimum distances:

$$a_{1,\min} = 5 \times d = 60 \text{ mm}$$

$$a_{2,\min} = 3 \times d = 36 \text{ mm}$$

$$a_{3t,\min} = 7 \times d = 84 \text{ mm}$$

$$a_{4c,\min} = 3 \times d = 36 \text{ mm}$$

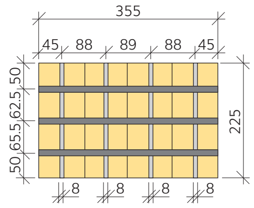


In each members three rows of dowels are placed:

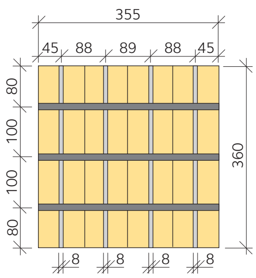
$$n_r = 3$$

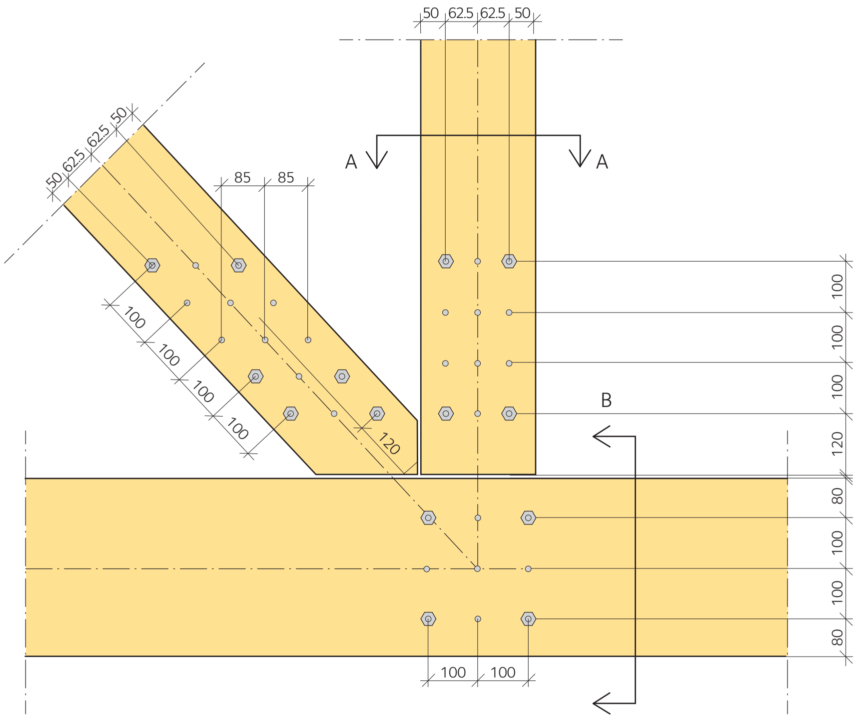
Final geometry of the node:

Vertical
Section A-A



Bottom chord
Section B-B





a) Verification of the doweled connection

Number of dowels in each row

Diagonal:

$$n_{\text{diagonal}} = 5 \qquad n_{\text{ef,diagonal}} = n_{\text{diagonal}}{}^{0.9} \times \sqrt[4]{\frac{a_1}{13 \times d}} = 5^{0.9} \times \sqrt[4]{\frac{100}{13 \times 12}} = 3.8$$

Vertical:

$$n_{\text{vertical}} = 4 \qquad n_{\text{ef,vertical}} = n_{\text{vertical}}{}^{0.9} \times \sqrt[4]{\frac{a_1}{13 \times d}} = 4^{0.9} \times \sqrt[4]{\frac{100}{13 \times 12}} = 3.1$$

Bottom chord:

$$n_{\text{chord}} = 3 \qquad n_{\text{ef,chord}} = n_{\text{chord}}{}^{0.9} \times \sqrt[4]{\frac{a_1}{13 \times d}} = 3^{0.9} \times \sqrt[4]{\frac{100}{13 \times 12}} = 2.4$$

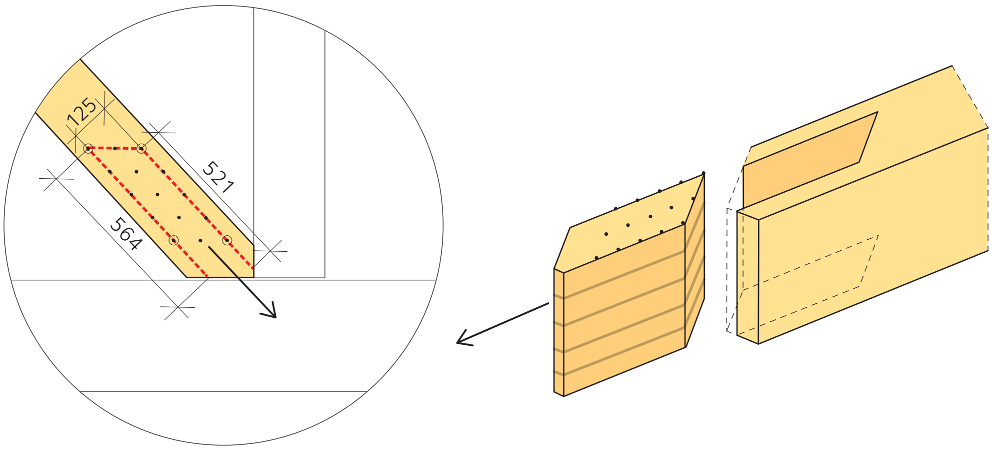
Verification of the doweled connection:

$$\frac{F_{\text{vertical}}}{n_r \times n_{\text{ef,vertical}} \times R_d} = \frac{424}{3 \times 3.12 \times 60.11} = 0.75 \qquad < 1 \quad \textbf{OK}$$

$$\frac{F_{\text{diagonal}}}{n_r \times n_{\text{ef,diagonal}} \times R_d} = \frac{581}{3 \times 3.81 \times 60.11} = 0.84 \qquad < 1 \quad \textbf{OK}$$

$$\frac{F_{\text{chord}}}{n_r \times n_{\text{ef,chord}} \times R_d} = \frac{397}{3 \times 2.41 \times 60.11} = 0.91 \qquad < 1 \quad \textbf{OK}$$

b) Block-shear verification



Load carrying capacity due to fracture along the perimeter of the fastener area:

$$F_{\text{bs,Rd}} = \max\left(1.5 \times A_{\text{net,t}} \times f_{t,0,d}\,,\ 0.7 \times A_{\text{net,v}} \times f_{v,d}\right) = \max\left(1.5 \times 31815 \times 12.48\,,\ 0.7 \times 303975 \times 2.24\right) = 595577 \text{ N}$$

where:

$$l_{\text{net,t}} = 125 - 2 \times \frac{d}{2} - d = 101 \text{ mm}$$

$$l_{\text{net,v}} = 564 - 5 \times d + 521 - 5 \times d = 965 \text{ mm}$$

$$\Sigma t = 355 - 4 \times (t + 2) = 355 - 4 \times (8 + 2) = 315 \text{ mm}$$

$$A_{\text{net,t}} = l_{\text{net,t}} \times \Sigma t = 101 \times 315 = 31815 \text{ mm}^2$$

$$A_{\text{net,v}} = l_{\text{net,v}} \times \Sigma t = 965 \times 315 = 303975 \text{ mm}^2$$

Block shear failure verification (*EN 1995-1-1, annex A*):

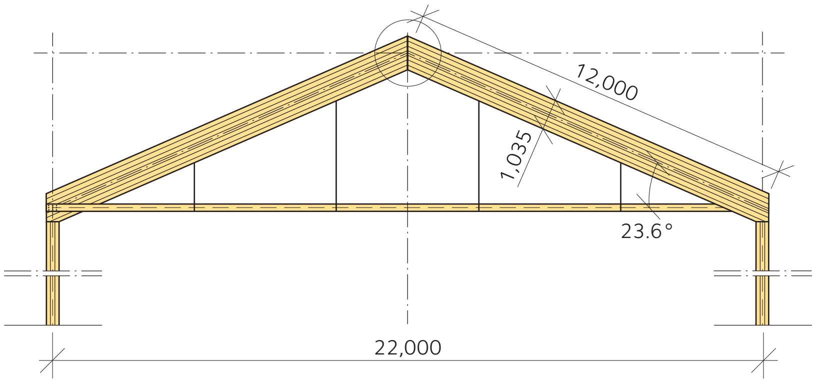
$$\frac{F_{\text{diagonal}}}{F_{\text{bs,Rd}}} = \frac{581}{595.58} = 0.97 \qquad < 1 \quad \textbf{OK}$$

The proposed connection has sufficient capacity. However, it is recommended to increase the number of dowels by 10 – 15 percent in each connector member in order to accounts for possible bending moments and/or eccentricities in the connection, *see The Glulam Handbook Volume 2, Chapter 8, page 119*.

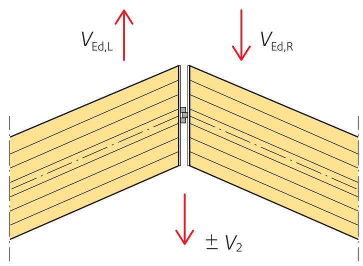
Example 22: Design of a pinned ridge joint

22.1 System, dimensions and design parameters

Design and verify the pinned ridge joint below.
The connection relates to the structure dimensioned in *example 5, page 107.*



The rafters are made of glulam, strength class	GL30c
Steel screws, strength class	10.9
Safety class 3	$\gamma_d = 1$
Service class 1	
Material partial factor for glulam	$\gamma_M = 1.25$
Material partial factors for steel	$\gamma_{M0} = 1.0$
	$\gamma_{M2} = 1.2$
Partial factor for connections	$\gamma_{MC} = 1.3$



22.2 Shear acting at the apex

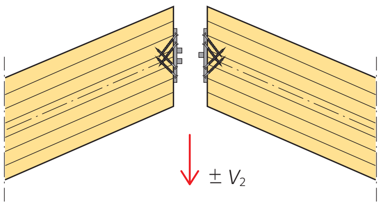
The shear force V_2 acting in the connection system is, *see example 5, page 107*:

Symmetric snow load:

$$V_2 = \frac{(q_1 - q_2) \times l_{tot}}{8} = \frac{(26.4 - 12.8) \times 22}{8} = 37.5 \text{ kN}$$

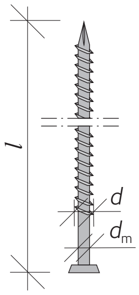
Symmetric load does not generate shear force at the apex.

22.3 Design of the connection

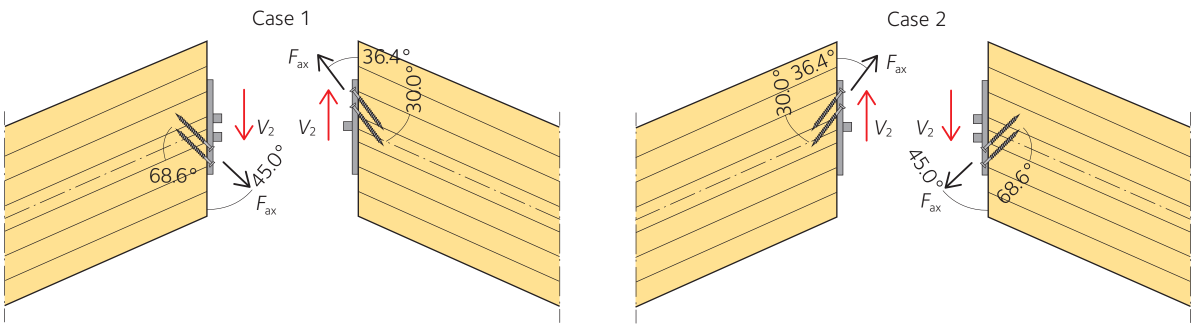


Fully threaded screws are used:

$l = 300$ mm
$d = 9$ mm
$d_m = 5.9$ mm
$f_u = 1000$ MPa



The shear force acting in a given part of the connection can be upwards or downwards, depending on which side of the roof is most loaded (i.e. left or right). Therefore, two different cases shall be considered.



The screws are only loaded axially:

$$F_{Ed,45°} = \frac{V_2}{\cos(45°)} = \frac{37.5}{\cos(45°)} = 53.0 \text{ kN}$$

$$F_{Ed,36.4°} = \frac{V_2}{\cos(36.4°)} = \frac{37.5}{\cos(36.4°)} = 46.5 \text{ kN}$$

Withdrawal capacity of a screw with an angle of 68.6° between screw axis and grain direction (*EN 1995-1-1, equation 8.38*):

$$l_{ad} = 270 \text{ mm}$$

$$f_{ax,k,s} = 0.52 \times d^{-0.5} l_{ad}{}^{-0.1} \times \rho_k{}^{0.8} = 0.52 \times 9^{-0.5} \times 270^{-0.1} \times 390^{0.8} = 11.71 \text{ MPa}$$

$$k_d = \min\left(1, \frac{d}{8}\right) = \min\left(1.0, \frac{9.0}{8.0}\right) = 1.0$$

$$F_{ax,k,rk} = \frac{f_{ax,k,s} \times d \times l_{ad} \times k_d}{1.2 \times \cos(\alpha)^2 + \sin(\alpha)^2} = \frac{11.7 \times 9 \times 270}{1.2 \times \cos\left(68.6°\right)^2 + \sin\left(68.6°\right)^2} = 27719.9 \text{ N}$$

Withdrawal capacity of a screw with an angle of 30° between screw axis and grain direction (*EN 1995-1-1, equation 8.38*):

$$l_{ad} = 265 \text{ mm}$$

$$f_{ax,k,s} = 0.52 \times d^{-0.5} l_{ad}{}^{-0.1} \times \rho_k{}^{0.8} = 0.52 \times 9^{-0.5} \times 265^{-0.1} \times 390^{0.8} = 11.73 \text{ MPa}$$

$$F_{ax,k,rk} = \frac{f_{ax,k,s} \times d \times l_{ad} \times k_d}{1.2 \times \cos(\alpha)^2 + \sin(\alpha)^2} = \frac{11.7 \times 9 \times 265}{1.2 \times \cos(30°)^2 + \sin(30°)^2} = 24333.2 \text{ N}$$

Tensile capacity of a screw, *see table 6.10, page 15*:

$$F_{t,Rk} = 0.9 f_u \times \pi \times \frac{d_m{}^2}{4} = 0.9 \times 1000 \times 3.14 \times \frac{5.9^2}{4} = 24593.3 \text{ N}$$

Design axial capacity of one screw with an angle of 68.6° between screw axis and grain direction:

$$F_{t,d,68.6} = \min\left(\frac{F_{ax,k,rk,68.6} \times k_{mod}}{\gamma_{MC}}, \frac{F_{t,s,k}}{\gamma_{M2}}\right) = \min\left(\frac{27.7 \times 0.8}{1.3}, \frac{24.6}{1.2}\right) = 17.1 \text{ kN}$$

Design axial capacity of one screw with an angle of 30° between screw axis and grain direction:

$$F_{t,d,30} = \min\left(\frac{F_{ax,k,rk,30} \times k_{mod}}{\gamma_{MC}}, \frac{F_{t,s,k}}{\gamma_{M2}}\right) = \min\left(\frac{24.3 \times 0.8}{1.3}, \frac{24.6}{1.2}\right) = 15 \text{ kN}$$

Spacing (*EN 1995-1-1, clause 8.7.2*):

$$a_{1,min} = 7 \times d = 7 \times 9 = 63 \text{ mm}$$

$$a_{2,min} = 5 \times d = 5 \times 9 = 45 \text{ mm}$$

$$a_{1CG,min} = 10 \times d = 90 \text{ mm}$$

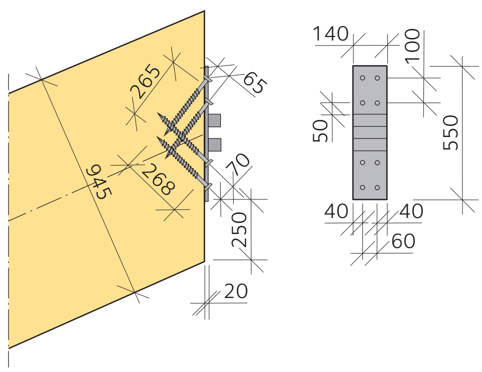
$$a_{2CG,min} = 4 \times d = 36 \text{ mm}$$

The number of vertical screw rows which can fit in the plate is $n_{rows} = 2$. Thus, the required number of horizontal rows is:

$$n_{row} = 2$$

$$n_{68.6} = 2 \qquad n_{ef,68.6} = n_{68.6}{}^{0.9} = 1.87$$

$$n_{30} = 2 \qquad n_{ef,30} = n_{30}{}^{0.9} = 1.87$$

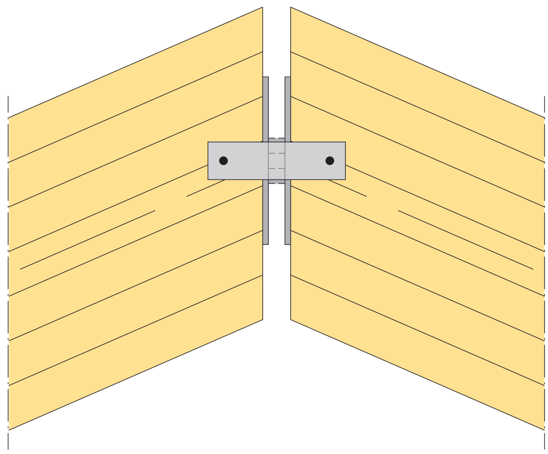


Shear verification:

$$\frac{F_{Ed,45°}}{n_{row} \times n_{ef,68.6} \times F_{t,d,68.6}} = \frac{53.0}{2 \times 1.87 \times 17.06} = 0.83 \quad \textbf{OK}$$

$$\frac{F_{Ed,36.4°}}{n_{row} \times n_{ef,30} \times F_{t,d,30}} = \frac{46.5}{2 \times 1.87 \times 14.97} = 0.83 \quad \textbf{OK}$$

Two steel plates on each side of the beam are used to secure the two halves of the truss and to take any tension which may result from wind forces.



Symbols

Symbol	Explanation
Latin upper case letters	
A	Cross-sectional area
A_{ef}	Effective area of the total contact surface between a punched metal plate fastener and the timber; effective total contact surface perpendicular to the grain
A_{f}	Cross-sectional area of flange
$A_{net,t}$	Net cross-sectional area perpendicular to the grain
$A_{net,v}$	Net shear area parallel to the grain
C	Spring stiffness
$E_{0,05}$	Fifth percentile value of modulus of elasticity
E_{d}	Design value of modulus of elasticity
E_{mean}	Mean value of modulus of elasticity
$E_{mean,fin}$	Final mean value of modulus of elasticity
F	Force
$F_{A,Ed}$	Design force acting on a punched metal plate fastener at the centroid of the effective area
$F_{A,min,d}$	Minimum design force acting on a punched metal plate fastener at the centroid of the effective area
$F_{ax,Ed}$	Design axial force on fastener
$F_{ax,Rd}$	Design value of axial withdrawal capacity of the fastener
$F_{ax,Rk}$	Characteristic axial withdrawal capacity of the fastener
F_{c}	Compressive force
F_{d}	Design force
$F_{d,ser}$	Design force at the serviceability limit state
$F_{f,Rd}$	Design load-carrying capacity per fastener in wall diaphragm
$F_{i,c,Ed}$	Design compressive reaction force at end of shear wall
$F_{i,t,Ed}$	Design tensile reaction force at end of shear wall
$F_{i,vert,Ed}$	Vertical load on wall
$F_{i,v,Rd}$	Design racking resistance of panel *i* or wall *i*
F_{la}	Lateral load
$F_{M,Ed}$	Design force from a design moment
F_{t}	Tensile force
$F_{t,Rk}$	Characteristic tensile capacity of a connection
$F_{v,0,Rk}$	Characteristic load-carrying capacity of a connector along the grain
$F_{v,Ed}$	Design shear force per shear plane of fastener; Horizontal design effect on wall diaphragm
$F_{v,Rd}$	Design load-carrying capacity per shear plane per fastener; Design racking load capacity
$F_{v,Rk}$	Characteristic load-carrying capacity per shear plane per fastener
$F_{v,w,Ed}$	Design shear force acting on web
$F_{x,Ed}$	Design value of a force in *x*-direction
$F_{y,Ed}$	Design value of a force in *y*-direction
$F_{x,Rd}$	Design value of plate capacity in *x*-direction
$F_{y,Rd}$	Design value of plate capacity in *y*-direction
$F_{x,Rk}$	Characteristic plate capacity in *x*-direction
$F_{y,Rk}$	Characteristic plate capacity in *y*-direction
$G_{0,05}$	Fifth percentile value of shear modulus
G_{d}	Design value of shear modulus
G_{mean}	Mean value of shear modulus
H	Overall rise of a truss
I_{f}	Second moment of area of flange
I_{tor}	Torsional moment of inertia
I_{z}	Second moment of area about the weak axis
K_{ser}	Slip modulus
$K_{ser,fin}$	Final slip modulus
K_{u}	Instantaneous slip modulus for ultimate limit states
$L_{net,t}$	Net width of the cross-section perpendicular to the grain
$L_{net,v}$	Net length of the fracture area in shear
$M_{A,Ed}$	Design moment acting on a punched metal plate fastener
$M_{ap,d}$	Design moment at apex zone
M_{d}	Design moment
$M_{y,Rk}$	Characteristic yield moment of fastener
N	Axial force
$R_{90,d}$	Design splitting capacity
$R_{90,k}$	Characteristic splitting capacity
$R_{ax,d}$	Design load-carrying capacity of an axially loaded connection
$R_{ax,k}$	Characteristic load-carrying capacity of an axially loaded connection
$R_{ax,\alpha,k}$	Characteristic load-carrying capacity at an angle α to grain
R_{d}	Design value of a load-carrying capacity
$R_{ef,k}$	Effective characteristic load-carrying capacity of a connection
$R_{iv,d}$	Design racking capacity of a wall
R_{k}	Characteristic load-carrying capacity
$R_{sp,k}$	Characteristic splitting capacity

$R_{to,k}$	Characteristic load-carrying capacity of a toothed plate connector
$R_{v,d}$	Design racking capacity of a wall diaphragm
V	Shear force; volume
V_u, V_l	Shear forces in upper and lower part of beam with a hole
W_y	Section modulus about axis y
X_d	Design value of a strength property
X_k	Characteristic value of a strength property
Latin lower case letters	
a	Distance
a_1	Spacing, parallel to grain, of fasteners within one row
$a_{1,CG}$	Minimum end distance to the centre of gravity of the screw in each timber member
a_2	Spacing, perpendicular to grain, between rows of fasteners
$a_{2,CG}$	Minimum edge distance to the centre of gravity of the screw in each timber member
$a_{3,c}$	Distance between fastener and unloaded end
$a_{3,t}$	Distance between fastener and loaded end
$a_{4,c}$	Distance between fastener and unloaded edge
$a_{4,t}$	Distance between fastener and loaded edge
a_{bow}	Maximum bow of truss member
$a_{bow,perm}$	Maximum permitted bow of truss member
a_{dev}	Maximum deviation of truss
$a_{dev,perm}$	Maximum permitted deviation of truss
b	Width
b_i	Width of panel i or wall i
b_{net}	Clear distance between studs
b_w	Web width
d	Diameter; outer thread diameter
d_1	Diameter of centre hole of connector; inner thread diameter
d_c	Connector diameter
d_{ef}	Effective diameter
d_h	Head diameter of connector
$f_{h,i,k}$	Characteristic embedment strength of timber member i
$f_{a,0,0}$	Characteristic anchorage capacity per unit area for $\alpha = 0°$ and $\beta = 0°$
$f_{a,90,90}$	Characteristic anchorage capacity per unit area for $\alpha = 90°$ and $\beta = 90°$
$f_{a,\alpha,\beta,k}$	Characteristic anchorage strength
$f_{ax,k}$	Characteristic withdrawal parameter for nails
$f_{c,0,d}$	Design compressive strength along the grain
$f_{c,w,d}$	Design compressive strength of web
$f_{f,c,d}$	Design compressive strength of flange
$f_{c,90,k}$	Characteristic compressive strength perpendicular to grain

$f_{f,t,d}$	Design tensile strength of flange
$f_{h,k}$	Characteristic embedment strength
$f_{head,k}$	Characteristic pull-through parameter for nails
f_1	Fundamental frequency
$f_{m,k}$	Characteristic bending strength
$f_{m,y,d}$	Design bending strength about the principal y-axis
$f_{m,z,d}$	Design bending strength about the principal z-axis
$f_{m,\alpha,d}$	Design bending strength at an angle α to the grain
$f_{t,0,d}$	Design tensile strength along the grain
$f_{t,0,k}$	Characteristic tensile strength along the grain
$f_{t,90,d}$	Design tensile strength perpendicular to the grain
$f_{t,w,d}$	Design tensile strength of the web
$f_{u,k}$	Characteristic tensile strength of bolts
$f_{v,0,d}$	Design panel shear strength
$f_{v,ax,\alpha,k}$	Characteristic withdrawal strength at an angle α to grain
$f_{v,ax,90,k}$	Characteristic withdrawal strength perpendicular to grain
$f_{v,d}$	Design shear strength
h	Depth; height of wall
h_{ap}	Depth of the apex zone
h_d	Hole depth
h_e	Embedment depth; loaded edge distance
h_{ef}	Effective depth
$h_{f,c}$	Depth of compression flange
$h_{f,t}$	Depth of tension flange
$h_{r\ell}$	Distance from lower edge of hole to bottom of member
h_{ru}	Distance from upper edge of hole to top of member
h_w	Web depth
i	Notch inclination
$k_{c,y}$, $k_{c,z}$	Instability factor
k_{cr}	Crack factor for shear resistance
k_{crit}	Factor used for lateral buckling
k_d	Dimension factor for panel
k_{def}	Deformation factor
k_{dis}	Factor taking into account the distribution of stresses in an apex zone
$k_{f,1}$, $k_{f,2}$, $k_{f,3}$	Modification factors for bracing resistance
k_h	Depth factor
$k_{i,q}$	Uniformly distributed load factor
k_m	Factor considering re-distribution of bending stresses in a cross-section
k_{mod}	Modification factor for duration of load and moisture content
k_n	Sheathing material factor

k_r	Reduction factor
$k_{R,red}$	Reduction factor for load-carrying capacity
k_s	Fastener spacing factor; modification factor for spring stiffness
$k_{s,red}$	Reduction factor for spacing
k_{shape}	Factor depending on the shape of the cross-section
k_{sys}	System strength factor
k_v	Reduction factor for notched beams
k_{vol}	Volume factor
k_y or k_z	Instability factor
$l_{a,min}$	Minimum anchorage length for a glued-in rod
l	Span; contact length
l_A	Distance from a hole to the centre of the member support
l_{ef}	Effective length; Effective length of distribution
l_V	Distance from a hole to the end of the member
l_Z	Spacing between holes
m	Mass per unit area
n_{40}	Number of frequencies below 40 Hz
n_{ef}	Effective number of fasteners
p_d	Distributed load
q_i	Equivalent uniformly distributed load
r	Radius of curvature
s	Spacing
s_0	Basic fastener spacing
r_{in}	Inner radius
t	Thickness
t_{pen}	Penetration depth
u_{creep}	Creep deformation
u_{fin}	Final deformation
$u_{fin,G}$	Final deformation for a permanent action G
$u_{fin,Q,1}$	Final deformation for the leading variable action Q_1
$u_{fin,Q,i}$	Final deformation for accompanying variable actions Q_i
u_{inst}	Instantaneous deformation
$u_{inst,G}$	Instantaneous deformation for a permanent action G
$u_{inst,Q,1}$	Instantaneous deformation for the leading variable action Q_1
$u_{inst,Q,i}$	Instantaneous deformation for accompanying variable actions Q_i
w_c	Precamber
w_{creep}	Creep deflection
w_{fin}	Final deflection
w_{inst}	Instantaneous deflection
$w_{net,fin}$	Net final deflection
v	Unit impulse velocity response

Greek lower case letters	
α	Angle between the x-direction and the force for a punched metal plate; Angle between the direction of the load and the loaded edge (or end)
β	Angle between the grain direction and the force for a punched metal plate
β_c	Straightness factor
γ	Angle between the x-direction and the timber connection line for a punched metal plate
γ_M	Partial factor for material properties, also accounting for model uncertainties and dimensional variations
λ_y	Slenderness ratio corresponding to bending about the y-axis
λ_z	Slenderness ratio corresponding to bending about the z-axis
$\lambda_{rel,y}$	Relative slenderness ratio corresponding to bending about the y-axis
$\lambda_{rel,z}$	Relative slenderness ratio corresponding to bending about the z-axis
ρ_k	Characteristic density
ρ_m	Mean density
$\sigma_{c,0,d}$	Design compressive stress along the grain
$\sigma_{c,\alpha,d}$	Design compressive stress at an angle α to the grain
$\sigma_{f,c,d}$	Mean design compressive stress of flange
$\sigma_{f,c,max,d}$	Design compressive stress of extreme fibres of flange
$\sigma_{f,t,d}$	Mean design tensile stress of flange
$\sigma_{f,t,max,d}$	Design tensile stress of extreme fibres of flange
$\sigma_{m,crit}$	Critical bending stress
$\sigma_{m,y,d}$	Design bending stress about the principal y-axis
$\sigma_{m,z,d}$	Design bending stress about the principal z-axis
$\sigma_{m,\alpha,d}$	Design bending stress at an angle α to the grain
σ_N	Axial stress
$\sigma_{t,0,d}$	Design tensile stress along the grain
$\sigma_{t,90,d}$	Design tensile stress perpendicular to the grain
$\sigma_{w,c,d}$	Design compressive stress of web
$\sigma_{w,t,d}$	Design tensile stress of web
τ_d	Design shear stress
$\tau_{F,d}$	Design anchorage stress from axial force
$\tau_{M,d}$	Design anchorage stress from moment
$\tau_{tor,d}$	Design shear stress from torsion
ψ_0	Factor for combination value of a variable action
ψ_1	Factor for frequent value of a variable action
ψ_2	Factor for quasi-permanent value of a variable action
ζ	Modal damping ratio

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Swedish glulam industry

The raw materials come from Swedish forests and the finished products meet the European standard for CE-marked glulam. All the glulam manufacturers have an environmental declaration and are certified by accredited certification bodies.

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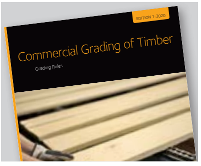

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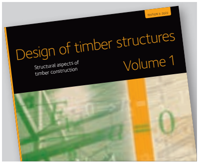

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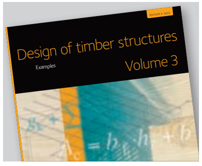

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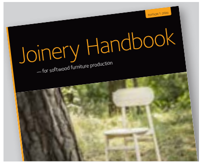

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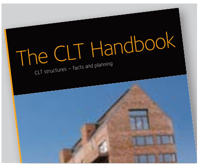

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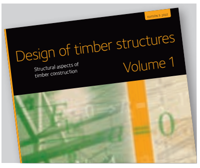

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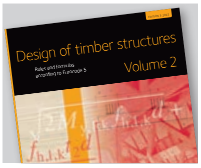

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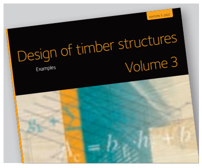

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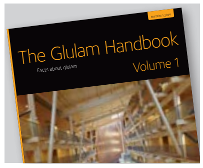

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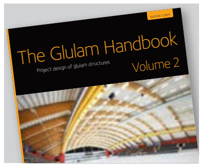

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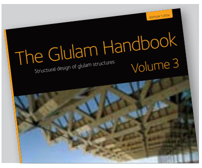

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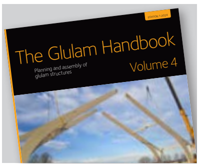

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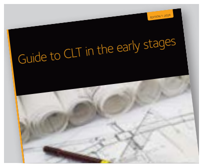

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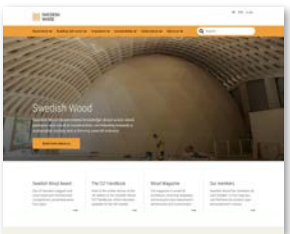

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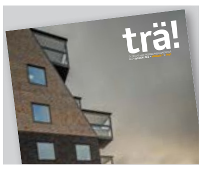

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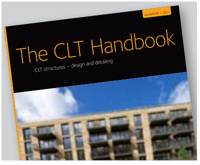

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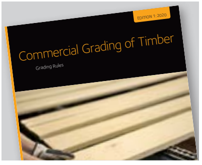

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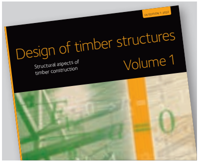

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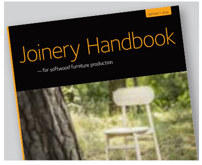

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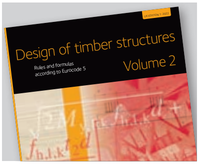

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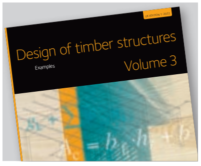

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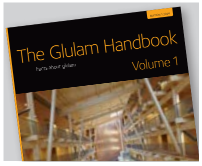

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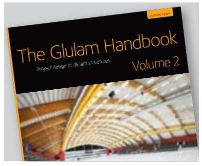

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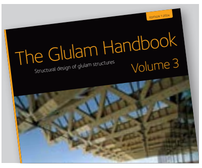

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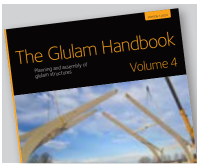

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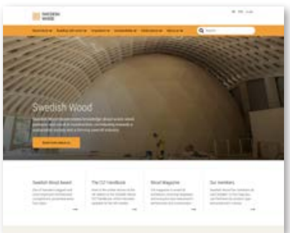

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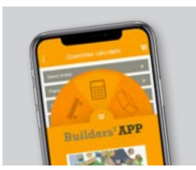

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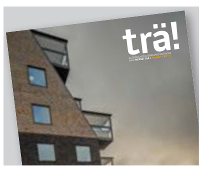

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